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Multi-State Markov Net Premium Valuation for Chronic Illness Riders with Care Benefits in Indonesia

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ABSTRACT

This study develops a net premium valuation model for chronic illness riders with care benefits in Indonesia using a discrete-time multi-state Markov framework. Indonesian 2023 prevalence, mortality, and population data are used as inputs. Age-group prevalence of stroke, chronic kidney disease, and hypertension is interpolated to single ages and used to derive model-implied transition probabilities under Markov assumptions. These probabilities are combined with a population-weighted unisex mortality basis to calculate net premiums under the actuarial equivalence principle. Results show that the probability of remaining healthy decreases with age, while illness and death probabilities increase. Hypertension gives the largest contribution to illness transitions. For entry ages 35–65, the annual net premium rises from IDR 1.62 million to a peak of IDR 3.25 million at age 61, then declines slightly. The model provides a population-level basis for diagnosis-based chronic illness rider pricing in settings without longitudinal incidence data or claims histories.

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1. INTRODUCTION

Increasing life expectancy and the growing burden of chronic diseases have created a need for insurance products that account not only for mortality risk but also for morbidity risk. Traditional life insurance valuation commonly represents the insured through survival and death states, whereas chronic disease protection requires a richer state structure because the insured may remain healthy, enter a chronic illness state, receive care benefits, and eventually die. Therefore, multi-state models are relevant for actuarial valuation when benefits depend on future health status. Recent actuarial studies have shown that multi-state and Markov frameworks are suitable for representing health-status transitions and valuing long-term care or chronic illness-related products [1], [2].

In Indonesia, transition-based models have begun to develop in the valuation of long-term care and related health insurance products. Previous national studies show that multi-state models can be used to determine premiums when the insured may move among several health or care-related states [3], [4]. However, further refinement is needed when diagnosis-based chronic illness states are used as benefit triggers. In particular, a chronic illness rider triggered by stroke, chronic kidney disease, or hypertension should be distinguished from pure long-term care insurance based on functional dependency.

The availability of recent Indonesian health, mortality, and demographic data provides an opportunity to develop a context-specific valuation model. The 2023 Indonesian Health Survey provides age-group prevalence information for chronic diseases, including stroke, chronic kidney disease, and hypertension [5]. The 2023 Indonesian mortality and morbidity table made available by the Persatuan Aktuaris Indonesia using BPJS Kesehatan data provides annual death probabilities [6]. Population data from Statistics Indonesia support the construction of a unisex mortality basis that reflects the demographic composition of the Indonesian population [7]. These sources allow chronic disease prevalence and mortality information to be combined within an actuarial transition model.

Recent international studies have shown that long-term care and health-contingent insurance modelling increasingly uses transition-based multi-state structures to represent movements among health, disability, care-dependency, and death states. Matrix-based Markov models have been applied to functional disability, mortality pooling, long-term care insurance, care annuities, and related health-contingent insurance products because they can represent changes in policyholder condition more realistically than single-decrement models [8], [9], [10], [11], [12], [13], [14], [15]. Related empirical and actuarial studies further confirm that morbidity, dependency, and mortality should be distinguished when valuing care-related benefits [16], [17], [18], [19], [20], [21]. However, many of these studies rely on international care systems, retirement-product designs, or dependency classifications that differ from the Indonesian disease-prevalence data used in this study.

Existing Indonesian applications of multi-state models for long-term care insurance mainly formulate transitions among health or care-dependency states and are closer to functional long-term care coverage. In contrast, the present study uses diagnosis-based national prevalence data for stroke, chronic kidney disease, and hypertension as morbidity inputs for a chronic illness rider. This distinction is important because the available Indonesian morbidity data are cross-sectional prevalence data, not longitudinal incidence data. In a Markov valuation model, however, state probabilities must be propagated using one-year transition probabilities $p_{ij}(x)$, not directly using prevalence proportions $\pi_j(x)$. Therefore, when longitudinal incidence observations are unavailable, prevalence must be used to derive model-implied transition probabilities under explicit assumptions, such as no recovery, mutually exclusive operational illness states, and specified illness-state mortality. This calibration step prevents cross-sectional prevalence from being misinterpreted as an observed annual incidence probability, while still allowing national morbidity information to be incorporated into the actuarial transition matrix.

This study contributes to the actuarial literature in two ways. First, it positions the product as a chronic illness rider with care benefits, thereby clarifying the interpretation of the benefit trigger. Second, it constructs model-implied annual transition probabilities from age-specific prevalence and mortality information, so that prevalence is not treated directly as a transition probability. This approach provides a population-level valuation framework for chronic illness rider pricing in Indonesia. The objective of this study is to develop a multi-state Markov net premium valuation model for chronic illness riders with care benefits using Indonesian morbidity, mortality, and population data. Specifically, this study estimates annual net premiums for entry ages 35–65 and evaluates the effects of entry age, discount rate, and illness-state mortality assumptions on the resulting premiums.

2. RESEARCH METHOD

2.1 Research Design

This study was a quantitative-computational study in actuarial mathematics. The objective was to construct a premium valuation model using a discrete-time multi-state Markov framework and to calculate the expected present value of benefits and premiums under the actuarial equivalence principle. A multi-state Markov model was selected because the product value depends on transitions among several states, including healthy, chronic illness, and death states. This approach is consistent with recent actuarial applications of Markov and matrix-based models for disability insurance, long-term care insurance, and health-state dependent products [1], [2], [8], [14].

The model was formulated in discrete annual time steps. This choice was consistent with the structure of the available data, which consisted of annual death probabilities and age-group prevalence. The product under consideration was not treated as pure long-term care insurance based on functional dependency. Instead, it was interpreted as a chronic illness rider with long-term care benefits because the benefit was triggered by entry into one of the specified chronic disease states.

2.2 Data Sources and Variables

This study used secondary data from three sources. The first source was the 2023 Indonesian Health Survey published by the Ministry of Health of the Republic of Indonesia [5]. This source provided age-group prevalence data for stroke, chronic kidney disease, and hypertension. These diseases were selected because they are relevant to chronic illness protection and were available in national tabulations.

The second source was the 2023 Indonesian mortality and morbidity table made available by the Persatuan Aktuaris Indonesia using BPJS Kesehatan data [6]. This source was used to obtain age-specific annual death

probabilities. The baseline mortality was applied to the healthy state, while mortality in illness states was constructed by applying excess mortality factors to the baseline death probability.

The third source was the official population table by age group and sex published by Statistics Indonesia [7]. These data were used to construct a population-weighted unisex mortality basis. The population variables were not used to estimate mortality directly, but to combine male and female mortality rates into a single unisex basis consistent with the demographic composition of Indonesia in 2023.

The input variables were age-specific prevalence of stroke, chronic kidney disease, and hypertension, annual death probabilities by age and sex, and population counts by age and sex. The output variables were the net single premium and annual net premium for the chronic illness rider with long-term care benefits. A summary of the data sources, variables, model usage, and main limitations is presented in Table 1.

Table 1. Data Sources, Variables, Model Usage, and Limitations

Source	Variable	Year	Usage in the model	Main limitations
Indonesian Health Survey, Ministry of Health [5]	Age-group prevalence of stroke, chronic kidney disease, and hypertension	2023	Interpolated to single ages and used to derive model-implied transition probabilities from the healthy state to illness states	Cross-sectional prevalence; no longitudinal incidence, recovery, or comorbidity histories
Indonesian mortality and morbidity table, Persatuan Aktuaris Indonesia/BPJS Kesehatan [6]	Age-specific annual death probabilities by sex	2023	Used to construct baseline mortality and illness-state mortality through excess mortality multipliers	Not disease-specific longitudinal mortality for the modelled illness states
Badan Pusat Statistik (BPS; Statistics Indonesia) [7]	Population counts by age group and sex	2023	Used as population weights to construct the unisex mortality basis	Population weights may differ from the insured population of a commercial portfolio

As shown in Table 1, the main data limitation is that the morbidity information from SKI 2023 is cross-sectional prevalence rather than longitudinal transition data. Therefore, the transition probabilities used in this study were calibrated as model-implied annual transition probabilities, not directly observed incidence probabilities. The model should consequently be interpreted as a synthetic cohort valuation framework under the assumptions of no recovery, mutually exclusive operational illness states, and specified illness-state mortality.

A key data limitation is that the morbidity information from SKI 2023 is cross-sectional prevalence rather than longitudinal transition data. Therefore, the transition probabilities used in this study were calibrated as model-implied annual transition probabilities, not directly observed incidence probabilities. The model should consequently be interpreted as a synthetic cohort valuation framework under the assumptions of no recovery, mutually exclusive operational illness states, and specified illness-state mortality.

2.3 Contract Assumptions and Valuation Parameters

The entry age of the insured was analysed from 35 to 65 years. The maximum valuation age was set at 75 years. The premium-paying period was 10 years, and the annual care benefit was payable for a maximum of 10 years. The death benefit was fixed at IDR 100,000,000, while the annual care benefit was set at 10% of the death benefit, namely IDR 10,000,000.

The baseline annual effective discount rate was 5%. Sensitivity analysis was then conducted using alternative discount rates of 3%, 7%, and 10%. The model excluded acquisition expenses, maintenance expenses, lapse rates, underwriting selection, solvency capital requirements, and profit margins. Therefore, the resulting premiums should be interpreted as net premiums rather than commercial premiums.

In practical commercial pricing, the present net premium framework may be extended by incorporating expense loadings, lapse behaviour, underwriting selection, and profit margins into the valuation equation. Let P_x^g denote the annual gross premium for an insured aged x, where the superscript g indicates a gross premium. If A_x^{ℓ} denotes the lapse-adjusted actuarial present value of benefits, $\ddot{a}_{x:\overline{10}|}^{\ell}$ denotes the lapse-adjusted premium annuity factor over the 10-year premium-paying period, $E_x^{m,\ell}$ and $E_x^{c,\ell}$ denote the present values of maintenance and claim-related expenses, e_0 denotes the initial acquisition expense, α denotes the scalar premium-proportional expense loading with $0 \leq \alpha < 1$, and M_x denotes the provision for profit and risk margin, the gross premium may be expressed as $P_x^g(1-\alpha)\ddot{a}_{x:\overline{10}|}^{\ell} = A_x^{\ell} + E_x^{m,\ell} + E_x^{c,\ell} + e_0 + M_x$. In this notation, the superscript

l indicates lapse-adjusted present values, while m and c refer to maintenance and claim-related expenses, respectively. The double dot in $\ddot{a}^{\ell}_{x:\overline{10}|}$ indicates an annuity-due, consistent with premiums payable at the beginning of each policy year, and $\overline{10}|$ ndicates the 10-year premium-paying period. Lapse behaviour may be incorporated by augmenting the transition matrix with a lapse state, while underwriting selection may be represented through duration-specific selected transition probabilities. These extensions are not implemented in the present study but provide a mathematical basis for adapting the net premium framework to commercial product pricing.

2.4 State Structure

This study used a discrete-time multi-state Markov model consisting of eight states. The state space is defined as $\mathcal{S} = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$, where S_0 denotes the healthy state, S_1 denotes stroke, S_2 denotes death after stroke, S_3 denotes chronic kidney disease, S_4 denotes death after chronic kidney disease, S_5 denotes hypertension, S_6 denotes death after hypertension, and S_7 denotes death from other causes. The transition structure among these states is presented in Figure 1. The active states in the model are S_0, S_1, S_3, and S_5, corresponding respectively to the healthy state, stroke, chronic kidney disease, and hypertension. The death states S_2, S_4, S_6, and $S_7$are absorbing states. Therefore, once an insured individual enters one of the death states, no further transition to another state is possible. The model also assumes no recovery from illness states S_1, S_3, and S_5 to the healthy state S_0. This assumption was adopted because the available aggregate data did not provide recovery probabilities or longitudinal disease histories. The three illness states S_1, S_3, and S_5 were treated as mutually exclusive operational states for valuation purposes. This assumption does not imply that stroke, chronic kidney disease, and hypertension are clinically mutually exclusive in the population. Rather, it means that once an insured individual enters the illness component of the model, the individual is classified according to the primary qualifying chronic disease represented in the model. This simplifying assumption was required because the available prevalence data did not contain joint prevalence or individual-level comorbidity histories.

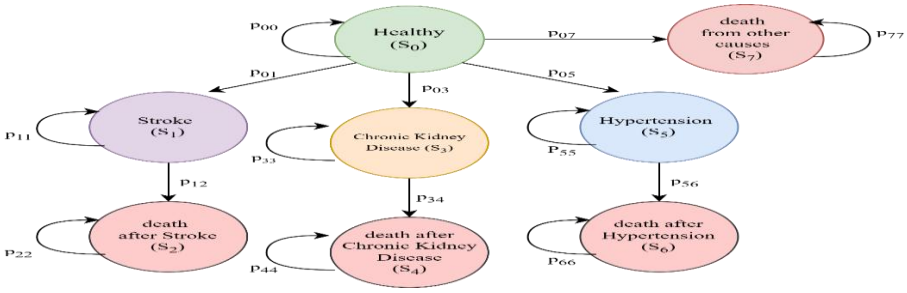


Figure 1. Multi-state Transition Structure for Chronic Illness Rider Valuation

Figure 1 illustrates the transition mechanism used in the chronic illness rider valuation model. The insured is initially assumed to be in the healthy state and may subsequently move into one of the diagnosis-based illness states, namely stroke, chronic kidney disease, or hypertension. Each illness state is associated with a corresponding absorbing death state, while death from other causes is represented separately. Care benefits are linked to occupancy of the active illness states, whereas death benefits are linked to transitions into absorbing death states.

Let X_t denote the state of the insured at policy duration t, where $X_t \in \mathcal{S}$. For $i, j \in \{0, 1, \dots, 7\}$, the Markov assumption is expressed as Equation (1):

$$\Pr(X_{t+1} = S_j \mid X_t = S_i, X_{t-1}, \dots, X_0) = \Pr(X_{t+1} = S_j \mid X_t = S_i) = p_{ij}(x + t), \tag{1}$$

where $p_{ij}(x + t)$ denotes the one-year transition probability from state S_i to state S_j at attained age $x + t$. Since the transition probabilities depend on attained age, the model is a time-inhomogeneous Markov model. This framework is appropriate for chronic illness rider valuation because benefit payments depend on the insured's future health state, not merely on survival or death [2], [8], [14].

2.5 Construction of the Unisex Mortality Basis

Let $q_x^m$and q_x^f denote the annual death probabilities for males and females at age x, respectively. Let N_x^m and N_x^f denote the corresponding male and female population counts. The unisex annual death probability at age x was calculated as Equation (2):

$$q_x = \frac{N_x^m q_x^m + N_x^f q_x^f}{N_x^m + N_x^f}. \tag{2}$$

In the multi-state model, q_x represents the baseline unisex mortality probability and is used as the transition probability from the healthy state S_0 to death from other causes S_7, that is, $p_{07}(x) = q_x$. Equation (2) is a population-weighted average. It was used to align the mortality basis with the sex composition of the Indonesian population in 2023. In the multi-state model, $q_x$represents the baseline unisex mortality probability and is subsequently used to define the transition probability from the healthy state $S_0$to death from other causes S_7, namely $p_{07}(x) = q_x$. The mortality probabilities were obtained from the PAI/BPJS mortality basis, while the population weights were obtained from Statistics Indonesia [6], [7].

2.6 Interpolation of Disease Prevalence

The SKI 2023 prevalence data were available by age groups. To apply the annual Markov model, age-group prevalence values were converted into single-age prevalence values. Let $\mathcal{K} = \{1, 3, 5\}$ denote the set of illness-state indices, where S_1 represents stroke, S_3 represents chronic kidney disease, and S_5 represents hypertension. For $k \in \mathcal{K}$, let $\pi_x^{(k)}$ denote the prevalence at age x associated with illness state S_k. Let $\left(a_1, \pi_{a_1}^{(k)}\right)$ and $\left(a_2, \pi_{a_2}^{(k)}\right)$ be two adjacent age points surrounding age x. The interpolated prevalence at age x was calculated by linear interpolation as Equation (3):

$$\pi_x^{(k)} = \pi_{a_1}^{(k)} + \frac{x - a_1}{a_2 - a_1}\left(\pi_{a_2}^{(k)} - \pi_{a_1}^{(k)}\right), \qquad k \in \{1,3,5\}. \tag{3}$$

This interpolation was applied separately to the prevalence of stroke (S_1), chronic kidney disease (S_3), and hypertension (S_5). The interpolated values were used only as morbidity inputs for transition calibration, not as transition probabilities themselves.

2.7 Derivation of Model-Implied Transition Probabilities from Prevalence

Prevalence represents the proportion of individuals already in an illness state at a given age, whereas a transition probability represents the probability of entering that illness state during one year. Therefore, prevalence cannot be used directly as a one-year transition probability. To address this issue, the transition probabilities from the healthy state S_0 to the illness states S_1, S_3, and S_5 were derived from the age-specific prevalence pattern. Let $\pi_x^{(k)}$ denote the prevalence at age x associated with illness state S_k, where $k \in \mathcal{K} = \{1, 3, 5\}$. Here, $k = 1$ corresponds to stroke, $k = 3$ corresponds to chronic kidney disease, and $k = 5$ corresponds to hypertension. Under the operational assumption of mutually exclusive illness states, the proportion free from the three modelled illness states at age x, and therefore assigned to the healthy state S_0 for transition calibration, is defined as Equation (4):

$$H_x = 1 - \sum_{k \in \{1,3,5\}} \pi_x^{(k)}. \tag{4}$$

Let $p_{0k}(x)$ denote the annual transition probability from the healthy state S_0 to illness state S_k at age x, for $k \in \{1, 3, 5\}$. Let $q_x^{(k)}$denote the annual death probability for individuals in illness state S_k, which is specified in Section 2.8. Under the no-recovery assumption, the prevalence of illness state S_k at age $x + 1$ can be approximated by the surviving prevalent cases in S_k at age x and new transitions from the healthy state S_0 to illness state S_k. Hence,

$$\pi_{x+1}^{(k)} = \pi_x^{(k)}\left(1 - q_x^{(k)}\right) + H_x p_{0k}(x) \quad \Leftrightarrow \quad p_{0k}(x) = \frac{\pi_{x+1}^{(k)} - \pi_x^{(k)}\left(1 - q_x^{(k)}\right)}{H_x}, \quad k \in \{1,3,5\}. \tag{5}$$

The transition probabilities obtained from Equation (5) should not be interpreted as directly observed incidence rates. They are model-implied annual transition probabilities constructed to reconcile cross-sectional prevalence information with the Markov state structure. This distinction is necessary because the SKI 2023 data provide age-specific prevalence, namely the proportion of individuals who are already in a disease condition at a given age, whereas the Markov model requires annual probabilities of movement from the healthy state to illness states. Therefore, the calibration is conditional on the assumptions of no recovery, mutually exclusive operational illness states, and specified illness-state mortality. Under these assumptions, the model produces a synthetic cohort representation of disease transitions for actuarial valuation purposes [1],[2]. Each derived transition probability was checked to satisfy $0 \le p_{0k}(x) \le 1, k \in \{1, 3, 5\}$. In addition, the total probability of leaving the healthy state $S_0$was required to satisfy $p_{01}(x) + p_{03}(x) + p_{05}(x) + p_{07}(x) \le 1$. These checks ensured that the derived transition probabilities formed a valid probability structure for the transition matrix P_x.

2.8 Illness-State Mortality and Transition Matrix

Following the relative-risk mortality adjustment used in multi-state long-term care insurance modelling [22], illness-state mortality may be specified on the mortality-intensity scale as $\mu_x^{(k)} = \theta_k \mu_x^{(0)}$, where θ_k is the excess mortality factor for illness state S_k, $\mu_k^{(0)}$ denotes the baseline force of mortality at age, and $\mu_x^{(k)}$ denotes the force

of mortality forindividual in illness state S_k, for $k \in \{1,2,3\}$. In the present annual discrete-time model, this relative-risk structure is implemented as the approximation in Equation (6):

$$q_x^{(k)} = \theta_k q_x, \tag{6}$$

where q_x denotes the baseline one-year mortality probability at attained age x. Since the relative-risk assumption is more naturally defined on the mortality-intensity scale, the discrete-time approximation in Equation (6) is used only when the baseline mortality probability is sufficiently small. A bounded alternative is $q_x^{(k)} = 1 - (1 - q_x)^{\theta_k}$, which follows from the relationship $q_x = 1 - \exp(-\mu_x)$ when the force of mortality in illness state $S_k$is $\mu_x^{(k)} = \theta_k \mu_x$. In the present study, the approximation is retained for simplicity, while sensitivity analysis is used to evaluate the effect of alternative excess mortality assumptions. In the baseline scenario, the excess mortality multipliers were set as $\theta_1 = 1.10$ for stroke, $\theta_3 = 1.15$ for chronic kidney disease, and $\theta_5 = 1.05$ for hypertension. Since Indonesian longitudinal claims data linking each illness state to subsequent mortality were not available, these values were treated as conservative actuarial assumptions rather than direct estimates from Indonesian experience data. The choice $\theta_k > 1$is supported by international evidence showing excess all-cause mortality among individuals with stroke, chronic kidney disease, and hypertension. For chronic kidney disease, a collaborative meta-analysis of general population cohorts reported adjusted hazard ratios of 1.18 for eGFR 60 mL/min/1.73 m^2 and 1.57 for eGFR 45 mL/min/1.73 m^2, relative to eGFR 95 mL/min/1.73 m^2 [23]. For hypertension, a cohort study of community-dwelling older adults reported an all-cause mortality hazard ratio of 1.10 for the hypertension group relative to the non-hypertension group, and 1.23 for stage 2–3 hypertension [24]. For stroke, long-term follow-up evidence shows substantially higher mortality among stroke survivors than among stroke-free controls, with one study reporting a hazard ratio of 2.2 [25]. Therefore, the selected multipliers, particularly $\theta_1 = 1.10$ for stroke, should be interpreted as conservative baseline loadings rather than direct clinical hazard ratios. The uncertainty surrounding these multipliers was addressed through the low, medium, and high excess mortality scenarios in the sensitivity analysis.

Specifically, $q_x^{(1)}$, $q_x^{(3)}$, and $q_x^{(5)}$represent the death probabilities for stroke, chronic kidney disease, and hypertension, respectively. For each attained age x, the one-step transition probability matrix is defined as $\mathbf{P}_x = [p_{ij}(x)]_{8\times 8}$, where $p_{ij}(x)$ denotes the probability of moving from state S_i to state S_j over one policy year. Using the state ordering $(S_0,\ S_1,\ S_2,\ S_3,\ S_4,\ S_5,\ S_6,\ S_7)$, the transition matrix is given by

$$\mathbf{P}_x = \begin{pmatrix} p_{00}(x) & p_{01}(x) & 0 & p_{03}(x) & 0 & p_{05}(x) & 0 & p_{07}(x) \\ 0 & p_{11}(x) & p_{12}(x) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{33}(x) & p_{34}(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{55}(x) & p_{56}(x) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The first row corresponds to the healthy state S_0. The transition probabilities $p_{01}(x)$, $p_{03}(x)$, $p_{05}(x)$, and $p_{07}(x)$ represent movement from S_0 to stroke S_1, chronic kidney disease S_3, hypertension S_5, and death from other causes S_7, respectively. Therefore, $p_{00}(x) = 1 - p_{01}(x) - p_{03}(x) - p_{05}(x) - p_{07}(x)$. For the stroke state S_1, $p_{11}(x) = 1 - q_x^{(1)}, p_{12}(x) = q_x^{(1)}$. For the chronic kidney disease state S_3, $p_{33}(x) = 1 - q_x^{(3)}, p_{34}(x) = q_x^{(3)}$. For the hypertension state S_5, $p_{55}(x) = 1 - q_x^{(5)}, p_{56}(x) = q_x^{(5)}$. The death states S_2, S_4, S_6, and S_7 are absorbing states, so that $p_{22}(x) = p_{44}(x) = p_{66}(x) = p_{77}(x) = 1$. All other entries not specified in the matrix are equal to zero. The matrix formulation follows the standard matrix-based treatment of multi-state insurance valuation, where health-state probabilities and expected cash flows are computed through products of age-specific transition matrices [2], [8], [14].

Because the transition probabilities depend on attained age, the t-year transition matrix for sn insured entering at age x is denoted by ${}_t\mathbf{P}_x$. It is obtained through the age-dependent Chapman–Kolmogorov product:

$${}_t\mathbf{P}_x = \mathbf{P}_x \mathbf{P}_{x+1} \cdots \mathbf{P}_{x+t-1}, t \geq 1, \tag{7}$$

with ${}_0\mathbf{P}_x = \mathbf{I}_8$, where $\mathbf{I}_8$ is the 8×8 identity matrix. The (i,j)-th element of ${}_t\mathbf{P}_x$ gives the probability that an insured aged x who is initially in state S_i will be in state S_j after t policy years. In the numerical implementation, the sequential products in Equation (8) were checked to ensure probability mass conservation over the valuation period. Since each one-year transition matrix P_{x+t} is row-stochastic, the product matrix ${}_t\mathbf{P}_x$ should also remain row-stochastic and satisfy ${}_t\mathbf{P}_x.\mathbf{1} = \mathbf{1}$, where $\mathbf{1}$ is an 8×1 vector of ones [26]. Therefore, for each entry age and duration t, the calculation was validated by checking $\left\| {}_t\mathbf{P}_x.\mathbf{1} - \mathbf{1} \right\|_\infty < \varepsilon$, for a small numerical tolerance ε. The corresponding state probability vector $\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_0\ {}_t\mathbf{P}_x$ was also required to satisfy $\sum_{j=0}^{7} \boldsymbol{\alpha}_t(j) = 1,\ \boldsymbol{\alpha}_t(j) \geq 0,\ j = 0, 1, \ldots, 7$. The absorbing states S_2, S_4, S_6, and S_7 were handled through unit diagonal entries, ensuring that no

probability mass leaves the absorbing states during repeated matrix multiplication. These checks reduce the risk of numerical drift when applying the age-dependent matrix products over the valuation period [27], [28].

2.9 Premium Valuation

Let B_D denote the death benefit and B_C denote the annual care benefit. Let i be the annual effective discount rate and $v = (1+i)^{-1}$ be the annual discount factor. All cash flows are valued at policy inception, that is, at time 0. Policy year $t+1$ refers to the interval $[t, t+1)$, for $t = 0,1, \ldots$. Premiums are assumed to be payable at the beginning of each policy year, whereas death and care benefits are assumed to be payable at the end of the relevant policy year. At policy inception, the insured is assumed to be in the healthy state S_0. Hence, the initial state probability vector is $\boldsymbol{\alpha}_0 = (1,0,0,0,0,0,0,0)$. The state probability vector at policy duration t is defined as $\boldsymbol{\alpha}_t = \boldsymbol{\alpha}_0 \ {}_t\mathbf{P}_x$, where ${}_t\mathbf{P}_x$ is the t-year transition matrix from entry age x. Let $\alpha_t(j)$ denote the j-th component of $\boldsymbol{\alpha}_t$ for $j = 0, 1, \ldots, 7$. Thus, $\alpha_t(j)$ represents the probability that the insured occupies state S_j at policy duration t, that is, $\alpha_t(j) = \Pr(X_t = S_j \mid X_0 = S_0)$. Since premiums are paid at the beginning of each policy year while the insured remains in the healthy state S_0, the expected present value of the healthy-state premium annuity-due over the m-year premium-paying period is given by Equation (8):

$$\ddot{a}^{(0)}_{x:\bar{m}|} = \sum_{t=0}^{m-1} v^t \, \alpha_t(0). \tag{8}$$

The term $v^t \alpha_t(0)$ reflects a premium paid at time t, conditional on the insured being in state S_0 at that time. In particular, the first premium at policy inception is not discounted because $v^0 = 1$.

The annual care benefit is assumed to be payable at the end of policy year t, for $t = 1, \ldots, r$, if the insured occupies one of the active illness states S_1, S_3, or S_5 at that payment time. Therefore, the expected present value of care benefits is given by Equation (9):

$$\mathrm{EPV}_C = B_C \sum_{t=1}^{r} v^t \, [\alpha_t(1) + \alpha_t(3) + \alpha_t(5)]. \tag{9}$$

This expression values the care benefit in arrears because the benefit at policy duration t is discounted by v^t. Let d_t denote the probability of death during the policy year $[t, t+1]$, conditional on the state probabilities at policy duration t. This probability is calculated from death transitions out of the healthy state and the active illness states during the one-year interval starting at attained age $x+t$ is given by Equation (10):

$$d_t = \alpha_t(0)p_{07}(x+t) + \alpha_t(1)p_{12}(x+t) + \alpha_t(3)p_{34}(x+t) + \alpha_t(5)p_{56}(x+t) \tag{10}$$

In Equation (10), $p_{07}(x+t)$is the transition probability from the healthy state $S_0$to death from other causes S_7. Meanwhile, $p_{12}(x+t)$, $p_{34}(x+t)$, and $p_{56}(x+t)$are the transition probabilities from the illness states S_1, S_3, and $S_5$to their corresponding death states S_2, S_4, and S_6, respectively. Since the death benefit is assumed to be payable at the end of the policy year of death, the expected present value of death benefits, EPV_D, is given by Equation (11):

$$\mathrm{EPV}_D = B_D \sum_{t=0}^{n-1} v^{t+1} \, d_t. \tag{11}$$

where n is the valuation period from entry age $x$to the maximum valuation age. The discount factor v^{t+1} is used because deaths occurring during policy year $t+1$, that is, during $[t, t+1)$, are valued as payments at time $t+1$. The net single premium (NSP) is then defined as Equation (12)

$$\mathrm{NSP}_x = \mathrm{EPV}_C + \mathrm{EPV}_D. \tag{12}$$

By the actuarial equivalence principle, the expected present value of future net premiums must equal the expected present value of future benefits. Hence, the annual net premium ANP_x isgiven by Equation (13):

$$\mathrm{ANP}_x = \frac{\mathrm{NSP}_x}{\ddot{a}^{(0)}_{x:\bar{m}|}}. \tag{13}$$

Equations (8)–(13) therefore apply a consistent timing convention: premiums are paid at the beginning of each policy year while the insured is in the healthy state S_0, care benefits are paid at the end of the relevant policy year when the insured occupies one of the illness states S_1, S_3, or S_5, and death benefits are paid at the end of the policy year of death. This structure follows the standard actuarial equivalence principle, where expected present values of benefits and premiums are computed using state probabilities and discounted to the valuation date [2], [8], [14].

2.10 Model Validation and Sensitivity Analysis

Model validation was conducted by checking the transition probability matrix at each attained age x. Here, $p_{ij}(x)$denotes the one-year transition probability from state $S_i$to state S_j, for $i, j = 0,1, \dots ,7$. Each entry of the matrix was required to satisfy $0 \leq p_{ij}(x) \leq 1, \ \ i,j \in \{0,1, \dots ,7\}$, and each row was required to sum to one: $\sum_{j=0}^{7} p_{ij}\,(x) = 1, i \in \{0,1, \dots ,7\}$. For the healthy state S_0, the validation condition was $p_{00}(x) + p_{01}(x) + p_{03}(x) + p_{05}(x) + p_{07}(x) = 1$. For the illness states S_1, S_3, and S_5, the validation conditions were $p_{11}(x) + p_{12}(x) = 1, p_{33}(x) + p_{34}(x) = 1, p_{55}(x) + p_{56}(x) = 1$. For the absorbing death states S_2, S_4, S_6, and S_7, the validation conditions were $p_{22}(x) = p_{44}(x) = p_{66}(x) = p_{77}(x) = 1$, with all other transition probabilities from these absorbing states equal to zero.

Sensitivity analysis was performed with respect to the annual effective discount rate $i$and the excess mortality factors θ_k, where $k \in \{1,3,5\}$ corresponds to the illness states S_1, S_3, and S_5, respectively. The discount-rate scenarios were 3%, 5%, 7%, and 10%. The excess mortality scenarios were classified into low, medium, and high assumptions. For each scenario, the annual net premium ANP_x was recalculated using Equation (13). This analysis was used to evaluate the stability of premium estimates under alternative economic and morbidity-mortality assumptions. All data processing and numerical calculations were conducted in R. Since the study used aggregated secondary data from publicly available official and institutional sources [5], [6], [7], without personal identifiers or direct respondent contact, no specific ethical approval was required.

3. RESULT AND ANALYSIS

3.1 Indonesian Unisex Mortality Basis

The first stage of the analysis produced the 2023 Indonesian unisex mortality basis. Male and female mortality probabilities from the PAI/BPJS mortality table were combined using population weights from Statistics Indonesia. This mortality basis was applied as the healthy-state death probability and as the baseline for illness-state mortality. The constructed unisex mortality basis increased with age and lay between the male and female mortality curves, consistent with its population-weighted construction. This basis was then used as the healthy-state death probability and as the baseline for illness-state mortality in the transition matrix.

3.2 Single-Age Disease Prevalence

The prevalence data for stroke, chronic kidney disease, and hypertension were obtained from SKI 2023 and interpolated to single ages using Equation (3). The interpolation was required because the Markov model uses annual age increments, while the original morbidity data were reported by age groups. Figure 2 shows that the prevalence of the three diseases increased with age, although their magnitudes differed. Hypertension had the highest prevalence across most ages, followed by stroke and chronic kidney disease. This pattern indicates that hypertension contributes the largest morbidity component in the model. Stroke became more relevant at middle and older ages, while chronic kidney disease remained lower but still increased with age. From an actuarial perspective, this finding implies that transition probabilities from the healthy state should not be assumed equal across diseases. The morbidity structure must reflect the observed differences in disease prevalence. However, because prevalence is not an annual transition probability, the interpolated prevalence values were used only as inputs for calibration.

3.3 Derived Transition Probabilities

The transition probabilities from the healthy state to the three illness states were calibrated using Equation (5). This procedure produced model-implied annual transition probabilities consistent with the age-specific prevalence pattern, illness-state mortality assumptions, and the no-recovery assumption.

Table 2. One-Step Transition Probabilities from the Healthy State at Selected Ages

Age (x)	Probability of Remaining Healthy ($p_{00}(x)$)	Probability of Stroke ($p_{01}(x)$)	Probability of Chronic Kidney Disease ($p_{03}(x)$)	Probability of Hypertension ($p_{05}(x)$)	Probability of Other-Cause Death ($p_{07}(x)$)
20	0.997627607	4.02669E-05	5.04063E-05	0.001509222	0.000772498
30	0.994988159	0.000154066	4.19701E-05	0.0035007	0.001315105
40	0.989474	0.000739862	0.000162905	0.007164182	0.002459052
50	0.982893187	0.001773597	0.000183814	0.00888728	0.006262123
60	0.973221592	0.001999581	0.000149345	0.010192934	0.014436547
70	0.962930961	0.001705276	0.000251539	0.010054097	0.025058127

Table 2 shows that the probability of remaining healthy declined with age. At younger ages, the probability of remaining in the healthy state was close to one, whereas the probabilities of entering illness states or death were relatively small. At older ages, the decline in the healthy-state probability became more apparent because morbidity and mortality risks increased simultaneously. Among the illness transitions, hypertension had the largest transition probability at all selected ages. This result is consistent with the prevalence pattern in Figure 2a. Stroke and chronic kidney disease had smaller transition probabilities, although stroke became more important at older ages.

3.4 Net Single Premium and Annual Net Premium

The premium valuation was conducted for entry ages 35–65. The benefit structure consisted of a death benefit of IDR 100,000,000 and an annual care benefit of IDR 10,000,000. The premium-paying period and care-benefit period were each set at 10 years. The baseline annual effective discount rate was 5%.

Table 3 presents the expected present value components and the resulting annual net premiums. The EPV annuity declined from age 35 to age 65. This reflects the shorter expected period during which premiums are paid while the insured remains healthy. In contrast, the EPV of care benefits generally increased with entry age because older insureds faced higher probabilities of entering chronic illness states.

Table 3. Net Single Premium and Annual Net Premium by Entry Age

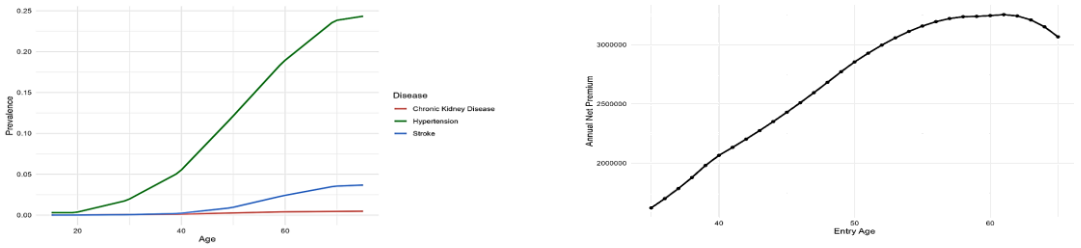
Entry Age	EPV annuity	EPV care (IDR)	EPV death (IDR)	NSP (IDR)	ANP (IDR)
35	7.879	1,602,858	11,185,383	12,788,241	1,623,142
⋮	⋮	⋮	⋮	⋮	⋮
40	7.737	2,673,291	13,316,110	15,989,401	2,066,556
⋮	⋮	⋮	⋮	⋮	⋮
45	7.645	2,978,054	15,595,667	18,573,721	2,429,468
⋮	⋮	⋮	⋮	⋮	⋮
50	7.481	3,609,146	17,744,034	21,353,181	2,854,288
⋮	⋮	⋮	⋮	⋮	⋮
55	7.318	3,949,021	19,155,970	23,104,990	3,157,196
⋮	⋮	⋮	⋮	⋮	⋮
60	7.160	4,038,265	19,196,592	23,234,857	3,245,113
⋮	⋮	⋮	⋮	⋮	⋮
65	6.968	4,366,762	16,996,150	21,362,913	3,066,045

The EPV of death benefits increased up to the late 50s and then declined. This pattern arises from two competing effects. Mortality risk increases with age, which tends to increase the expected value of death benefits. At the same time, older entry ages have a shorter valuation period up to age 75, which reduces the number of future years over which death benefits may be paid. The combination of these effects produces a non-monotonic pattern in the EPV of death benefits.

The net single premium increased from IDR 12.79 million at entry age 35 to approximately IDR 23.38 million at entry age 58, before declining at older entry ages. The annual net premium increased from IDR 1.62 million at entry age 35 to a peak of IDR 3.25 million at entry age 61, before decreasing slightly to IDR 3.07 million at entry age 65.

As an external plausibility check, the resulting annual net premiums correspond to approximately 1.62%–3.25% of the IDR 100,000,000 death benefit. Published Indonesian critical illness product materials show that commercial premiums are strongly affected by age, sum insured, benefit design, underwriting, premium refund features, commissions, and other expense loadings. For instance, one Indonesian critical illness illustration with a sum insured of IDR 500,000,000 and a 10-year premium-paying period reports a total annual premium of IDR 18,364,539, or approximately 3.67% of the sum insured [29]. Another Indonesian critical illness product explicitly states that the premium includes acquisition, administration, fund management, bank commissions, and marketing staff commissions [30]. These comparisons suggest that the modelled premiums are within a plausible order of magnitude, while remaining appropriately lower than commercial tariffs because the present study estimates net premiums and excludes commercial loadings.

Figure 2b shows that the annual net premium increased with entry age until the early 60s and then declined slightly, a pattern that is consistent with the interaction between increasing morbidity-mortality risk and the shortening valuation period at older entry ages. This pattern is actuarially reasonable because morbidity and mortality risks rise with age, while the remaining valuation period becomes shorter at older entry ages. Hence, the premium is not determined solely by increasing risk but also by the remaining period over which benefits and premiums are evaluated.



(a) (b)

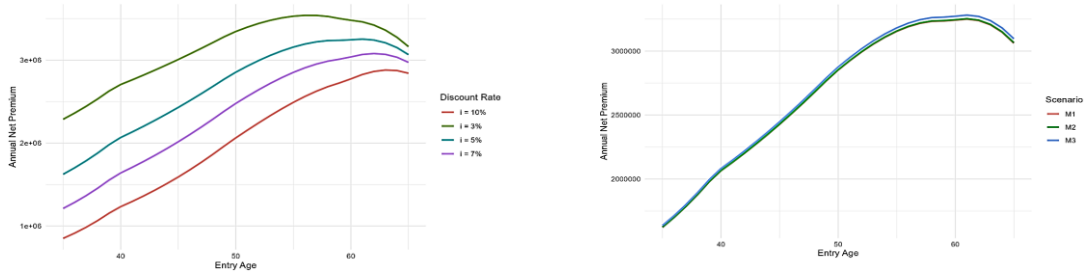
Figure 2. Age-related morbidity and premium patterns: (a) interpolated single-age prevalence curves used as morbidity inputs, not direct transition probabilities; and (b) annual net premium by entry age. The premium pattern reflects the interaction between age-related morbidity-mortality risk, discounting, and the shorter remaining valuation period at older entry ages.

3.5 Sensitivity to Discount Rate

Sensitivity analysis was first conducted by varying the annual effective discount rate. The scenarios considered were 3%, 5%, 7%, and 10%. The annual net premium was recalculated for each entry age using the same transition probability structure.

Figure 3a shows that higher discount rates produced lower annual net premiums. This result follows directly from present value theory. When the discount rate increases, future benefit payments have a lower present value. Since the net premium is calculated from the expected present value of benefits divided by the expected present value of the premium annuity, a lower present value of benefits generally reduces the annual net premium.

Across all discount-rate scenarios, the premium retained the same general age pattern: it increased from younger entry ages to older entry ages and then levelled off or declined slightly near the oldest entry ages. This indicates that the effect of entry age is robust to changes in the discount-rate assumption.



(a) (b)

Figure 3. Sensitivity analysis of annual net premiums by entry age: (a) higher discount rates reduce the present value of future benefits and therefore lower annual net premiums; and (b) alternative illness-state excess mortality assumptions affect premiums through the balance between expected care-benefit duration and death-benefit payment risk.

3.6 Sensitivity to Illness-State Mortality

The second sensitivity analysis evaluated the effect of alternative excess mortality factors in the illness states. The baseline model assumed higher mortality for individuals in stroke, chronic kidney disease, and hypertension states than for healthy individuals at the same age. In the baseline scenario, the excess mortality factors were set at 1.10 for stroke, 1.15 for chronic kidney disease, and 1.05 for hypertension. These values were treated as model assumptions and evaluated through sensitivity analysis, not as empirically estimated parameters. This should be clearly stated because the available data did not contain disease-specific longitudinal mortality experience.

Figure 3b shows that the effect of illness-state mortality on annual net premiums depends on the balance between care benefits and death benefits. If the death benefit dominates the benefit structure, higher illness-state mortality can increase the premium because the probability of death benefit payment rises. Conversely, if the care benefit is more influential, higher illness-state mortality may reduce the expected duration of care-benefit payments. Therefore, the direction and magnitude of the premium change must be interpreted in relation to the product design.

This result highlights the importance of illness-state mortality assumptions in chronic illness rider pricing. The model uses fixed excess mortality factors as a simplifying assumption. In commercial applications, these factors should ideally be estimated from portfolio-specific claims data or longitudinal health data.

3.7 Discussion of Model Implications and Limitations

The results show that a discrete-time multi-state Markov framework can represent the interaction among morbidity, mortality, benefit timing, discounting, and premium valuation in a chronic illness rider with care benefits. The premium pattern is not determined by age-related morbidity alone. It also depends on the healthy-state premium annuity, the probability of occupying illness states, the probability of death benefit payment, and the remaining valuation period up to the maximum valuation age. This explains why the annual net premium increases from younger entry ages to the early 60s and then declines slightly at older entry ages.

A central methodological implication is the separation between prevalence inputs, transition probabilities, and state occupancy probabilities. Cross-sectional prevalence $\pi_j(x)$ measures the proportion of individuals already in illness state j at age x, whereas the Markov transition probability $p_{0j}(x)$ represents movement from the healthy state to illness state j during one year. Therefore, the transition probabilities used in this study should be interpreted as model-implied parameters that reproduce the age-specific prevalence pattern under the stated assumptions, not as directly observed incidence probabilities. This distinction preserves the actuarial interpretation of the transition matrix.

Several limitations should be noted. The model assumes no recovery from illness states, treats stroke, chronic kidney disease, and hypertension as mutually exclusive operational states, and uses fixed excess mortality multipliers rather than disease-specific mortality estimated from Indonesian longitudinal claims data. Allowing recovery would add transition probabilities from illness states back to the healthy state and would generally reduce expected care-benefit payments while increasing the expected premium annuity. Incorporating comorbidity would require an expanded state space containing joint disease states and may increase expected benefits if comorbid states are assigned higher mortality or benefit intensities.

The use of population-weighted unisex mortality should also be interpreted carefully. The resulting mortality basis reflects the demographic composition of the Indonesian population in 2023, not the mortality experience of an insured portfolio. In commercial insurance pricing, insured lives may differ from the general population because of underwriting selection, socioeconomic profile, access to medical care, policyholder behaviour, and product eligibility rules. Therefore, the mortality basis should ideally be recalibrated using insurer-specific experience data or an insured-population mortality table for commercial implementation.

The premiums estimated in this study should be interpreted as synthetic-cohort net premiums rather than empirically estimated actuarial rates. The estimated premiums are net premiums because expenses, lapse behaviour, underwriting selection, solvency capital requirements, and profit margins are excluded. Thus, the results should be interpreted as a population-level actuarial basis for valuation, not as final commercial premiums. Future work should extend the framework using longitudinal claims or health panel data, recovery and comorbidity states, gross premium loadings, and uncertainty quantification through stochastic sensitivity analysis, bootstrap intervals, or Bayesian calibration.

4. CONCLUSION

This study developed a discrete-time multi-state Markov net premium valuation model for chronic illness riders with care benefits in Indonesia using national morbidity, mortality, and population data. The model represents age-dependent transitions among healthy, illness, and death states and applies the actuarial equivalence principle to calculate net premiums. The results indicate that entry age is a key determinant of premium valuation because morbidity risk, mortality risk, the healthy-state premium annuity, and the remaining valuation period change jointly across ages.

The main applied mathematics contribution of this study is the construction of a prevalence-based synthetic cohort Markov valuation framework. Instead of treating age-specific prevalence as annual incidence, the model derives model-implied transition probabilities from prevalence patterns under explicit Markov, no-recovery, illness-state, and mortality assumptions, providing a mathematically consistent way to use cross-sectional national morbidity data when longitudinal incidence or claims histories are unavailable. Consequently, the derived transition probabilities are not empirical incidence estimates but model-implied probabilities under explicit actuarial assumptions; this limitation defines the scope of interpretation, namely that the estimated premiums provide a population-level net premium basis for diagnosis-based chronic illness rider pricing in Indonesia, not final commercial premium rates. Future research may extend the model by incorporating recovery, comorbidity, individual heterogeneity, longitudinal claims or health panel data, expenses, lapse behaviour, underwriting selection, solvency capital requirements, profit margins, and uncertainty quantification through stochastic simulation, bootstrap intervals, or Bayesian calibration

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