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Variation Of Rotation In Chaos Game By Modifying The Rules

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ABSTRACT

The core concept of fractals is the process of rearranging identical components that have a large amount of self-similarity. One example of fractals is the Sierpinski trianglecan be generated using the chaos game method. This method is a form of play in drawing points on triangles that have certain rules and are repeated iteratively. This research will modify the rules of chaos game triangle with the addition of various rotationswith the center of rotation at one, two, three, four, and five reference points. The visual results obtained are in the form of fractals because they have self-similarity properties and a collection of new points formed experiences rotation with the center of rotation based on the selected reference point with the direction of rotation based on the rules. The visual results of the rotation θ angle are visually symmetrical about the axis-y with the visual results of the rotation 360^0-θ angle at one, three, four, and five reference points as the center of rotation. At two reference points as the center of rotation it is obtained that there are two parts that are visually symmetrical about a certain line. Visual results of rotation 360^0 angles at one, two, three reference points as the center of rotation have a shape similar to the Sierpinski triangle. Whereas at four and five points of reference as the center of rotation has a shape similar to the Sierpinski triangle.

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1. INTRODUCTION

Fractal is one of the studies in mathematics that studies geometrical forms. The essence of the fractal concept is the process of rearranging identical components, which have a large amount of self-similarity [5]. One example of fractals is the Sierpinski triangleis a linear fractal by having an identical self-similarity from one iteration to the next iteration [1]. The Sierpinski Triangle can be generated using the chaos game method. This method is a form of play in drawing points on triangles that have certain rules and are repeated iteratively.

Researchers [2] have discussedmodification of chaos rules game generation of Sierpinski triangles by adding rotational geometry transformations. Researchers [3] have modified the rules of chaos game triangle with the addition of onearbitrary point inside the triangle. Researcher[4] in his research discussed the formation of quadrilateral chaos games which are rotated at an angle using an affine transformation with a compression ratio of two. Researcher[6] has reviewed the rules of non-random chaos games on triangles. The results obtained are if the reference point is chosen non-randomly, then the new points produced do not produce a fractal shape. A collection of dots formed by rules*non-random*it will converge at certain coordinate points. The location of the starting point inside or outside the triangle will produce a new set of points inside the triangle in the next iteration [7]. Meanwhile, researchers [8] have modified the game chaos using four reference points in the form of convex polygons. Visual results obtained are that there are four collections of new points around the reference point. The four collections of new points have a shape similar to the shape of the outer part of the convex polygon.

Based on the results of these studies, this study will discuss the variation of rotation in the triangle chaos game by modifying the rules. Modification of the triangle chaos game rules in this study is in the form of additional rotations with the center of rotation at one, two, three, four, and five reference points. Visual results from modifying the chaos game rules can be known with the help of the program. The program will be created using MATLAB software. The aim of this research is θ to know the visual outcome of varying rotation in a chaotic triangle game by modifying the rules in the form of additional rotations with the center of rotation on one, two, three, four, and five points.

2. RESEARCH METHODE

The steps in generating chaotic triangle games by modifying the rules in the form of additional rotations in this study are as follows.

a. Determine three reference points namely point $P_1(x_1, y_1)$ $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$ so it forms the triangle shown in Figure 1.

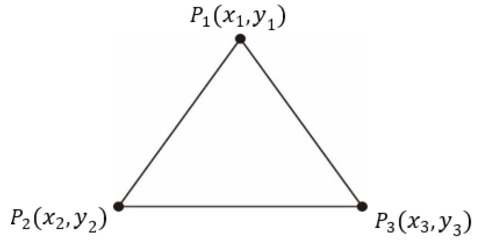


Figure 1. Three points of reference

b. Determine the starting point $Q_0(x_4, y_4)$ which is inside the triangle and shown in Figure 2.

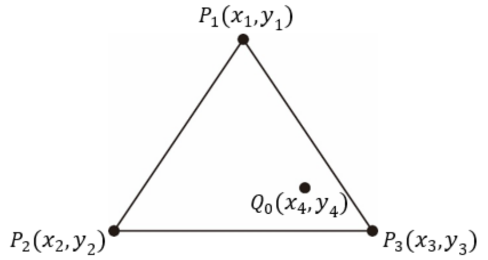


Figure 2 . Determination of starting point Q_0

c. Choose one random reference point to connect with the starting point $Q_0(x_4, y_4)$. Suppose the selected reference point is the point $P_2(x_2, y_2)$ shown in Figure 3.

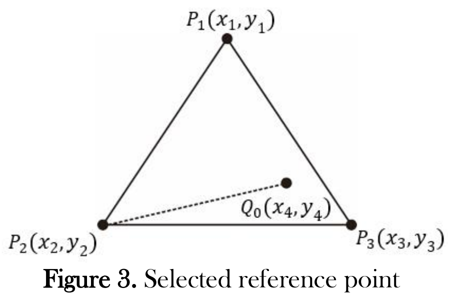


Figure 3. Selected reference point

d. Determine a new point $Q_1(x_5, y_5)$ which is the midpoint of the segment $\overline{Q_0P_2}$. The formula for determining the value X_5 and y_5 is as follows.

$$x_5 = \frac{x_4 + x_2}{2} \tag{3.1}$$

$$y_5 = \frac{y_4 + y_2}{2} \tag{3.2}$$

The new point $Q_1(x_5, y_5)$ is shown in Figure 4.

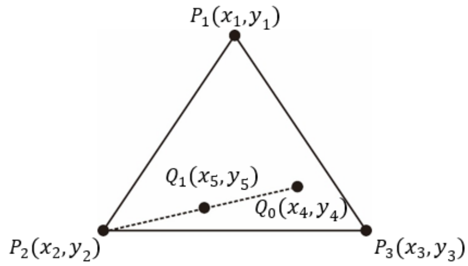


Figure 4. New point Q_1

e. The new point $Q_1(x_5,y_5)$ is rotated θ for example in a counterclockwise rotation with the center of rotation that is the selected reference point and is shown in Figure 5. The counterclockwise θ rotation formula is that is $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ and the formula is clockwise θ rotation that is $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos(\theta) & sin(\theta) \\ -sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$ with the description (x,y) is the point being rotated (x', y'), is the point that has been rotated, θ is the angle of rotation, and (x_0, y_0) is the center of rotation [9].

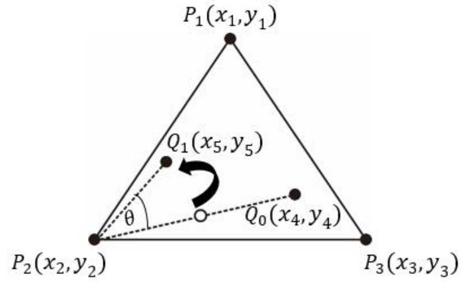


Figure 5. New point rotation Q_1

f. The point obtained in step-e is used as the starting point.

g. Repeat step-c to step-f so that it produces new points with the desired iteration.

While the modification of the chaos game triangle rules with the addition of rotation is divided into five types based on the number of reference points that will become the center of rotation, as follows.

a. One Reference Point as a Center Point of Rotation

Suppose a reference point that will be the center of rotation is the point $P_1(x_1,y_1)$. The rule used at a reference point as the center of rotation is if the selected reference point is a point $P_1(x_1,y_1)$, then the new point is rotated counterclockwise equal θ to the center of rotation located at the point $P_1(x_1,y_1)$. If the selected reference point is a point $P_2(x_2,y_2)$ or point $P_3(x_3,y_3)$, then the new point is not rotated [2].

b. Two Reference Points as Center Points of Rotation

For example, the two reference points that will become the center of rotation are the reference points $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$. The rule used at the two reference points as the center of rotation is if the selected reference point is a point $P_1(x_1,y_1)$, then the new point is not rotated. If the selected reference point is a point, then the new point is rotated clockwise at the point of rotation of the selected reference point. Meanwhile $P_2(x_2,y_2)$, if the selected reference point is a point, then the new point is $P_3(x_3,y_3)$, rotated counterclockwise equal to the center of rotation at the selected reference point [2].

c. Three Reference Points as Center Points of Rotation

For example, three reference points will be the center of rotation, the reference point $P_1(x_1,y_1)$, $P_2(x_2,y_2)$, and $P_3(x_3,y_3)$. The rule used at the three reference points as the center of rotation if the selected reference point is a point $P_1(x_1,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$, then the new point will be counterclockwise rotation of clockwise θ with the center of rotation based on the selected reference point.

d. Four Reference Points as Rotation Center Points

For example, the four reference points that will become the center of rotation are the reference points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3)$ and one additional reference point $P_4(x_4, y_4)$ which is inside the triangle. The rule used at the four reference points as the center of rotation is if the selected reference point is a point $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3)$, then the new point will be rotated $P_4(x_4, y_4)$ anticlockwise of θ with the center of rotation at the selected reference point.

e. Five Point of Reference as the Center Point of Rotation

For example, the five reference points that will become the center of rotation are the three reference points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3)$ and two additional reference points, which are points $P_4(x_4, y_4)$ and $P_5(x_5, y_5)$ which is inside the triangle $P_5(x_5, y_5)$. The rule used at the five reference points as rotation angles is if the selected reference point is a $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3)$, $P_4(x_4, y_4)$, or $P_5(x_5, y_5)$ then the new point will be rotated by θ counterclockwise with the center of rotation at the selected reference point.

3. RESULT AND ANALYSIS

a. One Reference Point as a Center Point of Rotation

The reference point coordinates used are located at the point $P_1(11{,}21)$, $P_2(1{,}1)$, and $P_3(21{,}1)$.. While the coordinates of the starting point are located at the point $Q_0(11{,}11)$. One visual result of the modification of the rules of the chaos game triangle with one reference point as the center of rotation at a point $P_1(11{,}21)$ that is rotated $\theta = 10°$ with 15.000 iterations is shown in Figure 6.

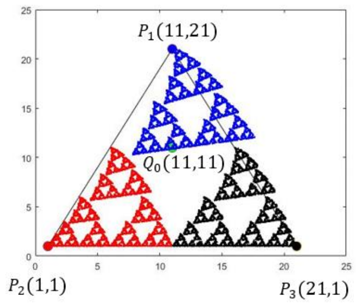


Figure 6. Visual results at one reference point as the center of rotation with a rotation angle of $\theta = 10°$

In Figure 6, if the reference point chosen is a new point $P_1(11{,}21)$ formed blue. If the reference point is chosen point $P_2(1{,}1)$, then the new point formed red. Meanwhile, if the reference point is chosen point $P_3(21{,}1)$, then the new point formed is black. The new points will collect based on the selected reference point. Because there are three reference points, there are three new points.$P_1(11{,}21)P_2(1{,}1)P_3(21{,}1)$

The visual result at one reference point as the center of rotation is a fractal form because it has the self-similarity characteristic shown in Figure 7. The shape of the part,, and resembles the shape of the part A_1, A_2, and A_3 so on.

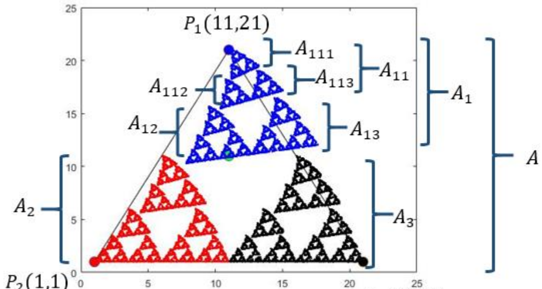


Figure 7. *Self-similarity* at one reference point as the center of rotation

At one reference point as the center of rotation, the visual results of the rotation θ angle are visually symmetrical about the axis-y with the visual results of the rotation angle of $360° - \theta$. At an angle of rotation equal θ to using the splitting point with points $T_1(x_1, y_1)$, points $T_2(x_2, y_2)$, and points $T_3(x_3, y_3)$. Whereas, the rotation angle is equal to using the splitting point with points $T_1'(x_1', y_1')$, points $T_2'(x_2', y_2')$, and points $T_3'(x_3', y_3')$ Based on this, the visual results of the rotation $\theta = 10°$ angle are symmetrical visually to the axis-y with the visual results of the rotation $\theta = 350°$ angle as shown in Figure 8.

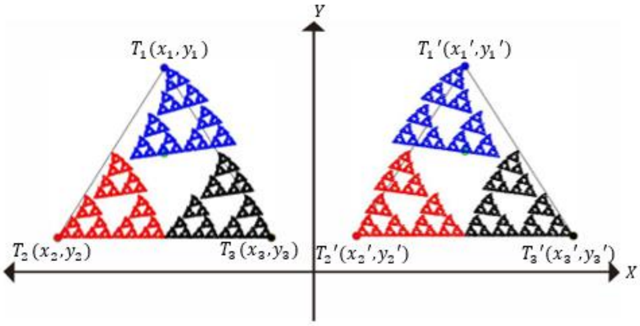
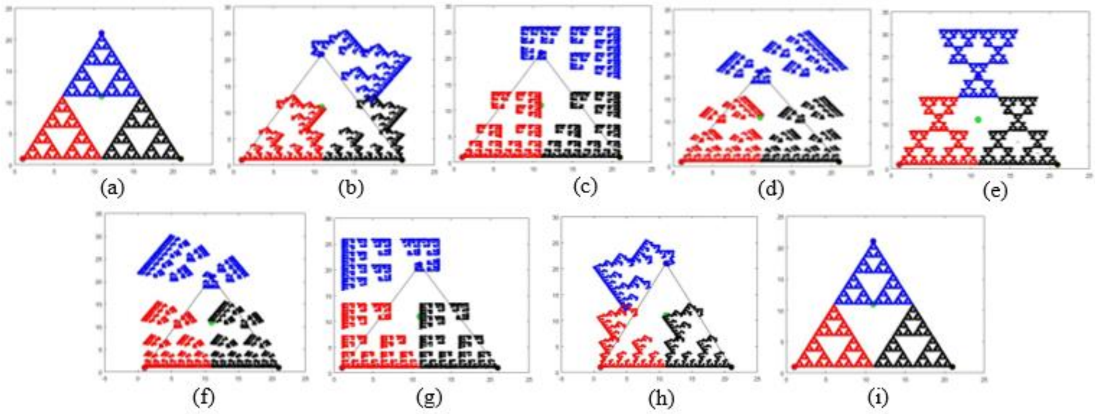


Figure 8. Visual results $\theta = 10°$ are symmetrical about the axis - with visual $\theta = 350°$ results at one reference point as the center of rotation

Meanwhile, in Figure 9 it can be seen that the new collection of blue dots that have formed rotates counter-clockwise with the center of rotation in that point $P_1(11,21)$. Whereas the new collection of red and black dots that are formed does not occur or remain constant rotation. The visual results of the rotation $\theta = 0°$ angle are the same as the visual results of the rotation $\theta = 360°$ angle and this also applies to the two, three, four and five reference points as the center of rotation. Based on the visual results of the rotation $\theta = 360°$ angle in Figure 9 (i) shows that there are three sets of new points that accumulate based on their reference points. The third set of new points has a shape resembling the shape of the outer portion. The visual results $\theta = 360°$ of this angle are similar to the Sierpinski triangle.



(a) $\theta = 0°$*; (b)* $\theta = 60°$*; (c)* $\theta = 90°$*; (d)* $\theta = 120°$*; (e)* $\theta = 180°$*; (f)* $\theta = 240°$*; (g)* $\theta = 270°$*; (h)* $\theta = 300°$*; (i)* $\theta = 360°$

Figure 9. Visual results at one reference point as the center of rotation

b. Two Reference Points as Rotas Center Points

Visual results at two reference points as the center of rotation with a rotation $\theta = 10°$ angle of as shown in Figure 10.

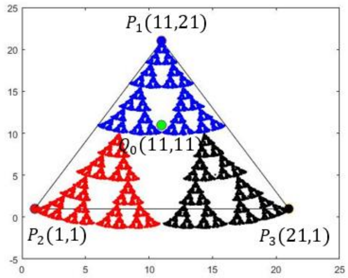


Figure 10. Visual results at two reference points as the center of rotation with a rotation angle of $\theta = 10°$

Based on Figure 10, if the selected reference point is at a point $P_1(11,21)$, then the new point will be blue. If the reference point is selected at the point $P_2(1,1)$, then the new point will be red. Meanwhile, if the reference point is selected at the point $P_3(21,1)$, then the new point will be black. Because there are three reference points, there are three new points. The visual result of this rule is a fractal object because it has the nature of self-similarity shown in Figure 11. The shape of a part A_1, A_2, A_3 resembles the shape of a part and so on.

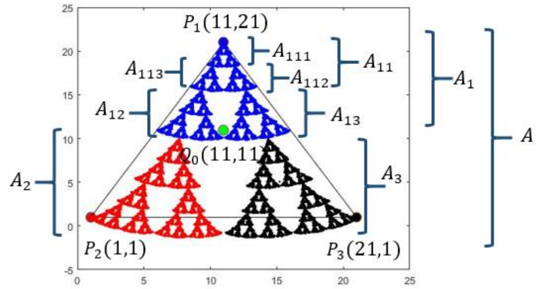


Figure 11. Self-similarity at two reference points as the center of rotation

Figure 12 shows the visual symmetry between the fractal parts of a particular line. In the picture it appears that the A_1 simteris part is visually related to a certain line with the part A_2.

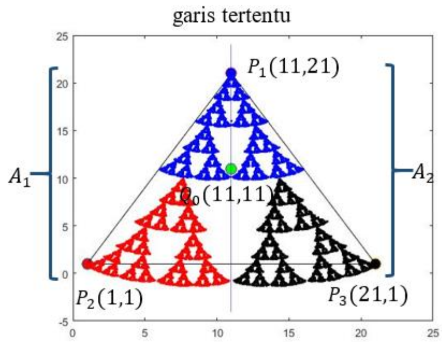
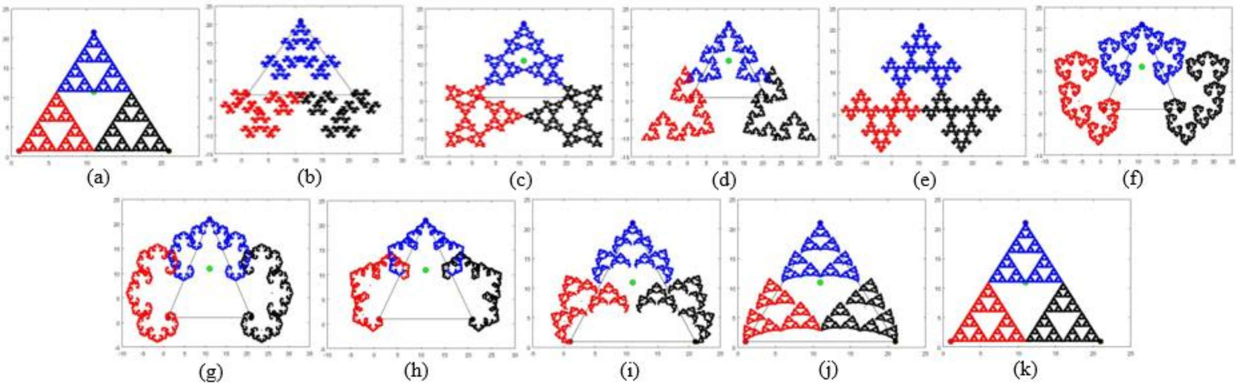


Figure 12. The A_1 symmetrical part is visually with the part B_2 against a certain line

Whereas in Figure 13, a collection of new red dots formed rotates clockwise with the center of rotation at that point $P_2(1,1)$. A collection of new black dots that are formed rotate counter-clockwise with the center of rotation at the point $P_3(21,1)$. Whereas the new collection of blue dots that are formed does not occur or remain constant rotation. Based on the visual $\theta = 360°$ results in Figure 13 (k) shows that there are three collections of new points that accumulate based on their reference points. The third set of new points has a shape resembling the shape of the outer portion. The visual results $\theta = 360°$ of this angle are similar to the Sierpinski triangle.



(a) $\theta = 0°$; *(b)* $\theta = 60°$; *(c)* $\theta = 90°$; *(d)* $\theta = 120°$; *(e)* $\theta = 180°$; *(f)* $\theta = 240°$; *(g)* $\theta = 270°$; *(h)* $\theta = 300°$; *(i)* $\theta = 330°$; *(j)* $\theta = 350°$; *(k)* $\theta = 360°$

Figure 13. Visual results at two reference points as the center of rotation

c. Three Reference Points as Center Points of Rotation

The visual results of a triangle chaos game with three reference points as the center of rotation are shown in Figure 14. The iteration used is 15.000 iterations.

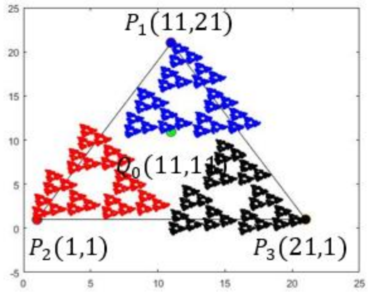


Figure 14. Visual results at the three reference points as the center of rotation with a rotation angle of $\theta = 10°$

In Figure 14, the blue, red, and black dots are new dots that collect based on the selected reference point. This visual result has the same explanation as one and two reference points as the center of rotation. Because there are three reference points, there are three new points. The visual results for the three reference points as the center of rotation are fractal because they have the self-similarity properties shown in Figure 15. The shape of the part A_1, A_2, A_3 resembles the shape of the part and so on.

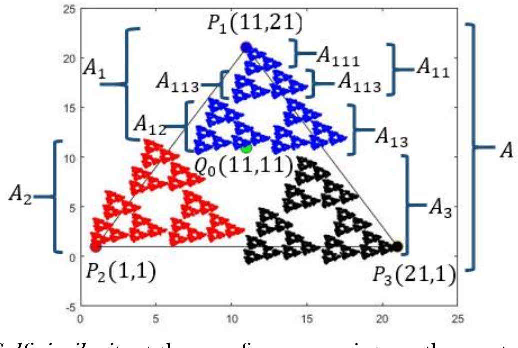


Figure 15. *Self-similarity* at three reference points as the center of rotation

At the three reference points as the center of rotation, the visual results of the rotation angle θ are visually symmetrical about the axis-y with the visual results of the rotation angle of $360° - \theta$. Based on this, the visual results of rotation $\theta = 10°$ angles are visually symmetrical with respect to the axes-y with visual results of rotation $\theta = 350°$ angles as shown in Figure 16. Examples of the points have the same explanation at one kind of rotation center point.

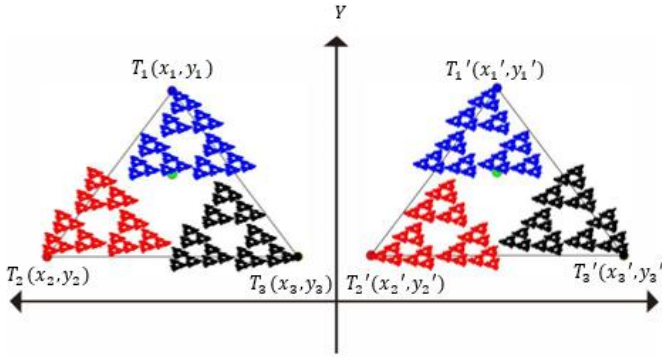
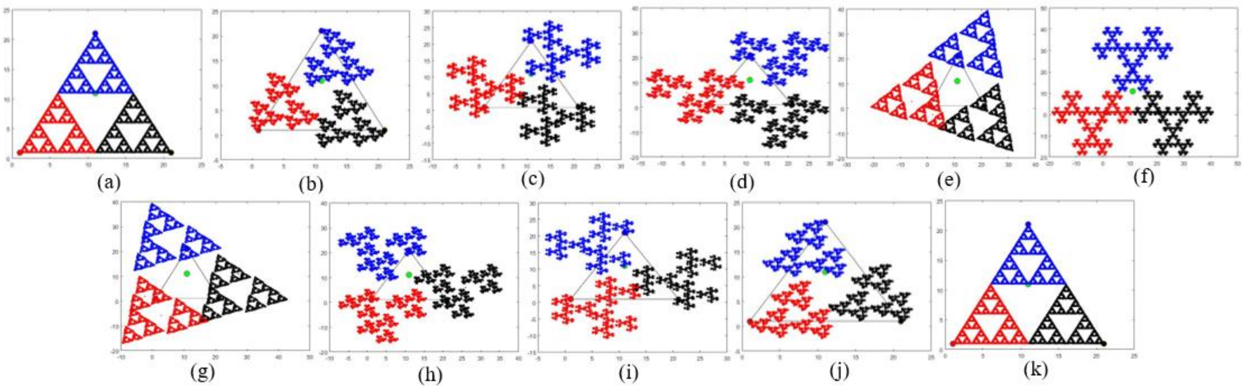


Figure 16. Visual results are $\theta = 10°$ symmetrical about the axis - with visual $\theta = 350°$ results at the three reference points as the center of rotation

Based on Figure 17, the new collection of blue dots formed rotates with the center of rotation at the point$P_1(11,21)$. A collection of new red dots rotates at a point $P_2(1,1)$ and a collection of new black dots rotates at a dot $P_3(21,1)$. All three rotate counter-clockwise. Based on visual $\theta = 360°$ results in Figure 17 (k) shows that there are three collections of new points that collect based on the reference point. The third set of new points has a shape resembling the shape of the outer portion. The visual results of $\theta = 360°$ this angle are similar to the Sierpinski triangle.



(a) $\theta = 0°$; *(b)* $\theta = 20°$; *(c)* $\theta = 60°$; *(d)* $\theta = 90°$; *(e)* $\theta = 120°$; *(f)* $\theta = 180°$; *(g)* $\theta = 240°$; *(h)* $\theta = 270°$; *(i)* $\theta = 300°$; *(j)* $\theta = 340°$; *(k)* $\theta = 360°$

Figure 17. Results at the three reference points as the center of rotation

d. Four Points of Reference as a Center of Rotation

The location of the three reference points used are points $P_1(11,21)$, points $P_2(1,1)$, points $P_3(21,1)$, and one additional reference point $P_4(11,8)$ which is the point inside the triangle. And the location of the coordinates of the starting point $Q_0(11,11)$. The iteration used 20.000 iterations. In Figure 18, if the selected reference points are points $P_1(11,21)$, dots $P_2(1,1)$, dots $P_3(21,1)$, or points $P_4(8,11)$then the new dots will turn blue, red, black, and magenta. The new points formed will gather at the selected point. Because there are four reference points, so there are four collections of new points.

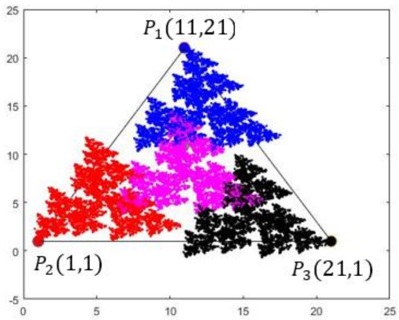


Figure 18. Visual results at four reference points as the center of rotation with a rotation angle of $\theta = 10°$

The visual results at these four types of reference points are fractal because they have the self-similarity properties shown in Figure 19. The shape of parts,,, resembles the shape of parts A_1, A_2, A_3, A_4 and so on.

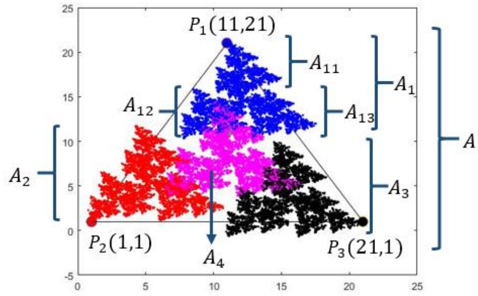


Figure 19. *Self-similarity* at four reference points as the center of rotation

At the four reference points as the center of rotation, the visual result of the rotation θ angle is visually symmetrical about the axis-y with the visual result of the rotation $360° - \theta$ angle of. Based on this, the visual results of rotation $\theta = 10°$ angles are visually symmetrical about the axes-y with the visual results of the rotation $\theta = 350°$ angles as shown in Figure 20. Examples of the points have the same explanation at one reference point as the center of rotation.

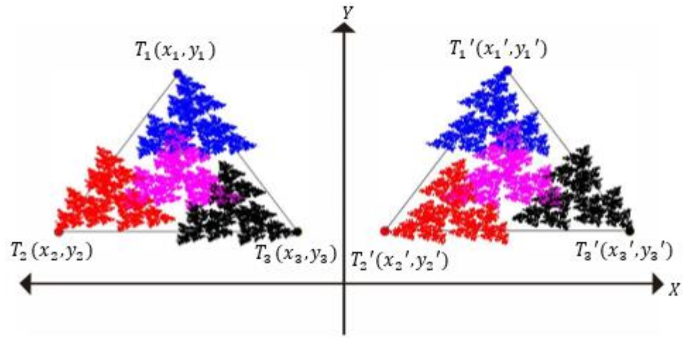
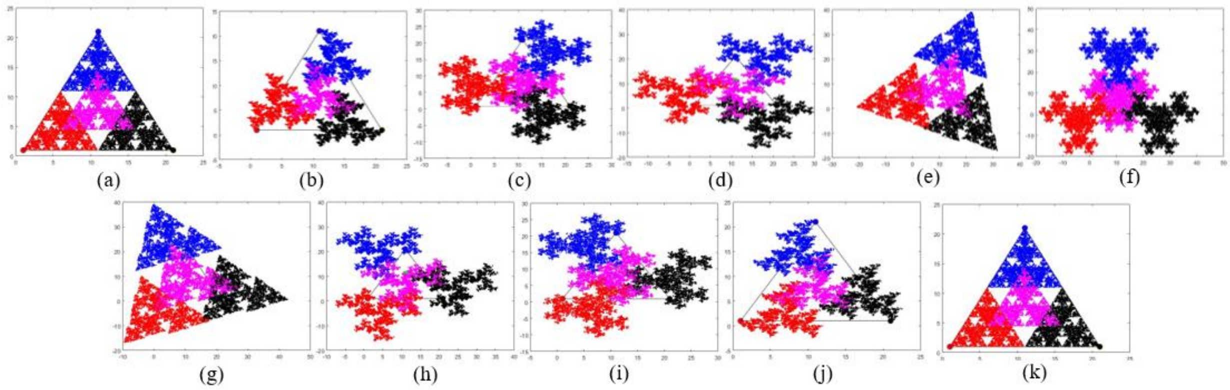


Figure 20. Visual results are $\theta = 10°$ symmetrical about the axis-y with visual $\theta = 350°$ results at four reference points as the center of rotation

The visual results at the four reference points as the center of rotation with variations in the rotation angle of 0°, 20°, 60°, 90°, 120°, 180°, 240°, 270°, 300°, 340°, and 360° are shown in Figure 21. A collection of new dots that are blue, red, black, and magenta that form rotates in opposite directions clockwise. Sequentially with the center of rotation at points $P_1(1,1)$, $P_2(11,21)$, $P_3(21,1)$, and $P_4(8,11)$. The visual results of the rotational $\theta = 360°$ angles in Figure 21 (k) state that there are four new sets of dots that gather based on their reference points. The four collections of new points have a shape resembling the shape of the outer portion. The visual results of $\theta = 360°$ this angle are almost similar to the Sierpinski triangle.



(a) $\theta = 0°$; *(b)* $\theta = 20°$; *(c)* $\theta = 60°$; *(d)* $\theta = 90°$; *(e)* $\theta = 120°$; *(f)* $\theta = 180°$; *(g)* $\theta = 240°$; *(h)* $\theta = 270°$; *(i)* $\theta = 300°$; *(j)* $\theta = 340°$; *(k)* $\theta = 360°$

Figure 21. Visual results at four reference points as the center of rotation

e. Five Point of Reference as the Center Point of Rotation

The location of the three reference points used are points$P_1(11{,}21)$, dots $P_2(1{,}1)$, and dots $P_3(21{,}1)$. Two additional reference points are located at points $P_4(6{,}6)$ and points $P_5(16{,}6)$. While the starting point is located at the point $Q_0(11{,}11)$. The visual results for the five reference points as the center of rotation $\theta = 10°$ with rotation angles are shown in Figure 22. The number of iterations used is 25.000 iterations. New dots are shown in blue, red, black, magenta, and cyan dots. If the selected points are points $P_1(11{,}21)$, points $P_2(1{,}1)$, points $P_3(21{,}1)$, points $P_4(6{,}6)$, or $P_5(16{,}6)$, then sequentially new points are formed in blue, red, black, magenta, and cyan colors. The new points collect based on the selected point. Because there are five points of reference, so there are five sets of new points.

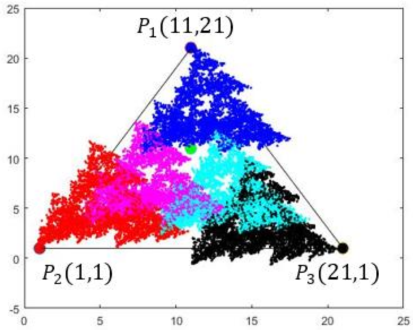


Figure 22. Visual results at five reference points as the center of rotation with a rotation angle of $\theta = 10°$

Visual results at five types of reference points are fractal because they have self-similarity. The shape of a part A_1, part A_2, part A_3, part A_4, and part A_5 resembles the shape of a part A. Look at Figure 23.

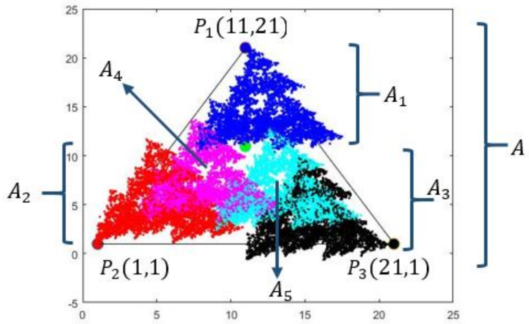


Figure 23.*Self-similarity* at five kinds of rotation center points

At five kinds of rotation center points, the visual results of rotation θ angles are visually symmetrical about the axes-y with visual results of rotation angles of $360° - \theta$. Based on this, the visual results of rotation angles are visually $\theta = 10°$ symmetrical about the axes-y with the visual results of rotation angles $\theta = 350°$ as shown in Figure 24. Examples of the points have the same explanation at one kind of rotation center point.

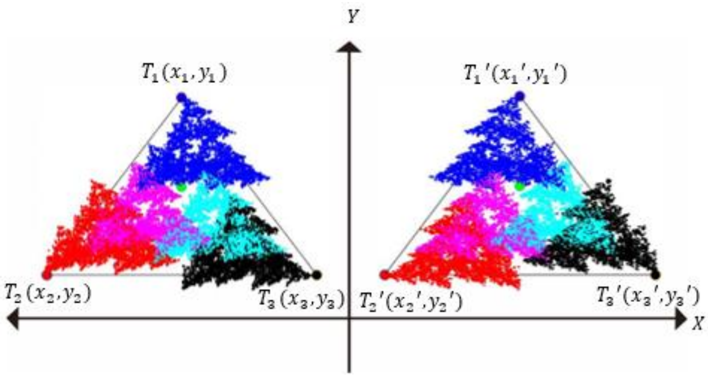
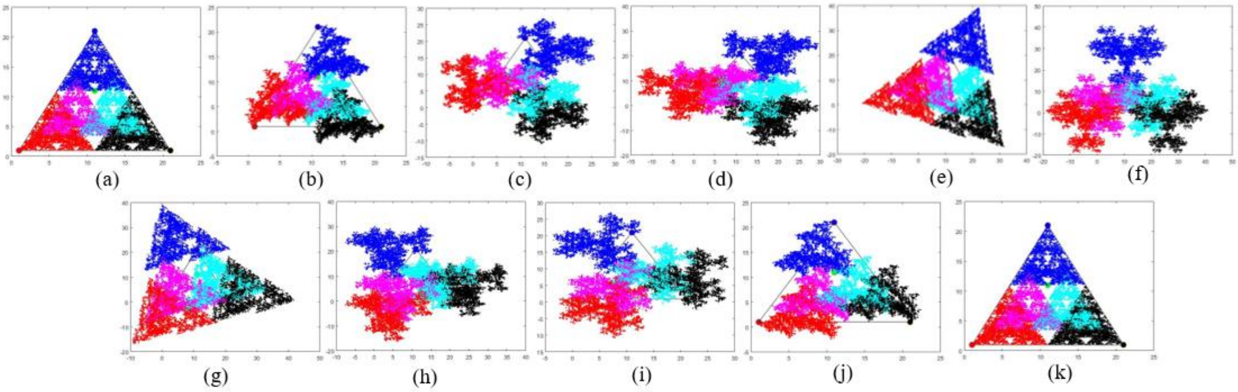


Figure 24. Visual results are $\theta = 10°$ symmetrical about the axis-y with visual results $\theta = 350°$ at five reference points as the center of rotation.

Based on Figure 25, a collection of new dots in blue, red, black, magenta, and cyan spin counterclockwise. Sequentially based on these colors the rotation is centered at the dot $P_1(11,21)$, $P_2(1,1)$, $P_3(21,11)$, $P_4(6,6)$, and $P_5(16,6)$. The visual results of the rotational $\theta = 360°$ angles in Figure 25 (k) show that there are five new collections of dots that gather according to their reference points. The five sets of new points have a shape resembling the shape of the outer portion. The visual results of $\theta = 360°$ this angle are almost similar to the Sierpinski triangle.



(a) $\theta = 0°$*; (b)* $\theta = 20°$*; (c)* $\theta = 60°$*; (d)* $\theta = 90°$*; (e)* $\theta = 120°$*; (f)* $\theta = 180°$*; (g)* $\theta = 240°$*; (h)* $\theta = 270°$*; (i)* $\theta = 300°$*; (j)* $\theta = 340°$*; (i)* $\theta = 360°$

Figure 25. Visual results at five reference points as the center of rotation

4. CONCLUSION

The conclusions obtained from are as follows.

a. Visual results that are formed from varying the angle of rotation at one, two, three, four, and five reference points as the center of rotation are fractal forms because they have self-similarity properties and the collection of new points formed has rotated with the center of rotation based on the point the reference chosen with the direction of rotation based on the rules.

b. The visual results of the rotation angle are visually symmetrical about the axis - with the visual results of the rotation angle at one, three, four, and five reference points as the center of rotation.θy360°-θ

c. At two reference points as the center of rotation it is obtained that there are two parts that are visually symmetrical about a certain line.

d. The visual results of rotation angles at one, two, and three reference points as the center of rotation indicate that there are three new collections of points that collect based on their reference points in the shape of the outermost shape andθ=360° This visual result is similar to the Sierpinski triangle.

e. Visual results of the rotation angle at the four reference points as the center of rotation there are four sets of new points and at the five reference points as the center of rotation there are five groups of new points that gather based on the reference point with a shape resembling the shape of the outer portion andθ=360° This visual result is almost similar to the Sierpinski triangle.

References

[1]. Sampurno, J. and Irfana Diah Faryuni, Fractal Analysis Method, Yogyakarta: Deepublish, 2016.
[2]. Purnomo, K. D, Generation of Sierpinski Triangle with Affine Transformation Based on several geometric objects, Proceedings of the National Mathematics Seminar, Jember: Department of Mathematics, Faculty of Mathematics and Natural Sciences, University of Jember, Pages: 365-375, 2014.
[3]. Devaney, RL, Fractal Patterns and Chaos Games, Boston: Department of Mathematics Boston University, 2003.
[4]. Ratna, E. N, Modifying Chaos Game Rules by Utilizing the Center of Strength of Triangle, Thesis, Jember: Faculty of Mathematics and Natural Sciences, University of Jember, 2018.
[5]. Larasati, I, Modification of Chaos Games with Rotation Variations in Quadrilateral, Scripts, Jember: Faculty of Mathematics and Natural Sciences, University of Jember, 2019.
[6]. Yunaning, F, Study of Non-Random Chaos Game Rules on Triangles, Thesis, Jember: Faculty of Mathematics and Natural Sciences University of Jember, 2018.
[7]. Purnomo, KD, Rere FA, and Kusno, Study of the Formation of the Sierpinski Triangle on the Chaos Game Problem by Utilizing Affine Transformation, Mathematical Journal, Vol. 6, No. 2, Hal: 86-92, 2016.
[8]. Dikara, NA, Modification of Chaos Games with Reference Points Forming Non-Convex Polygons, Thesis, Jember: Faculty of Mathematics and Natural Sciences University of Jember, 2019.
[9]. Kusno, Geometry of Building Design and Design and Modeling of Objects with Compute-Assisted Curves and Surfaces, Jember: Faculty of Mathematics and Natural Sciences, University of Jember, 2003.