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A Mixed Integer Linear Programming Based Scheduling Model for Cost Minimization in Sea Tollway Vessel Operations

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ABSTRACT

The Sea Tollway Program plays a critical role in Indonesia's national logistics system by improving maritime connectivity and reducing regional disparities in goods distribution. However, operational inefficiencies in vessel scheduling and prolonged berthing times continue to limit its effectiveness. This study addresses these challenges by formulating a deterministic Mixed Integer Linear Programming (MILP) based vessel scheduling model with capacity and cargo flow constraints aimed at minimizing time-dependent operational cost and improving berthing time efficiency. A case study is conducted on Sea Tollway Route H-1 using operational data from the first semester of 2025. The optimization model is implemented using the PuLP library and solved with the CBC solver. The results show that the optimized schedules consistently reduce operational costs by approximately 5–8% per voyage and decrease berthing time by about 12–17%, corresponding to an average reduction of two hours per voyage. Statistical significance testing confirms that these improvements are not due to random variation, while sensitivity analysis demonstrates the robustness of the optimized solutions under changes in key operational parameters. Overall, the proposed MILP-based framework provides a mathematically sound and practically applicable decision-support tool for improving vessel scheduling and operational efficiency in Sea Tollway maritime logistics.

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1. INTRODUCTION

Indonesia's Sea Tollway Program plays a strategic role in strengthening national logistics connectivity by improving maritime transportation links and reducing disparities in goods distribution across the archipelago. As an island nation with high dependence on sea-based logistics, Indonesia relies on the Sea Tollway to support the availability of essential goods, enhance accessibility to remote regions, and promote integration within the national supply chain system [1], [2]. From a logistics perspective, the program functions not only as a transportation corridor but also as a public service instrument aimed at improving efficiency and regional economic equity [3], [4].

Despite its strategic importance, Sea Tollway operations continue to face operational inefficiencies, particularly in vessel scheduling and port service coordination. Unsynchronized sailing schedules, prolonged loading and unloading processes, and congestion at intermediate ports have been reported to increase operational costs and reduce service reliability [3], [5], [6]. These inefficiencies can propagate along the supply chain, leading to delayed deliveries, underutilized vessel capacity, and higher logistics costs, especially for regions that depend heavily on scheduled Sea Tollway services [2], [4].

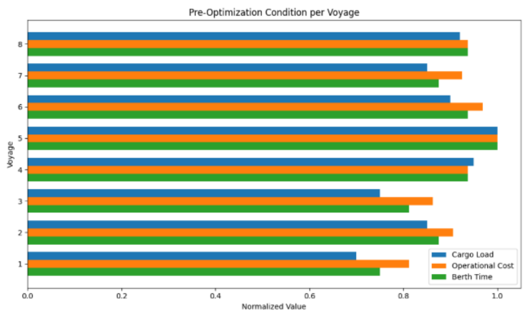

Figure 1. Pre-Optimization Operational Performance of the H-1 Sea Tollway Route

As shown in Figure 1, the pre-optimization operational conditions of the Sea Tollway route examined in this study exhibit considerable variation in cargo load, operational cost, and berthing time across voyages. Prior to optimization, operational performance appears uneven, indicating inefficiencies in capacity utilization and port service coordination. These conditions highlight the need for a systematic and data-driven approach to improving vessel scheduling and port operation efficiency.

To address such challenges, optimization-based approaches have been widely applied in maritime transportation systems. Mixed Integer Linear Programming (MILP), in particular, has proven effective for modeling vessel routing and scheduling problems by integrating discrete decision variables, capacity constraints, and operational requirements within a unified mathematical framework [5], [7], [8], [9]. Previous studies demonstrate that MILP-based models can reduce transportation costs and improve operational efficiency in shipping networks [10], [11]. However, in the context of the Indonesian Sea Tollway Program, existing research has largely focused on descriptive evaluations or general optimization models, with limited emphasis on rigorous statistical validation and robustness assessment of the optimization outcomes [2], [4], [6]. Similar optimization frameworks have been extensively developed in international maritime logistics research, particularly in liner shipping network design and integrated routing-scheduling models, demonstrating the scalability and analytical rigor of MILP-based formulations in complex shipping environments [7], [9], [12], [13].

From an applied mathematics perspective, this gap is significant. Optimization results that are not accompanied by quantitative validation may be difficult to assess in terms of reliability and practical relevance. Therefore, there is a need for an applied optimization framework that not only generates cost-efficient scheduling solutions but also evaluates their consistency, statistical significance, and robustness under real operational conditions.

The main contribution of this study lies in the integration of a Mixed Integer Linear Programming–based vessel scheduling model with statistical significance testing and sensitivity analysis within an operational Sea Tollway case study. Unlike many existing studies that focus solely on obtaining optimal solutions, this research combines optimization with post-optimization validation to assess the reliability and robustness of the resulting schedules. This integrated approach enhances the analytical rigor of the study and aligns with the scope of applied mathematics and operations research in maritime logistics.

Accordingly, this study proposes a MILP-based vessel scheduling model aimed at minimizing operational costs and improving berthing time efficiency in Sea Tollway operations. Using a case study of Sea Tollway Route H-1 and operational data from the first semester of 2025, the research seeks to (i) develop a scheduling model that reflects actual operational constraints, (ii) validate the optimization results through statistical significance testing, and (iii) assess solution robustness using sensitivity analysis.

2. RESEARCH METHOD

This study adopts a case study approach focusing on Sea Tollway Route H-1 operated by PT Pelayaran Nasional Indonesia (PELNI). The research methodology is structured sequentially, beginning with the collection and processing of operational data as the basis for defining model parameters. The vessel scheduling problem is then formulated using a Mixed Integer Linear Programming (MILP) framework that captures vessel capacity constraints, cargo flow balance, and port service characteristics. The optimization model is subsequently solved using a computational MILP solver. The resulting solutions are evaluated through a comparison of operational performance before and after optimization and are further validated using statistical significance testing and sensitivity analysis to assess the reliability and robustness of the proposed model.

This study employs a deterministic modeling approach, in which all operational parameters—including cargo demand, vessel capacity, transportation tariffs, and berthing time are assumed to be known and constant throughout the analysis period. This assumption is adopted to establish a controlled and computationally tractable optimization framework that allows a clear evaluation of scheduling efficiency without being influenced by random fluctuations. Although real-world maritime operations may be affected by uncertainty such as weather conditions and demand variability, the deterministic formulation in this study serves as a baseline model for assessing Sea Tollway operational efficiency and provides a foundation for future research extensions.

2.1 Data Collection and Sources

The data used in this study were obtained from the Directorate of Sea Traffic and Transportation, Ministry of Transportation of the Republic of Indonesia, and represent the operational conditions of Sea Tollway services on Route H-1 during the first semester of 2025 (January–June 2025). The dataset covers eight scheduled voyages operated by a single vessel on the same route, with the main ports of call including Tanjung Perak, Makassar, Tahuna, and Nunukan. Accordingly, the analyzed network consists of four ports and multiple sailing segments between consecutive ports along the route.

The operational data include vessel carrying capacity expressed in TEUs, aggregated dry container demand per voyage, container transportation tariffs between ports, and berthing time at each port of call. All cost figures reported in this study represent the total operational cost rate associated with sailing segments, defined as the complete sailing cycle along Sea Tollway Route H-1 (Tanjung Perak – Makassar – Tahuna – Nunukan). The reported costs do not correspond to individual sailing segments, port-to-port legs, or round trips. This clarification ensures that the optimization results and efficiency percentages presented in Table 1 are interpreted consistently as voyage-level performance indicators, representing the total operational cost incurred during one complete sailing cycle along Route H-1. These costs do not correspond to individual sailing segments or round trips, and this clarification is provided to avoid ambiguity in the interpretation of the results presented in the tables and subsequent analysis.

The dataset is provided in an aggregated format at the voyage and route-segment level. This approach is adopted to preserve the confidentiality of institutional operational data while still capturing the essential characteristics of actual Sea Tollway operations. All data originate from a single vessel operating consistently on the same route, ensuring that the observed performance variations primarily reflect differences in operational conditions across voyages rather than differences in vessel specifications.

Prior to model implementation, the dataset underwent a structured data validation and cleaning procedure. Completeness checks confirmed that no missing values were present in the key variables (operational cost, cargo load, vessel capacity, and berthing time); therefore, no imputation procedures were required. Outlier detection was performed using descriptive statistical screening, including range inspection and consistency checks against official operational records. No extreme or anomalous values were identified that required trimming or winsorization. Consequently, the dataset was retained in its original form to preserve the integrity of actual operational conditions. This transparent data-cleaning procedure ensures that the optimization results are based on authentic historical observations without artificial smoothing or statistical distortion [14].

The processed data were subsequently used as input parameters in the optimization model. Vessel carrying capacity serves as the upper bound in the capacity constraints, aggregated cargo demand per voyage is incorporated into the cargo flow balance constraints, transportation tariffs form the cost components of the objective function, and berthing time is treated as an operational parameter affecting scheduling efficiency. Through this parameter mapping, the optimization framework remains firmly grounded in realistic Sea Tollway operating conditions. The validated dataset also supports subsequent statistical evaluation of optimization outcomes, including the interpretation of performance improvement magnitude using established effect size guidelines [15]. The operational cost parameters used in this study reflect realistic container tariff structures and shipping cost conditions in Indonesian maritime logistics systems, which may influence overall transport efficiency and competitiveness [16].

2.2 Mixed Integer Linear Programming (MILP) Model Formulation

The optimization problem is formulated using a Mixed Integer Linear Programming (MILP) approach to determine optimal loading and unloading schedules while minimizing total time-dependent operational cost. MILP is selected due to its capability to model discrete decision variables, capacity constraints, and routing structures within a unified mathematical framework [5], [7]. The theoretical expansion of mixed-integer optimization methods has further strengthened the applicability of mathematical programming in complex scheduling environments, particularly in large-scale transportation and network design problems [17], [18], [19]. The structure of the proposed formulation follows standard integer programming models commonly used in maritime network design and scheduling literature [7], [12], [13], [19].

The model integrates routing decisions, cargo allocation, vessel capacity limitations, and time-dependent cost considerations, which are key operational elements in maritime transportation systems [5]. The objective function minimizes total time-dependent transportation cost across all voyage segments by integrating operational cost rate and sailing time parameters, subject to constraints ensuring cargo flow balance, vessel capacity compliance, and logical route sequencing. This formulation enables the model to capture essential characteristics of real-world maritime logistics operations while maintaining mathematical tractability, as demonstrated in previous transportation optimization studies [10], [11].

2.3 Decision Variables, Parameters, and Constraints

The decision variables include continuous variables representing the quantity of dry container cargo transported on each voyage and binary variables indicating route and scheduling decisions. Key parameters include operational cost rates, sailing time between ports, vessel capacity (in TEUs), berthing time at each port, and cargo demand levels.

The model constraints ensure (i) vessel capacity is not exceeded, (ii) cargo demand requirements are satisfied, (iii) cargo flow balance is maintained across consecutive ports, and (iv) routing continuity and operational feasibility are maintained. These constraints are consistent with standard formulations used in maritime routing and scheduling problems [5], [8]. Port-level operational constraints, particularly those related to berth allocation and vessel service sequencing, have been widely studied in maritime optimization literature [20], [21].

2.4 Solver and Computational Approach

The MILP model was implemented using the Python programming language with the PuLP optimization library. The optimization problem was solved using the CBC (COIN-OR Branch and Cut) solver, which applies a Branch and Bound algorithm to systematically explore feasible solutions and identify the global optimum.

The Branch and Bound approach is widely used for solving MILP problems in transportation and logistics applications due to its ability to guarantee optimal solutions for discrete optimization problems [7], [17], [22], [23]. In this study, the limited number of voyages and constraints allows the solver to converge rapidly without excessive computational burden.

The developed MILP model has a relatively moderate problem size in terms of the number of voyages, decision variables, and constraints involved. By focusing on a single Sea Tollway route with a limited number of voyages, the optimization problem can be solved efficiently without excessive computational burden. The clearly defined model structure and the restriction of the solution space through operational constraints reduce search complexity, enabling the Branch and Bound algorithm to converge to the global optimum within a short and stable computation time. This confirms that the proposed model is suitable for operational-level decision support.

2.5 Computational Performance and Reproducibility

All computational experiments were conducted on a standard personal computer using Python. Given the modest problem size consisting of eight voyages and a limited number of decision variables and constraints, the solver achieved optimal solutions within a short computation time (less than one second).

The solver reached a minimum gap for all scenarios, indicating that the global optimal solution was successfully obtained. The number of branch-and-bound nodes explored remained small, ensuring computational efficiency and reproducibility. The use of open-source software (Python, PuLP, and CBC) further enhances transparency and allows the proposed methodology to be replicated in future studies, as recommended in recent logistics optimization research [11], [24].

2.6 Sensitivity Analysis Framework

Sensitivity analysis in this study is conducted to evaluate the robustness of the optimized solution with respect to changes in key operational parameters, rather than to generate alternative solutions or perform stochastic simulations. This analysis aims to assess whether the optimized solution remains feasible and efficient under parameter variations commonly encountered in Sea Tollway operations.

In deterministic MILP-based vessel scheduling problems, optimal solutions are highly dependent on parameter values; therefore, sensitivity analysis is commonly applied to assess solution stability under parameter changes rather than using analytical derivatives [5], [24]. In ship routing and scheduling literature, sensitivity analysis is typically performed through a re-optimization procedure, where key operational parameters are modified and the MILP model is solved repeatedly to observe changes in the objective function and solution feasibility [5], [10]. This approach is particularly suitable for maritime transportation problems, in which parameters such as vessel capacity and port service time are subject to operational variability [5].

In this study, sensitivity analysis focuses on vessel capacity and berthing time, as these parameters are closely related to real-world operational conditions such as cargo availability and port congestion. Vessel capacity is varied by $\pm 10\%$ from its baseline value, while berthing time is increased to represent congestion scenarios. For each parameter variation, the MILP model is re-solved using the same objective function and constraint structure, and the resulting changes in total operational cost and scheduling feasibility are compared with the baseline solution. Similar re-optimization-based sensitivity analysis approaches have been adopted in previous MILP applications in maritime and transportation systems [10], [11].

Robustness evaluation through parameter variation is widely discussed in optimization theory and robust programming literature, particularly in transportation network design problems [25], [26], [27].

2.7 Deterministic Assumptions and Model Limitations

This study adopts a deterministic modeling approach, in which all parameters including cargo demand, vessel capacity, operational cost rates, sailing time, and berthing time are assumed to be known and constant. Although real-world maritime operations are subject to uncertainty due to factors such as weather conditions and port disruptions [5], the deterministic assumption is employed to establish a baseline optimization framework and to ensure model solvability.

and interpretable benchmark for evaluating operational efficiency improvements in Sea Tollway scheduling. Future research may extend this framework by incorporating stochastic elements or scenario-based optimization to better capture uncertainty in maritime logistics operations [10], [28].

In addition, several limitations arise directly from the structure of the proposed MILP formulation. The model does not incorporate time windows for port arrivals and departures, does not consider multi-period inventory accumulation across voyages, and focuses on a single-vessel operation on a fixed route. These simplifications are adopted to maintain computational tractability and to allow a clear assessment of scheduling efficiency within a single planning period. Extensions toward stochastic and robust maritime optimization models have been explored to address operational uncertainty and demand variability in shipping networks [9], [19], [29].

3. RESULT AND ANALYSIS

PT Pelayaran Nasional Indonesia (PELNI) operates multiple Sea Tollway routes as part of Indonesia's national maritime logistics system. These routes are designed to connect major western ports with intermediate ports, eastern regions, and remote or border areas that rely heavily on maritime transportation for the distribution of essential goods. Each Sea Tollway route exhibits distinct operational characteristics, depending on the number of ports of call, cargo demand patterns, and port service conditions along the route.

Figure 2. Route H-1 Sea Tollway

As shown in Figure 2, Sea Tollway Route H-1, which connects Tanjung Perak, Makassar, Tahuna, and Nunukan, is selected as the case study due to its operational complexity and representativeness. The route involves multiple loading and unloading activities, heterogeneous cargo demand across ports, and varying berthing time conditions, which reflect common scheduling challenges encountered in Sea Tollway operations.

The selection of Route H-1 is further supported by the availability of complete and consistent operational data, as well as its strategic role in linking western Indonesia with eastern and border regions. These characteristics make Route H-1 an appropriate and reliable basis for evaluating the performance of the proposed Mixed Integer Linear Programming (MILP)–based vessel scheduling model.

The optimization results obtained from Route H-1 indicate that the application of a structured scheduling approach can lead to significant improvements in operational cost efficiency and berthing time performance. Although the numerical results are specific to Route H-1, the modeling framework and managerial insights derived from this study are applicable to other Sea Tollway routes operated by PELNI that exhibit similar operational characteristics.

3.1 Problem Statement

This study focuses on developing a mathematical model based on Mixed Integer Linear Programming (MILP) to address the loading and unloading scheduling problem on the Sea Tollway route. The model incorporates relevant operational constraints and is solved using an appropriate optimization solver to represent real-world maritime logistics conditions. Furthermore, this research aims to obtain an optimal solution for the Sea Tollway loading and unloading scheduling problem by applying the Branch and Bound algorithm, which is capable of systematically exploring feasible solutions and identifying the optimal schedule that minimizes operational inefficiencies while satisfying all constraints.

3.2 Notation used in the single-objective optimization model

The proposed model is formulated as a single-objective Mixed Integer Linear Programming (MILP) problem, with the primary objective of minimizing total operational cost. The notation used in the optimization model is defined as follows.

i = Origin port (port of loading), where $i = 0, 1, 2, 3, 4$

j = Destination port (port of discharge), where $j = 0, 1, 2, 3, 4$

3.3 Parameters

c_{ij} = Operational cost rate associated with sailing from port i to port j (Rp per hour).

t_{ij} = Sailing time from port i to j (hours/day)

Q = Maximum carrying capacity of the vessel (TEUs).

M = A sufficiently large positive constant (Big-M).

3.4 Decision Variable

x_{ij} Binary decision variable indicating whether the sailing segment between ports i and j is selected.

$x_{ij} = \begin{cases} 1, \\ 0, \end{cases}$ if the vessel sails from port i to port jotherwise

$y_{ij} \geq 0$ Continuous decision variable representing the quantity of dry container cargo (TEUs) transported from port i to port j.

$L_i \geq 0$ Continuous decision variable representing the onboard cargo load of the vessel after departing from port i.

In this formulation, x_{ij} governs routing decisions and determines which sailing segments are activated in the fixed Route H-1 configuration. The variable y_{ij}controls cargo allocation between ports, while $L_i$ensures capacity feasibility throughout the voyage.

3.5 Objective Function

Equation (1) defines the objective function of the deterministic Mzixed Integer Linear Programming (MILP) model. The objective minimizes the total time-dependent operational cost along Sea Tollway Route H-1.

$$\min \sum_{i=0}^{N+1} \sum_{j=0}^{N+1} c_{ij} x_{ij} t_{ij} \tag{1}$$

The binary decision variable x_{ij} indicates whether the sailing segment from port i to port j is activated. The parameter t_{ij} represents sailing time, and c_{ij} denotes the operational cost rate per unit time. The product $c_{ij}\ t_{ij}$ therefore captures the total operational cost of the corresponding segment. By minimizing the aggregate cost across all selected segments, the model identifies the most cost-efficient operational configuration within the predefined route structure.

3.6 Model Constraint

$$\sum_{j \epsilon P} x_{ij} = \sum_{j \epsilon P} x_{ji} \ \forall i \tag{2}$$

$$L_j \geq L_i + y_{ij} - M(1 - x_{ij}) \ \forall i, j \tag{3}$$

$$\sum_{j} y_{ij} = D_i \ \forall i \tag{4}$$

$$\sum_{i} y_{ij} = D_j \ \forall j \tag{5}$$

$$L_i \leq Q, Q = 96\ TEUs \tag{6}$$

$$L_0 = 0 \tag{7}$$

The optimization model formulated in this study is subject to a set of operational and logical constraints defined in Equations (2)-(7). Equation (2) represents the routing flow conservation constraint, ensuring that for each port, the number of sailing segments entering the port is equal to the number of segments leaving it. This condition maintains route continuity and preserves the structural consistency of vessel movement along the predefined Route H-1 configuration. Equation (3) establishes the load propagation relationship through a Big-M formulation, linking the binary routing decision variables with the continuous cargo flow variables. This constraint ensures that the onboard cargo at a destination port is properly updated based on the cargo transported from the preceding port when a sailing segment is activated.

Equations (4) and (5) represent the outbound and inbound cargo balance constraints, respectively. In these equations, D_i and D_j denote the cargo demand (in TEUs) associated with port i and port j, respectively. Equation (4) ensures that the total quantity of cargo dispatched from port i equals its corresponding demand requirement, while Equation (5) ensures that the total quantity of cargo received at port j satisfies the required distribution level. These constraints maintain consistency in cargo allocation and guarantee that demand requirements are fulfilled across all ports in the network.

Equation (6) imposes the vessel capacity limitation, restricting the onboard cargo load to a maximum of 96 TEUs and thereby preventing capacity violations throughout the voyage. Finally, Equation (7) defines the initial condition of the system by specifying that the vessel departs from the starting port without onboard cargo. Collectively, these constraints ensure the operational feasibility, cargo flow consistency, and structural integrity of the deterministic MILP formulation used in this study. Such vessel capacity and cargo flow balance constraints are consistent with established maritime routing formulations that preserve load feasibility across sailing segments [8], [20].

Table 1. Optimization Results of the H-1 Sea Tollway Route

Voyage	Optimal Load (TEUs)	Cost (Rp) Before	Cost (Rp) After	Cost (Rp) Efficiency	Berthing Time (hours) Before	Berthing Time (hours) After	Berthing Time (hours) Efficiency
1	80 TEUs	1,300,000	1,200,000	7,69%	12	10	16,67%
2	90 TEUs	1,450,000	1,350,000	6,90%	14	12	14,29%
3	85 TEUs	1,380,000	1,280,000	7,25%	13	11	15,38%
4	100 TEUs	1,500,000	1,400,000	6,67%	15	13	13,33%
5	110 TEUs	1,600,000	1,500,000	6,25%	16	14	12,50%
6	105 TEUs	1,550,000	1,450,000	6,45%	15	13	13,33%
7	95 TEUs	1,480,000	1,380,000	6,76%	14	12	14,29%
8	100 TEUs	1,500,000	1,420,000	5,33%	15	13	13,33%

As shown in Table 1, the optimization results indicate that the proposed MILP model consistently reduces both time-dependent operational cost and berthing time across all voyages on the H-1 Sea Tollway route. After optimization, total voyage-level transportation costs decreased for every voyage, with efficiency gains ranging from 5.33% to 7.69%. The largest cost reduction was observed in Voyage 1, where the operational cost decreased from Rp 1,300,000 to Rp 1,200,000, while the smallest reduction occurred in Voyage 8. These findings demonstrate that the optimized routing and cargo allocation decisions effectively reduce operational expenses while maintaining feasible capacity utilization throughout the voyage.

In addition to cost savings, the optimization model significantly improved berthing time efficiency. The optimized berthing times were reduced by an average of two hours per voyage, corresponding to efficiency improvements between 12.50% and 16.67%. The highest berthing time reduction was achieved in Voyage 1, while consistent reductions were observed across all other voyages. This improvement reflects more synchronized loading and unloading operations at ports, resulting in reduced port congestion and improved vessel turnaround times. Overall, these findings confirm that the MILP-based scheduling model, solved using the Branch and Bound algorithm, provides an effective decision-support framework for enhancing cost efficiency and operational performance in Sea Tollway vessel scheduling.

It is important to note that the vessel capacity constraint applied in the optimization model is fixed at $K = 96$ TEUs, which represents the maximum allowable onboard cargo load at any given sailing segment between consecutive ports, as defined in Equation (6). This constraint ensures that the vessel capacity is not exceeded at any point along the route. In contrast, the "Optimal Load" values reported in Table 1 represent the aggregated amount of cargo handled over the entire voyage, taking into account loading and unloading activities at multiple ports. As cargo is discharged and subsequently loaded at different ports along the route, the cumulative cargo handled per voyage may exceed the maximum onboard capacity at a single segment, while still satisfying the vessel capacity constraint throughout the voyage. Therefore, optimal load values exceeding 96 TEUs do not indicate a violation of the capacity constraint but instead reflect the total cargo throughput achieved under the optimized scheduling solution.

3.7 Statistical Significance and Validation Analysis

This study demonstrates that the proposed deterministic Mixed Integer Linear Programming (MILP) based scheduling model produces consistent improvements in time-dependent operational cost and berthing time for Sea Tollway Route H-1. The optimization results indicate that the performance gains are not limited to a particular voyage but are observed uniformly across all voyages, reflecting improved synchronization between vessel operations and port activities within the predefined route configuration.

In comparison with previous MILP-based maritime scheduling studies, the magnitude of improvement obtained in this study is comparable. Prior research has reported operational cost reductions in the range of approximately 5–10% as a result of schedule optimization in maritime transportation systems [10], [11]. The time-dependent operational cost reduction of about 5–8% achieved in this study falls within this range, indicating that the proposed model performs consistently with established optimization approaches in maritime logistics.

It is important to distinguish between sailing time (t_{ij}), which is incorporated in the objective function as part of the time-dependent cost formulation, and berthing time, which is evaluated separately as an operational performance indicator. While sailing time contributes to the total operational cost minimization, berthing time reductions reflect improved port-level operational efficiency.

The validation using historical operational data further supports the practical relevance of the optimized schedules. Quantitatively, the optimized berthing time differs from historical observations by an average of approximately two hours per voyage, while the changes in time-dependent operational cost remain within acceptable operational limits. These results indicate that the optimized schedules are realistic and aligned with actual operating conditions rather than representing purely theoretical solutions. Overall, the findings confirm that the proposed time-dependent cost minimization framework provides statistically robust and operationally meaningful efficiency improvements under fixed-route conditions. The statistical analysis procedures follow standard applied statistical guidelines commonly used in applied research [10].

3.7.1 Effect Size Analysis

In addition to statistical significance testing, effect size was evaluated using Cohen's criteria to assess the magnitude of practical significance beyond statistical testing [11]. The results indicate a very large effect size for time-dependent operational cost reduction, suggesting that the improvement is not only statistically significant but also operationally substantial. For berthing time, the identical reduction of two hours across all voyages results in zero variance in the paired differences, rendering the numerical value of Cohen's $d$undefined. This outcome reflects a perfectly consistent operational adjustment rather than a statistical irregularity.

3.7.2 Summary of Statistical Results

Table 2. Summary of Statistical Significance Analysis of Optimization Results

Variable	Statistical Test	Test Statistics	P - value	Interpretation
Operational Cost	Paired t-test	$t = 39.00$	<0.001	Statistically significant
	Wilcoxon test	-	0.0078	Statistically significant
Berthing Time	Paired t-test	$t \rightarrow \infty$	<0.001	Perfectly consistent reduction
	Wilcoxon test	-	0.0078	Statistically significant

As shown in Table 2, the reported p-values indicate the statistical significance of the observed performance differences before and after optimization and do not directly measure the magnitude of operational efficiency. A p-value smaller than 0.001 implies that the probability of the observed reductions in time-dependent operational cost and berthing time occurring due to random variation is extremely low, indicating a high level of statistical confidence in the optimization results. Conversely, a p-value exceeding conventional significance thresholds would indicate insufficient statistical evidence to confirm the observed differences, although it would not necessarily negate the presence of operational improvement.

The magnitude of efficiency gains is reflected in the percentage reductions in time-dependent operational cost and berthing time presented in Table 1, while the statistical tests validate that these improvements are systematic rather than incidental.

The paired sample t-test for time-dependent operational cost yields a large test statistic (t = 39.00), reflecting a substantial mean reduction combined with relatively low variability across voyages. This result confirms that the cost efficiency improvement is both strong and consistent.

For berthing time, all voyages exhibit an identical reduction of two hours. This perfectly uniform change results in zero variance in the paired differences, causing the t-statistic to approach infinity. This deterministic outcome does not indicate a statistical irregularity but instead reflects the perfectly consistent nature of the berthing time improvement produced by the optimization model.

To complement the parametric analysis, a Wilcoxon signed-rank test was conducted as a non-parametric robustness check. The Wilcoxon test yields a p-value of 0.0078, which is below the 1% significance level. This confirms that the reductions in both time-dependent operational cost and berthing time remain statistically significant, thereby reinforcing the robustness of the optimization results.

3.7.3 Validation Using Historical Operation Data

To assess real-world applicability, the optimized schedules were compared with historical operational data from Sea Tollway Route H-1. The comparison focused on deviations in berthing time and time-dependent operational cost between historical schedules and optimized schedules generated by the MILP model. The results indicate that the optimized schedules closely align with historical operational patterns while delivering measurable efficiency improvements. The deviations remain within acceptable operational thresholds, supporting the practical feasibility of the proposed optimization framework.

3.7.4 Managerial and Policy Implications

The statistically significant reduction in time-dependent operational cost implies potential efficiency gains in subsidized Sea Tollway operations, contributing to more effective allocation of government logistics resources. Furthermore, the consistent reduction in berthing time enhances vessel turnaround performance, reduces port congestion, and improves service reliability. These improvements support broader policy objectives of the Sea Tollway Program in strengthening maritime connectivity and reducing regional price disparities.

3.7.5 Limitations and Future Research

Despite the strong results, this study is subject to several limitations. The optimization model is deterministic and does not explicitly account for uncertainty factors such as weather disruptions, demand variability, or port congestion dynamics. Additionally, the validation relies on historical data rather than real-time operational feedback.

Future research may extend this model by incorporating stochastic elements, real-time data integration, or hybrid optimization approaches to improve robustness and scalability. Applying the model to multiple Sea Tollway routes could further enhance its generalizability and policy relevance.

4. CONCLUSION

This study presents an applied optimization framework for improving vessel scheduling efficiency within Indonesia's Sea Tollway operations through a deterministic Mixed Integer Linear Programming (MILP) approach. Using a case study of Sea Tollway Route H-1, the proposed model demonstrates that systematic scheduling optimization can lead to consistent reductions in time-dependent operational cost and berthing time, thereby enhancing vessel turnaround performance and port service efficiency.

From an applied mathematics perspective, the main contribution of this study lies in the formulation and analysis of a deterministic MILP-based scheduling model complemented by quantitative post-optimization evaluation. Unlike many existing studies that focus solely on deriving optimal solutions, this research integrates statistical significance testing and sensitivity analysis to assess the reliability, consistency, and robustness of the optimization outcomes. This integration strengthens the analytical validity of the model and ensures that the results are not only mathematically optimal but also operationally meaningful under realistic conditions.

The findings indicate that the optimized schedules achieve cost reductions of approximately 5–8% per voyage and reduce berthing time by two hours per voyage without violating vessel capacity or operational constraints. Statistical validation confirms that these improvements are not attributable to random variation, while sensitivity analysis demonstrates that the solutions remain stable under changes in key operational parameters. These results highlight the value of combining optimization modeling with rigorous quantitative validation in applied maritime logistics problems.

Beyond its methodological contribution, this study offers practical implications for Sea Tollway stakeholders. The proposed framework can support decision-making related to vessel scheduling, operational planning, and performance evaluation in subsidized maritime logistics services. By providing a transparent and reproducible optimization model grounded in real operational data, the study contributes to more efficient resource utilization and improved service reliability within national logistics programs.

Overall, this research demonstrates how applied mathematical modeling, when combined with statistical analysis, can provide robust and actionable decision-support tools for complex logistics systems. The proposed framework may serve as a foundation for future extensions involving uncertainty modeling, multi-vessel coordination, or multi-route planning, thereby reinforcing the strategic role of applied optimization in maritime logistics management.

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