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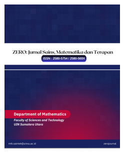

Numerical Pricing of European Options under Proportional Transaction Costs: A Semi-Discretization Approach to the Nonlinear Barles-Soner Model

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ABSTRACT

The classical Black–Scholes model assumes a frictionless market, which often leads to the undervaluation of option premiums when transaction costs are present. This study prices European call options under proportional transaction costs using the nonlinear Barles–Soner framework and a semi-discretization-based numerical approach. Using historical stock data from PT XYZ (an anonymized Indonesian equity), the results show that transaction costs significantly increase effective volatility and generate systematic deviations from classical Black–Scholes prices. In particular, option premiums increase by IDR 392.33 and IDR 776.66 for transaction cost parameters of 0.015 and 0.030, respectively, compared with the frictionless benchmark. These findings confirm that ignoring transaction costs leads to substantial underpricing and that the proposed framework provides a more realistic and conservative valuation for hedging and risk management in emerging markets.

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1. INTRODUCTION

In modern financial markets, derivative instruments, particularly European options, play a pivotal role in hedging and speculation. Since its introduction in 1973, the Black–Scholes (B–S) model has served as the standard framework for option pricing [1]. This model provides analytical solutions under ideal market assumptions, including constant volatility and frictionless trading. However, these assumptions are often violated in real market conditions, especially due to transaction costs arising from periodic portfolio rebalancing [2], [3].

Ignoring transaction costs can lead to significant pricing inaccuracies and discrepancies between theoretical values and actual market prices. To address this limitation, several studies have proposed modifying the pricing framework by incorporating effective volatility [2], [4]. A major advancement was introduced by Barles and Soner (1998), who formulated a nonlinear Black–Scholes equation based on an exponential utility approach, in which volatility becomes a function of the option price's second derivative.

However, the nonlinearity of the Barles–Soner model precludes closed-form analytical solutions and necessitates numerical approximation. Standard explicit finite difference methods are commonly employed, but they often suffer from strict stability conditions and convergence difficulties, particularly under the strong nonlinearity induced by transaction costs. [7] & [11] subsequent studies, such as [9], developed stable and consistent numerical schemes, these works primarily focused on methodological aspects using synthetic data or stylized parameter settings. Other investigations, including [12], explored alternative finite difference strategies and stability improvements, yet remained within abstract numerical frameworks without direct application to real market data.

Most of the existing literature concentrates on developed financial markets or purely numerical benchmarks. There is limited empirical evidence on how the Barles–Soner model behaves when applied to emerging markets, where transaction costs, liquidity frictions, and discrete trading effects are often more pronounced. Consequently, the practical relevance of nonlinear option pricing models for markets such as Indonesia remains underexplored.

To bridge this gap, this study applies a semi-discretization approach based on the Method of Lines to the nonlinear Barles–Soner equation and implements it using historical stock data from PT XYZ. Unlike previous works that primarily emphasize methodological development, this research focuses on empirical deployment and interpretation in an emerging-market setting.

The objective of this study is to evaluate European call option prices under proportional transaction costs using a robust numerical framework and to quantify the deviation from the classical Black–Scholes model when applied to Indonesian stock data. By doing so, this work demonstrates how neglecting transaction costs can lead to systematic underpricing and highlights the practical relevance of nonlinear option pricing models for risk management in emerging markets.

2. RESEARCH METHOD

2.1 Research Flow

This study is applied with an applied quantitative approach that focuses on the determination of European option pricing with consideration of transaction costs through the application of the semi-discretization method on the Barles-Soner nonlinear model (Barles and Soner, 1998; [7]. The research process is carried out systematically through five main stages, namely literature review, stock data collection, data processing and parameter estimation, mathematical model development, and numerical option assessment [7]. The research framework is displayed more clearly through the flowchart presented in Figure 1.

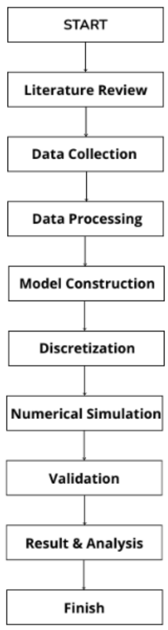

Figure 1. Research Flow

The research procedure is carried out by being able to follow the systematic workflow described in Figure 1. The study began with a literature review regarding option pricing and numerical methods then continued with the collection of data on the daily closing price of PT XYZ shares. Raw data is processed to estimate historical volatility parameters. Furthermore, the mathematical model is constructed using the Barles-Soner framework and solved numerically through the Difference Method with a semi-discretized scheme. The numerical model is first validated by providing a comparison of the results against the Black-Scholes analytical solution before it is used to simulate the option price that takes into account transaction costs [1], [7].

In addition to the procedural workflow, particular attention is given to the selection of numerical discretization parameters, as they directly affect the stability and accuracy of the simulation results.

In numerical simulations, the choice of spatial and temporal discretization plays a crucial role in ensuring stability and accuracy. Therefore, the grid parameters in this study are not selected arbitrarily. Several combinations of spatial grid sizes and time steps are tested in preliminary experiments. The spatial domain of the stock price is truncated to a sufficiently large interval [0, S_max], where S_max is chosen so that further enlargement of the domain produces no visible change in the computed option price.

For each domain setting, multiple pairs of $(\Delta S, \Delta t)$ are examined. The numerical scheme is considered acceptable when: (i) the solution remains stable over the entire time horizon, and (ii) further refinement of either the spatial grid or the time step results in only negligible changes in the option price. This empirical convergence criterion is adopted because explicit theoretical stability bounds for the nonlinear Barles–Soner equation are difficult to derive.

The final discretization parameters reported in this study correspond to the smallest grid sizes for which the numerical solution becomes grid-independent within a prescribed tolerance. This procedure ensures that the reported option prices are not artifacts of a particular discretization choice, but reflect stable and convergent numerical behavior.

2.2 Data Processing and Parameter Estimation

The primary data used in this study consist of the daily closing stock prices of PT XYZ, an anonymized Indonesian listed company, obtained from the Yahoo Finance database [16]. The dataset covers a one-year trading period from December 30, 2024, to December 24, 2025. This study focuses on a single underlying asset in order to provide a clear and controlled numerical illustration of the Barles–Soner model. Consequently, the results are asset-specific and should not be interpreted as universally representative of all stocks or market conditions.

To implement the Barles–Soner framework, the historical volatility parameter, which represents the magnitude of asset price fluctuations, must be estimated from the raw data. First, the daily logarithmic return is computed using the standard formula described in [17], as given in Equation (1). The standard deviation of the daily returns is then calculated and annualized using Equation (2) to obtain the baseline volatility σ_0.

The transaction cost parameter $a$in the Barles–Soner model is not directly observable in market data and is therefore treated as an exogenous control parameter. In this study, several representative values of $a$are considered to reflect different market frictions, namely the frictionless case ($a = 0$), a low transaction cost scenario, and a high transaction cost scenario. This scenario-based calibration follows common practice in the numerical literature on nonlinear Black–Scholes equations and allows the qualitative impact of transaction costs to be examined.

It is important to note that the present framework focuses solely on proportional transaction costs and does not explicitly account for other market frictions such as bid–ask spreads, liquidity constraints, or discrete trading effects. Moreover, robustness across different time windows or market regimes is not explored. These simplifications are adopted to maintain a tractable numerical setting and to highlight the core effect of transaction costs within the Barles–Soner model.

The daily logarithmic return is computed to measure relative price changes between consecutive trading days. The log-return for day $t$is defined by Equation (1).

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

Where S_t denotes the closing stock price on day and S_{t-1} is the closing price on the previous day. A sample of the daily log-return computation based on Equation (1) is presented in Table 1 using actual stock price data.

Table 1. Sample of Daily Stock Return Calculation for PT XYZ

Date	Closing Price (S_t)	Previous Price (S_{t-1})	Log Return $r_t = \ln(S_t/S_{t-1})$
30-Dec-2024	9675	9800	−0.0128
02-Jan-2025	9900	9675	0.0230
03-Jan-2025	9850	9900	−0.0051
06-Jan-2025	9675	9850	−0.0179
07-Jan-2025	9525	9675	−0.0156

After obtaining the daily returns, the standard deviation (s) is computed to measure the dispersion of the asset's price movements. The annualized volatility is computed to measure the dispersion of the asset's price movements [17]. The annualized volatility (σ) is then derived by scaling the standard deviation with the square root of the number of trading days in a year $(k \approx 252)$, following the volatility estimation technique in option pricing literature [16].

$$\sigma = s\sqrt{252} \tag{2}$$

Based on the computational results from the dataset, the annualized volatility is estimated at 29.49%. This value, along with the latest closing price (S_0) of IDR 8,025 and a risk-free interest rate (r) of 6% , serves as the baseline input for the numerical simulation.

2.3 Mathematical Model

2.3.1 Non-Linear Blach-Scholes Equation

The classical Black–Scholes model [9] derived under the assumption of a frictionless market with constant volatility and continuous hedging. To incorporate the influence of transaction costs, Barles and Soner (1998) introduced a nonlinear adjustment based on the principle of utility maximization. The modified partial differential equation (PDE) governing the price $V(S,t)$ of a European call option is given by Equation (3):

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2(S,t,V_{SS})S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{3}$$

where :
S : Underlying Asset Price,
t : time,
r : risk-free interest rate,
$\sigma(S,t,V_{SS})$: Nonlinear Volatility
$V_{SS} = \frac{\partial^2 V}{\partial S^2}$ denotes the gamma of the option

Equation (3) shows that the evolution of the option price is influenced by both the stochastic diffusion component arising from stock price fluctuations and the deterministic growth induced by the risk-free interest rate and discounting effects.

2.3.2 Volatility as a Representation of Transaction Costs

In the Barles–Soner model, volatility σno longer remains constant but becomes a nonlinear function of the second derivative of the option price (V_{SS}). Transaction costs are incorporated through an adjustment of volatility, which is expressed by Equation (4):

$$\sigma^2 = \sigma_0^2\left(1 + \Psi\left(e^{r(T-t)}a^2S^2V_{SS}\right)\right) \tag{4}$$

where :
σ_0 is the basic volatility without transaction costs,
$a = \mu\sqrt{\gamma N}$ is a parameter of transaction cost intensity and risk aversion,
$\Psi(\cdot)$ is a volatility correction function that satisfies an implicit nonlinear differential equation.

In the numerical implementation, the volatility correction function $\boldsymbol{\Psi}(\cdot)$is not solved as a continuous differential problem, but evaluated pointwise using the standard iterative procedure proposed by Barles and Soner (1998) and adopted in subsequent numerical studies [7]. At each time step and spatial node, the argument of $\boldsymbol{\Psi}$ is computed from the current gamma approximation, and the corresponding corrected volatility is obtained through a fixed-point iteration until convergence within a prescribed tolerance. In practice, only a few iterations are required, and the procedure remains computationally efficient while preserving the nonlinear structure of the model

2.3.3 Transformation of Initial Value Problems

To simplify the numerical problem solving, the boundary value problem (because the payoff condition is known at maturity $t = T$) is transformed into an initial value problem , as commonly applied in numerical solutions of nonlinear Black–Scholes equations using the semi-discretization approach [7]. The transformation is performed using the time variable $\tau = T - t$ and the function $U(S,\tau) = V(S,t)$ Which allows the problem to be solved backward in time in a stable numerical framework [7], [12].

Under this transformation, the governing equation can be rewritten in the form given by Equation (5):

$$U_\tau = \frac{S^2}{2}\sigma^2 U_{SS} + rSU_S - rU \tag{5}$$

For domain and $S > 0$ and $0 < \tau \leq T$

2.3.4 Initial and Boundary Conditions

Initial Conditions (Payoff Option)

Since the instrument analyzed in this study is a European call option, the option value at maturity depends solely on the difference between the stock price and the strike price. The payoff function at maturity is defined by Equation (6), following the standard Black–Scholes framework [1]:

$$V(S,T) = \max(S - E, 0) \tag{6}$$

where Edenotes the strike price

This condition is used as a starting point in numerical simulation because the calculation of the solution can be done backwards from the due date to the initial time.

Stock Price Limit Conditions

To ensure that the numerical solution remains realistic, boundary conditions are imposed on the stock price domain. As the stock price approaches zero, the option value should also approach zero. Conversely, when the stock price becomes sufficiently large, the option value should converge to its intrinsic value. These conditions are expressed by Equations (7) and (8), respectively, in accordance with standard practice in numerical option pricing [9], [10] :

$$V(0,t) = 0,\ \ 0 \leq t \leq T, \tag{7}$$

$$\lim_{S\to\infty} V(S,t) = S - Ee^{-r(T-t)},\ \ 0 \leq t \leq T \tag{8}$$

The right boundary condition is designed to be a representation of the asymptotic behavior of the option price. When the stock price is close to zero, the value of the option will have a tendency to be zero, while when the stock price increases to high enough, the value of the option will be close to its intrinsic value. The application of these conditions aims to ensure that the resulting numerical solutions remain in harmony with economically and financially reasonable behavior.

2.4 Numerical Method

2.4.1 Finite Difference Spatial Discretization

The spatial discretization of the governing equation is performed by approximating partial derivatives using Taylor series expansions [7], [11]. This approach, known as the Finite Difference Method, converts the continuous spatial domain into a set of discrete grid points so that the partial differential equation can be represented in algebraic form and solved numerically [11].

Let $h = \Delta S$ denote the grid spacing between adjacent spatial points. The Taylor series expansion of the function U around the point S_i are given by Equations (9) and (10):

$$U(S_{i+1}) = U(S_i) + h\frac{\partial U}{\partial S} + \frac{h^2}{2!}\frac{\partial^2 U}{\partial S^2} + O(h^3) \tag{9}$$

$$U(S_{i-1}) = U(S_i) - h\frac{\partial U}{\partial S} + \frac{h^2}{2!}\frac{\partial^2 U}{\partial S^2} - O(h^3) \tag{10}$$

By eliminating the first-derivative terms in Equations (9) and (10), second-order central difference schemes for the first and second spatial derivatives can be derived. The approximation for the first derivative is given by Equation (11):

$$\frac{\partial U}{\partial S}(S_i,\tau) \approx \frac{u_{i+1}(\tau) - u_{i-1}(\tau)}{2h} \tag{11}$$

and the approximation for the second derivative is given by Equation (12):

$$\frac{\partial^2 U}{\partial S^2}(S_i,\tau) \approx \frac{u_{i-1}(\tau) - 2u_i(\tau) + u_{i+1}(\tau)}{h^2} \tag{12}$$

2.4.2 Formation of a system of Ordinary Differential Equations

After substituting the finite difference approximations in Equations (11) and (12) into the transformed partial differential equation (5), the problem is reduced to a system of ordinary differential equations with respect to the time variable τ, which characterizes the Method of Lines approach [7]. The resulting semi-discrete equation at each interior grid point S_i is given by Equation (13):

$$\frac{du_i}{d\tau} = \frac{S_2^i\sigma_2^i}{2}\left(\frac{u_{i-1}-2u_i+u_{i+1}}{h^2}\right) + rS_i\frac{u_{i+1}-u_{i-1}}{2h} - ru_i, \tag{13}$$

For $i = 1,2,\dots,N-1$.

By grouping terms with respect to u_{i-1}, u_i and u_{i+1}, Equation (13) can be written in the compact form given by Equation (14):

$$\frac{du_i}{d\tau} = \alpha_i u_{i-1} + \beta_i u_i + \gamma_{iu_{i+1}} \tag{14}$$

where the coefficients are defined by

$$a_i = \frac{\sigma_i^2 s_i^2}{2h^2} - \frac{rS_i}{2h} \tag{15}$$

$$\beta_i = -\frac{\sigma_i^2 s_i^2}{h^2} - r \tag{16}$$

$$\gamma_i = \frac{\sigma_i^2 s_i^2}{2h^2} - \frac{rS_i}{2h} \tag{17}$$

In matrix form, the system of Equations (14)–(17) can be expressed as:

$$\frac{du(\tau)}{d\tau} = M(\tau)u(\tau) + w(\tau) \tag{18}$$

where $u(\tau) = (u_1(\tau), \dots, u_{N-1}(\tau))^T$,
$M(\tau)$ is a tridiagonal matrix whose entries depend on the nonlinear volatility, and $w(\tau)$ is a vector arising from the boundary conditions.

Here, $\sigma_i^2(\tau) = \sigma_0^2(1 + \Psi\left(e^{r\tau}a^2 S_2^i \Delta_i(u)\right)$and $\Delta_i(u)$ represents an approximation of the second derivative at the point S_i.

2.4.3 Numerical Boundary Condition Treatment

The system of ordinary differential equations in Equation (18) is solved with respect to time using the explicit Forward Euler scheme with time step $\tau\boldsymbol{k} = \Delta\boldsymbol{\tau}$. The time-marching formula used to compute the option price at time level $(\boldsymbol{n}+\mathbf{1})$ from the previous level n is given by Equation (19):

$$u^{n+1} = (I + kM^n)u^n \tag{19}$$

where $\mathbf{I}$ denotes the identity matrix and $\mathbf{M}^n$represents the system matrix evaluated at time level n.

At each time step, the nonlinear volatility σ_i^2 is updated based on the gamma approximation $A_i(\mathbf{u}^n)$ through the Barles–Soner volatility function defined in Equation (4).

Since the stock price domain is truncated to a finite interval, the values of the numerical solution at the boundary points must be approximated. To maintain second-order accuracy, the boundary values are computed using second-order Lagrange polynomial extrapolation. The left and right boundary approximations are given by Equations (20) and (21), respectively:

$$u_0 = 2u_1 - u_2, \tag{20}$$

$$u_N = 2u_{N-1} - u_{N-2}\,. \tag{21}$$

This treatment preserves the consistency and accuracy of the numerical scheme across the spatial domain.

An explicit Forward Euler scheme is adopted for time integration due to its simplicity, low computational cost, and ease of implementation within the Method of Lines framework. Although implicit or semi-implicit schemes offer unconditional stability, they require the solution of large nonlinear systems at each time step, which significantly increases computational complexity for the Barles–Soner equation. Since the present study focuses on empirical deployment and interpretability rather than on developing a new solver, the explicit scheme is chosen and combined with systematic step-size sensitivity tests to ensure that all simulations are performed within a stable regime.

2.4.4 Stability and Consistency Analysis

Numerical stability is assessed empirically through grid refinement and time-step sensitivity tests, as theoretical CFL-type bounds are difficult to derive for the nonlinear Barles–Soner equation.

To assess the consistency of the method, the local truncation error is derived using a Taylor series expansion. The resulting truncation error is expressed by Equation (22):

$$T = O(h^2) + O(k) \tag{22}$$

where $h$denotes the spatial step size and $k$represents the time step. Equation (22) indicates that the proposed scheme is second-order accurate in space and first-order accurate in time.

Since the time integration is performed using the explicit Forward Euler scheme, the numerical method is conditionally stable. In practice, this requires the time step to be sufficiently small relative to the spatial grid in order to prevent numerical divergence. Rather than assuming stability a priori, the discretization parameters are selected through preliminary experiments. Several combinations of spatial grids $N_S$and time steps $N_T$are tested, and unstable configurations are discarded.

Several combinations of spatial grids $N_S$and time steps $N_T$are tested in preliminary experiments, including $(N_S, N_T) = (50,2000), (100,5000), (200,10000)$. Unstable configurations, characterized by oscillatory or diverging numerical solutions, are discarded.

The final grid $(N_S = 100, N_T = 5000)$is selected because it satisfies two practical criteria: (i) the numerical solution remains stable throughout the entire time domain, and (ii) further refinement of either the spatial grid or the time step produces changes in the computed option price of less than 0.5%. This empirical convergence behavior is adopted as an operational stability criterion, since explicit CFL-type bounds and formal error estimates are difficult to derive for the nonlinear Barles–Soner equation.

Although the Forward Euler time integrator is conditionally stable, the above sensitivity tests ensure that the chosen discretization lies within a stable regime for all reported simulations.

To assess robustness, the numerical experiments are conducted under multiple grid resolutions and for different transaction cost parameters. In addition to the at-the-money (ATM) case, in-the-money (ITM) and out-of-the-money (OTM) scenarios are also computed. In all cases, the option prices exhibit consistent convergence patterns and monotonic behavior with respect to the transaction cost parameter, indicating that the reported results are not artifacts of a single discretization choice.

3. RESULT AND ANALYSIS

3.1 Research Data and Simulation Parameters

The data used in this simulation consist of the historical daily closing prices of PT XYZ, an anonymized Indonesian listed company in the banking sector. The dataset covers a one-year trading period. Based on these data, the latest closing price is recorded as IDR 8,025.

For the numerical simulation, the historical volatility $\sigma$of the underlying asset is first estimated. The daily log-returns are computed using Equation (1), and their standard deviation is then annualized using Equation (2). Based on the historical data of PT XYZ, the resulting annualized volatility is 29.49%.

The analysis is not limited to the at-the-money (ATM) case. In order to observe the behavior of the model under different market conditions, three scenarios are considered: In-the-Money (ITM), At-the-Money (ATM), and Out-of-the-Money (OTM).

The simulation parameters used in this study are summarized in Table 2, while a sample of the daily stock return calculation is presented in Table 1. The discretization parameters employed in the numerical experiments are selected based on the grid refinement and stability analysis described in Section 2.4.4.

In option pricing, volatility plays a dominant role in determining the magnitude of the option premium. Therefore, the sensitivity of the numerical results to changes in the volatility and transaction cost parameters is of particular interest. In this study, the baseline volatility is estimated from historical data, while the transaction cost parameter $a$is varied across representative levels. The numerical experiments indicate that small increases in either parameter led to systematic increases in the option price, with the effect being more pronounced for higher transaction cost intensities. This behavior reflects the nonlinear amplification of effective volatility in the Barles–Soner model and confirms that the computed option prices are highly sensitive to market uncertainty and trading frictions. Consequently, accurate estimation of volatility and careful calibration of transaction cost parameters are essential for obtaining reliable option valuations.

Table 2. Parameter Values

Parameters	**Symbol**	**Value**	**Information**
Initial Share Price	S_0	8025	Determination of Final Share Price of PT. XYZ
Execution Price	E	8025	At-The-Money (ATM) Scenario
Due date	T	1	1 Year
Risk-Free Interest Rate	t	0.06	Benchmark interest rate assumptions (6%)
Historical Volatility	σ_0	0.2960	Estimates from historical data of PT. XYZ (29,49%)
Transaction Cost Parameters	α	0,0.02,0.05	Independent Variable (Free = 0, Small Fee = 0.02, Big Costs = 0.02)
Space Steps	N	100	Number of stock grid points (S)
Time Step	M	5000	Number of time grid points (t)

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3.2 Numerical Convergence and Validation

Before analyzing the impact of transaction costs, the accuracy of the proposed numerical scheme is first validated under frictionless market conditions ($a = 0$). The numerical option prices obtained from the semi-discretization method are compared with the analytical solution of the classical Black–Scholes model.

To measure the accuracy of the numerical results, the relative error between the analytical and numerical solutions is defined by Equation (23):

$$E_R = \left|\frac{V_{analytical} - V_{numerical}}{V_{analytical}}\right| \times 100\% \quad (23)$$

For the parameter set (S_0, K, r, σ, T), the analytical Black–Scholes price is calculated as IDR 1,165.73. Table 3 reports the numerical option prices obtained under increasing spatial grid resolutions, together with the corresponding relative errors.

Table 3. Convergence of Numerical Solution towards Analytical Black-Scholes Price ($a = 0$)

Number of Spatial Grids (N)	**Numerical Option Price (IDR)**	**Analytical Option Price (IDR)**	**Relative Error (E_R)**
50	1169.33	1165.73	0.308%
100	1166.62	1165.73	0.076%
200	1165.96	1165.73	0.020%
400	1165.79	1165.73	0.005%

The results in Table 3 demonstrate that as the number of spatial grid points increases, the numerical solution converges rapidly toward the analytical Black–Scholes price. When $N = 50$, the relative error is 0.308%, which decreases to 0.076% for $N = 100$, 0.020% for $N = 200$, and only 0.005% for $N = 400$. This monotonic reduction confirms the consistency and convergence of the proposed semi-discretization scheme.

This validation step establishes a reliable baseline for subsequent simulations under transaction costs. Since the numerical method accurately reproduces the analytical benchmark when $a = 0$, any deviation observed in the nonlinear setting can be attributed to the effect of transaction costs rather than to numerical artifacts.

3.3 Effect of Transaction Costs on Option Prices

Based on the validated numerical framework, the simulation is then extended to incorporate proportional transaction costs using the Barles–Soner model.

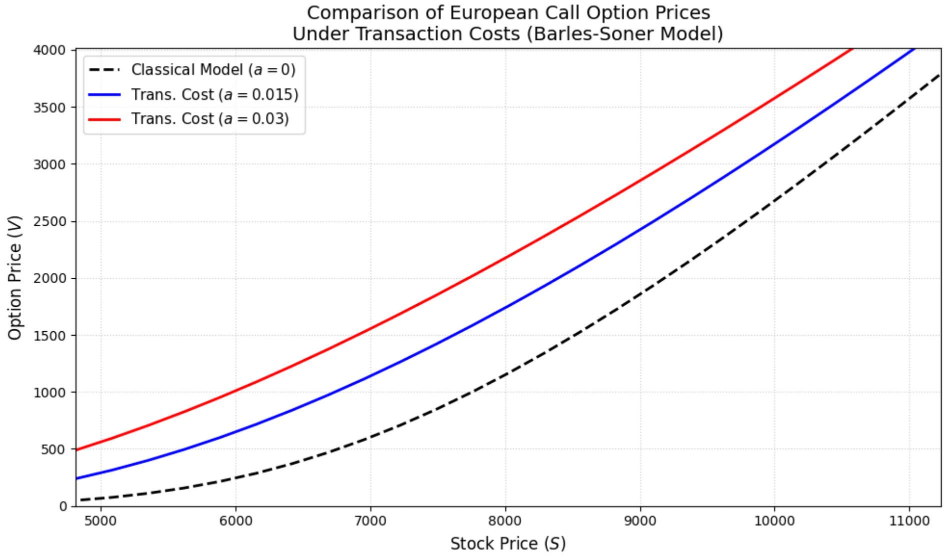

Figure 2. Numerical comparison of European call option prices between the classical Black-Scholes model and the Barles-Soner model with transaction cost parameters and $(a = 0)(a = 0.015)(a = 0.03)$

The dashed black curve represents the theoretical Black–Scholes price in a frictionless market $(a = 0)$. The solid blue and red curves correspond to the Barles–Soner prices under low and high transaction cost scenarios, respectively. A clear divergence pattern is observed across the entire stock price domain. The curves incorporating transaction costs consistently lie above the classical benchmark, with the red curve $(a = 0.030)$exhibiting the highest premiums.

This behavior is consistent with nonlinear option pricing theory under transaction costs (Barles and Soner, 1998). The presence of transaction costs amplifies the effective volatility through the nonlinear correction term, leading to systematically higher option prices. Since option values are positively correlated with volatility, a larger transaction cost parameter imposes a higher premium to compensate for the additional hedging costs and risks faced by option writers.

A detailed comparison of option prices at the at-the-money (ATM) position $(S_0 = K)$is presented in Table 4.

Table 4. Option Price Comparison under Transaction Costs

Scenario	Parameters (a)	Option Price (IDR)	Deviation from Classical
Model (Benchmark)	0	1165,79	–
Low Transaction Cost	0.015	1558.12	+392.33
High Transaction Cost	0.030	1942.45	+776.66

Under frictionless conditions, the option price is IDR 1,165.79. When transaction costs are introduced, the price increases to IDR 1,558.12 for $a = 0.015$and to IDR 1,942.45 for $a = 0.030$. These correspond to premium increases of IDR 392.33 and IDR 776.66, respectively, relative to the classical benchmark.

Similar monotonic behavior is observed for the ITM and OTM scenarios. In all cases, higher transaction cost intensities result in uniformly higher option prices. This consistency across moneyness levels confirms that the impact of transaction costs is structural rather than local, and that the Barles–Soner model systematically adjusts option values upward to reflect market frictions.

These results demonstrate that neglecting transaction costs leads to systematic underpricing when the classical Black–Scholes model is applied directly. The nonlinear framework therefore provides a more realistic valuation by internalizing hedging frictions and market imperfections, particularly in environments where transaction costs are non-negligible.

3.4 Practical Implications

The numerical results demonstrate that incorporating transaction costs leads to a substantial increase in option premiums. In particular, the Barles–Soner model produces systematically higher prices than the classical Black–Scholes model, confirming that neglecting transaction costs may result in significant underpricing, especially from the perspective of the option writer.

In practice, transaction costs are not constant across market conditions. During periods of high volatility or low liquidity, such as market stress or thin trading environments, effective transaction costs tend to increase due to wider bid–ask spreads and more frequent portfolio rebalancing. Under such conditions, the divergence between classical Black–Scholes prices and nonlinear prices becomes more pronounced. The Barles–Soner framework captures this effect through the nonlinear amplification of volatility, implying that the classical model becomes progressively less reliable as market frictions intensify.

From a practical standpoint, these findings indicate that investors and risk managers in emerging markets, such as Indonesia, should not rely solely on the standard Black–Scholes formula, especially in volatile or illiquid regimes. The nonlinear model provides a more conservative valuation that internalizes hedging frictions and therefore offers a safer benchmark for pricing and risk management. This is particularly relevant in markets where transaction costs and liquidity constraints are structurally higher than in developed markets.

Consequently, the proposed framework is not merely a numerical alternative, but a practical tool for adapting option valuation to different market regimes. By adjusting the transaction cost parameter, the model can reflect varying levels of market friction, allowing practitioners to assess how option prices respond to changing trading environments and to avoid systematic underestimation of risk. From a regulatory perspective, these observations are intended to be illustrative rather than prescriptive, highlighting how frictionless pricing assumptions may understate risk in practice and motivating more cautious stress-testing and valuation benchmarks in markets with significant trading frictions.

4. CONCLUSION

This study applies a semi-discretized numerical approach based on the Method of Lines to evaluate European call option prices under transaction costs using the nonlinear Barles–Soner model. Using historical data from PT XYZ, the numerical scheme is shown to be stable and consistent, and its accuracy is validated against the analytical Black–Scholes solution in the frictionless case.

The results demonstrate that incorporating transaction costs leads to systematically higher option prices than those produced by the classical Black–Scholes model, with the divergence increasing as transaction costs rise. This confirms that ignoring transaction costs can result in substantial underpricing in realistic market environments.

The scope of this study is limited to a single underlying asset with constant baseline volatility and a single observation period. Future research may extend this framework to multiple Indonesian stocks, incorporate more realistic market dynamics such as jumps or stochastic volatility, and explore higher-order numerical schemes. From a broader perspective, these findings illustrate that frictionless pricing assumptions may understate risk in markets with significant trading frictions. This observation is illustrative rather than prescriptive, but it highlights the potential value of nonlinear pricing perspectives for more cautious valuation and risk assessment in emerging markets.

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