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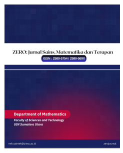

Numerical Pricing of European Stock Option Based on Black-Scholes Model Using Crank-Nicolson Method

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ABSTRACT

Accurate option pricing is an important issue in quantitative finance, especially in emerging financial markets, which are generally characterized by price volatility and limited historical data. This research evaluates the numerical performance of the classical Crank-Nicolson finite difference method in determining the price of European call options based on the Black-Scholes model using Indonesian stock market data. The Black-Scholes equation is discretized on a uniform spatial and temporal grid, and the numerical solution is verified by comparison with the Black-Scholes analytical solution as a mathematical reference. The numerical results show that the Crank-Nicolson method produces stable and convergent solutions, with a relative error of less than 1% at a sufficiently fine grid resolution. Furthermore, sensitivity analysis to volatility and temporal convergence tests demonstrate the consistency of the numerical solution's behavior to variations in the model's key parameters. These findings indicate that the Crank-Nicolson method provides a reliable numerical approach for evaluating European option pricing within the classical Black-Scholes framework under the analyzed market conditions.

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1. INTRODUCTION

The development of digital technology has changed the structure of modern financial markets through the acceleration of information flow, increased transaction efficiency, and increased complexity of financial instruments [1]. These changes have encouraged the widespread use of derivative instruments, both for investment and risk management purposes [2], [3].

Among various derivative instruments, option contracts play an important role because their value is highly sensitive to the price of the underlying asset, market volatility, and time to maturity [4], [5]. Therefore, accurate option pricing is a fundamental requirement, especially in emerging financial markets, which are generally characterized by higher volatility and relatively limited market efficiency compared to developed markets [6], [7].

The Black-Scholes model is a major milestone in option pricing theory because it provides a closed analytical solution based on ideal market assumptions, such as constant volatility and the absence of market friction [8], [9]. Although this model has strong theoretical significance, various studies show that these assumptions are often not fully met in practice, prompting the development of numerical approaches to solve the Black–Scholes model more flexibly [10], [11].

Finite difference methods are among the most widely used numerical approaches due to their simplicity of formulation and ease of implementation. Among these methods, the Crank–Nicolson scheme is known to have unconditional stability and second-order accuracy in both spatial and temporal discretization, making it particularly well-suited for parabolic partial differential equations such as the Black–Scholes equation [12]. Although various advanced numerical developments have been proposed, the classical Crank–Nicolson method remains relevant in applied studies because it offers a good balance between stability, accuracy, and computational complexity.

Although the Crank–Nicolson method has been tested theoretically, empirical studies that specifically evaluate the numerical performance and convergence behavior of this method using Indonesian stock market data in the context of European option pricing are still relatively limited [15]. This condition indicates a research gap, particularly in the provision of controlled numerical evaluations based on emerging financial market data.

Based on this gap, this research aims to apply the Crank–Nicolson finite difference method in solving the Black–Scholes model to determine the price of European call options using Indonesian stock market data. The research focuses on evaluating the numerical stability and convergence behavior of the solution by comparing it with the Black–Scholes analytical solution as a mathematical reference. With this approach, the research is expected to provide a clear numerical picture of the performance of the classic Crank–Nicolson method in the context of emerging financial markets.

2. RESEARCH METHOD

2.1 Research Design

In this research uses a numerical quantitative approach to determine the price of European call options based on the Black–Scholes model, which is solved using the Crank–Nicolson finite difference method. The numerical approach was chosen because it allows for the systematic solution of partial differential equations through space and time discretization, as well as controlled evaluation of the stability and convergence of solutions [16].

The Crank–Nicolson method was chosen because it is a semi-implicit finite difference scheme that has unconditional stability and second-order accuracy in time and space discretization, making it well suited for solving parabolic partial differential equations commonly encountered in quantitative financial modeling [17]. This stability advantage makes the method widely used in determining the price of derivative instruments because it does not require strict time step restrictions to maintain numerical stability [18].

This research is applied and computational in nature, focusing on the direct implementation of established numerical schemes to evaluate their performance on real stock market data from emerging financial markets. This approach aligns with previous numerical studies that emphasize the importance of validating numerical methods using empirical data to evaluate the reliability of solutions under realistic market conditions [19].

This research is applied and computational in nature, focusing on evaluating the numerical performance of the classical Crank–Nicolson method using Indonesian stock market data as a representation of emerging financial markets. This research does not aim to propose new methodological innovations, but rather to evaluate the numerical stability and convergence behavior of solutions through spatial and temporal convergence testing and limited sensitivity analysis of volatility, as was done in previous numerical studies.

2.2 Mathematical Model

Mathematical models applied in this research are based on the Black–Scholes partial differential equation that determines the pricing of European call options, which is formulated as follows:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \quad (1)$$

where $V(S,t)$ denotes the option price depends on the underlying asset price S and time t, σ represents the asset volatility, and r denotes the constant risk-free interest rate. Equation (1) is derived under ideal market assumptions and serves as the theoretical foundation for the numerical methods applied in this research [8].

Equation (1) is derived by assumption that the underlying stock price evolves according to a Geometric Brownian Motion (GBM), which is expressed as:

$$dS = \mu S\, dt + \sigma S\, dW_t \quad (2)$$

where μ denotes the expected stock price growth rate and W_t denotes a standard Brownian motion [4]. By applying the no-arbitrage principle and transforming the model into the risk-neutral probability measure, the drift parameter μ is replaced by the risk-free interest rate r, resulting in the Black–Scholes formulation [20][21].

For a European call option, the terminal condition at maturity $t = T$ are defined as

$$V(S,T) = \max(S - K, 0) \quad (3)$$

while the boundary conditions over the bounded asset price domain $S \in [0, S_{max}]$ are defined as:

$$V(0,t) = 0, V(S_{max}, t) = S_{max} - Ke^{-r(T-t)} \quad (4)$$

where K denotes the strike price and T is the time to maturity of the option. These boundary conditions reflect the theoretical behavior of the European call option value under extreme underlying asset prices, that is, when the stock price approaches zero and when the stock price is very high [22].

In the numerical implementation, the stock price domain is limited to the interval up to S_{max} to allow for limited spatial discretization. In this research, the upper limit of the stock price is explicitly set at $S_{max} = 4K$, which is numerically large enough to ensure that the upper bound solution approaches its theoretical asymptotic value [22]. The selection of such a limited price domain is a common practice in the numerical solution of Black–Scholes equations to minimize boundary truncation errors without significantly increasing the computational load [23][24].

2.3 Data Source and Parameter Estimation

The empirical data used in this research was obtained from Yahoo Finance, using daily closing stock prices of a large publicly listed company in Indonesia (referred to as PT XYZ). The dataset covers the observation period from January 3, 2025, to December 30, 2025, with a total of 235 daily observations. Choosing a single representative stock was intentional as part of the research design to maintain a controlled numerical environment for evaluating the performance of the Crank–Nicolson method. Focusing on a single asset allows for a clearer analysis of the numerical stability and convergence behavior of the scheme without being distorted by cross-asset parameter heterogeneity, particularly differences in volatility structure and price scale. Therefore, this research prioritizes in-depth and controlled numerical evaluation over cross-asset generalization, which is beyond the scope of this research and may be a direction for development in future research.

The model parameters include the most recent stock price (S_0), the strike price (K), the risk-free interest rate (r) obtained from official publications of Bank Indonesia, the annualized volatility (σ) estimated from daily return data, and the time to maturity (T) expressed in years. Although the observation period was limited to one year, this data range was considered sufficient to estimate consistent volatility parameters within the framework of the classic Black–Scholes model with the assumption of constant volatility.

The stock price volatility (σ) is computed based on the standard deviation of daily logarithmic returns. The logarithmic return is defined as

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \quad (5)$$

where S_t dan S_{t-1} denote the stock closing prices on day-t and the previous day, respectively [25]. The standard deviation of daily returns is computed as:

$$s = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \quad (6)$$

where n denotes the number of observation days and $\bar{r}$ represents the mean daily return [26]. The annualized volatility is obtained by adjusting the annualized standard deviation of daily returns using the square root of the annual number of trading days in a year ($k = 252$), that is,

$$\sigma = \sqrt{k}\, s \quad (7)$$

This approach a standard procedure in the analysis of financial market volatility based on discrete-time data [25][26]. The volatility value obtained is used as a basic parameter in numerical calculations, while limited variations around this value are considered in sensitivity analysis to evaluate the stability of numerical solutions against changes in volatility. The validation of the numerical solution is performed by comparing it with the Black–Scholes analytical solution, with the aim of assessing the accuracy and convergence behavior of the Crank–Nicolson scheme, rather than replicating empirical market option prices.

2.4 Time Transformation

To simplify numerical computation, a time-variable transformation is introduced by defining $\tau = T - t$, dan $U(S,\tau) = V(S, T - \tau)$ thereby reformulating the option pricing problem as a forward time-evolution problem. With this transformation, the time derivative can be written as

$$\frac{\partial V}{\partial t} = -\frac{\partial U}{\partial \tau} \quad (8)$$

Substituting equation (8) into equation (1) yields the forward-time formulation of the partial differential equation as

$$\frac{\partial U}{\partial \tau} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 U}{\partial S^2} + rS\frac{\partial U}{\partial S} - rU \quad (9)$$

which subsequently became as the basis for the application of the Crank–Nicolson finite difference scheme in this research.

2.5 Spatial Discretization

The spatial domain $[0, S_{max}]$ is discretized into M uniformly spaced grid points with a step size $\Delta S = \frac{S_{max}}{M}$. The value of the numerical solution function at the i-th spatial grid point and j-th time level is denoted as $U_i^j \approx U(S_i, \tau_j)$, with $S_i = i\Delta S$. To approximate the spatial derivatives in equation (9), second-order accurate central finite difference schemes are employed. The first-order spatial derivative at the grid point S_i is approximated as

$$\left.\frac{\partial U}{\partial S}\right|_{S_i} \approx \frac{U_{i+1}^j - U_{i-1}^j}{2\Delta S} \quad (10)$$

while the second-order spatial derivative is approximated as:

$$\left.\frac{\partial^2 U}{\partial S^2}\right|_{S_i} \approx \frac{U_{i+1}^j - 2U_i^j + U_{i-1}^j}{(\Delta S)^2} \quad (11)$$

This approach produces a symmetric and consistent spatial approximation, and is suitable for application to parabolic partial differential equations such as the Black–Scholes model. In this research, the number of spatial grid points M is varied to analyze the numerical convergence behavior. To maintain consistency between spatial and temporal discretization, the number of time steps N is chosen to be proportional to the number of spatial grids, while temporal convergence testing is performed by varying the time step size at a fixed spatial resolution.

2.6 Crank-Nicolson Time Discretization

Let the time interval $[0, T]$ be divided into N uniform time steps with a step size $\Delta\tau = \frac{T}{N}$. Using the spatial discretization in Subsection 2.4, the Crank–Nicolson scheme is applied to equation (9) by averaging the spatial differential operator over two consecutive time levels. This scheme is formulated as:

$$\frac{U_i^{j+1} - U_i^j}{\Delta\tau} = \frac{1}{2}\left[L\left(U_i^{j+1}\right) + L\left(U_i^j\right)\right] \quad (12)$$

where $L(\cdot)$ represents the spatial differential operator of the Black–Scholes equation. This discretization process generates a system of linear equations with tridiagonal coefficient matrices, which can then be written in matrix form as follows:

$$AU^{j+1} = BU^j + b \quad (13)$$

where matrices A and B are tridiagonal matrices that depend on the model parameters and grid sizes, while vector b contains contributions from boundary conditions [10]. System of linear equations generated is effectively solved using Thomas's Algorithm, considering its low computational complexity and good numerical stability for systems with tridiagonal structure [27]. The numerical iteration are performed backward from the maturity time to the initial time. Final value $V(S_0, 0)$ represents the estimated price of the European call option at the present time [10][12].

In the standard time discretization, temporal convergence testing was performed by setting a fixed spatial resolution and gradually varying the time step size. This approach was used to evaluate the consistency of numerical solutions with respect to time discretization and to ensure the temporal stability of the Crank–Nicolson scheme within the Black–Scholes model. This test is verifiable and is not intended to quantitatively estimate the order of convergence.

2.7 Validation and Evaluation

Numerical solutions are validated by comparing them with the Black–Scholes analytical solution for European call options, which is formulated as follows:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (14)$$

with

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T} \quad (15)$$

Where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution. An absolute difference between the numerically calculated option price and the analytical solution is used as an accuracy indicator to assess the convergence rate and accuracy of the applied numerical scheme. In temporal convergence testing, accuracy is evaluated by comparing numerical solutions obtained at various time step sizes against the Black–Scholes analytical solution as a reference.

2.8 Computational Implementation

Python was chosen based on the availability of reliable and widely tested numerical libraries in scientific computing, such as NumPy for numerical operations and Matplotlib for result visualization. Google Colab was chosen because it provides a stable, easily reproducible cloud-based computing environment that allows numerical computations to be executed without dependence on local hardware specifications. This combination supports transparency, reproducibility, and efficiency in the implementation of numerical methods in this research.

3. RESULT AND ANALYSIS

This research aims to evaluate the performance of the Crank–Nicolson finite difference method in pricing European call options within the Black–Scholes framework. The analysis focuses on empirical data processing, evaluation of numerical convergence behavior, and measurement of accuracy levels through systematic comparison between numerical solutions and Black–Scholes analytical solutions as mathematical benchmarks.

3.1 Empirical Data and Parameter Estimation

Daily closing stock prices are used as the basis for estimating parameters in the Black–Scholes model. Daily logarithmic returns are calculated from the ratio of consecutive closing prices, as presented in Table 1. The results show that stock returns fluctuate around zero without a dominant trend during the observation period, indicating relatively stable short-term price dynamics.

Table 1. Logarithmic Daily Returns of the Observed Stock Prices

t	Date	Closing Price on the -th Day (S_t) IDR	Closing Price on Day t−1 (S_{t-1}) IDR	Logarithmic Return (r_t)
0	January 03, 2025	9850	9900	−0.005063
1	January 06, 2025	9675	9850	−0.017926
2	January 07, 2025	9525	9675	−0.015625
⋮	⋮	⋮	⋮	⋮
233	December 29, 2025	8025	8025	0
234	December 29, 2025	8075	8025	0.006211

Based on descriptive statistical analysis of logarithmic returns, the Black–Scholes model parameters were obtained as summarized in Table 2. The estimated annual volatility of 29.37% is within the range commonly reported for banking sector stocks in the Indonesian capital market, reflecting moderate volatility conditions without extreme spikes during the observation period. These parameters are used as the basis for all numerical calculations in the following section.

Table 2. Estimated Parameters Used in the Black–Scholes Model

Parameter	Symbol	Value
Last stock price	S_0	8075 (IDR)
Strike price	K	10000 (IDR)
Risk-free rate	r	0.06/year
Mean daily return	$\bar{r}$	-0.000867
Daily standard deviation	s	0.018502
Annualized volatility	σ	0.293703/year
Time to maturity	T	0.25 (year)

3.2 Numerical Results and Convergence Analysis

Based on the parameters in Table 2, the price of European call options is calculated numerically using the Crank–Nicolson method and analytically using the Black–Scholes formula. The analytical solution is used as a reference to evaluate the accuracy of the numerical solution. To measure the degree of agreement between the two approaches, the relative error is defined as

$$ER = \frac{|C_A - C_N|}{C_A} \times 100\% \tag{16}$$

where C_A denotes the option price based on the analytical Black–Scholes solution and C_N denotes the option price computed using the Crank–Nicolson method.

In the context of numerical methods, the accuracy level refers to the rate of decrease in numerical error with respect to grid size refinement. The Crank–Nicolson scheme theoretically has second-order accuracy in both spatial and temporal discretization, which means that the numerical error is expected to decrease proportionally to the square of the grid step size when the spatial and temporal resolution is refined. A comparison between the numerical and analytical option prices for various grid sizes in spatial and temporal grid sizes $(M = N)$ is presented in Table 3. The numerical option price s are evaluated at the initial stock price and expressed as $V(S_0, 0)$.

Table 3. Comparison of Numerical and Analytical European Call Option Prices and Relative Errors

$M = N$	Numerical Price (IDR)	Analytical Pricing (IDR)	Relative Error (%)
50	62.538028	53.023359	17.944297
100	54.132018	53.023359	2.090887
150	53.599257	53.023359	1.086122
200	53.420202	53.023359	0.748431
250	53.352819	53.023359	0.621349
300	53.127669	53.023359	0.196725
400	53.156822	53.023359	0.251706
500	53.115226	53.023359	0.173257
600	53.061557	53.023359	0.072040

The results in Table 3 show that increasing the grid resolution causes the convergence of numerical option prices toward the analytical Black–Scholes value to be stable and consistent. On relatively coarse grids, the numerical solutions show significant deviations, but these errors decrease systematically as the number of grid points increases.

To reinforce the spatial convergence analysis, Figure 1 displays a log–log graph between relative error and spatial step size. This graph shows a consistent downward trend in error that is consistent with second-order behavior with respect to spatial step size. This visualization is used as qualitative verification of the convergence characteristics of the Crank–Nicolson scheme, without performing numerical slope estimates or making quantitative claims about the order of convergence.

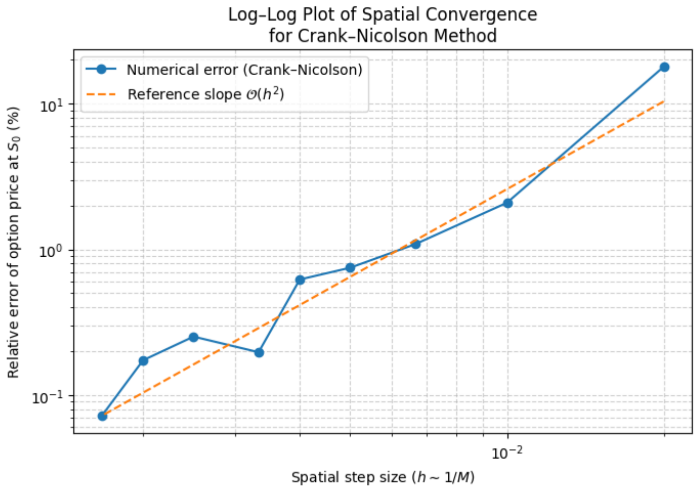

Figure 1. Log–Log Plot of Relative Errors of Crank–Nicolson Solutions Versus Spatial Step Size

In this research, convergence analysis focused primarily on spatial discretization, while temporal convergence was evaluated separately by keeping the spatial resolution constant. This approach is commonly used in numerical studies to ensure that the effects of spatial and temporal discretization can be analyzed in a controlled manner.

Furthermore, the tendency of numerical solutions on coarse grids to produce higher option prices than analytical solutions can be explained as a result of discretization errors and boundary truncation errors in limited spatial domains. This effect is significantly reduced when the grid resolution is refined, as reflected in the decrease in relative error in Table 3 and Figure 1.

From a numerical and practical standpoint, this stable and controlled convergence behavior indicates that the Crank–Nicolson method can be used as a reliable computational approach for option pricing within the Black–Scholes framework. In the context of the Indonesian stock market, which during the observation period did not show extreme fluctuations or significant volatility spikes, the assumption of constant volatility still provides an adequate approach for numerical evaluation. This condition also supports the stability of the resulting solution, given that the Crank–Nicolson scheme is sensitive to sharp changes in model parameters.

For financial practitioners, such as derivatives analysts and risk managers, this consistent convergence characteristic has practical implications in that option pricing can be performed numerically without relying on the selection of very small time steps. Thus, the Crank–Nicolson method has the potential to be efficiently integrated into computation-based analysis tools, such as Python implementations or numerical spreadsheets, as a means of initial option pricing and sensitivity analysis in emerging financial market environments.

3.3 Graphical comparison of Numerical and Analytical Solutions

Figure 2 shows a graphical comparison between the Crank–Nicolson numerical solution and the Black–Scholes analytical solution at various spatial grid resolutions. At relatively coarse grid resolutions, the numerical solution tends to produce higher option prices than the analytical solution.

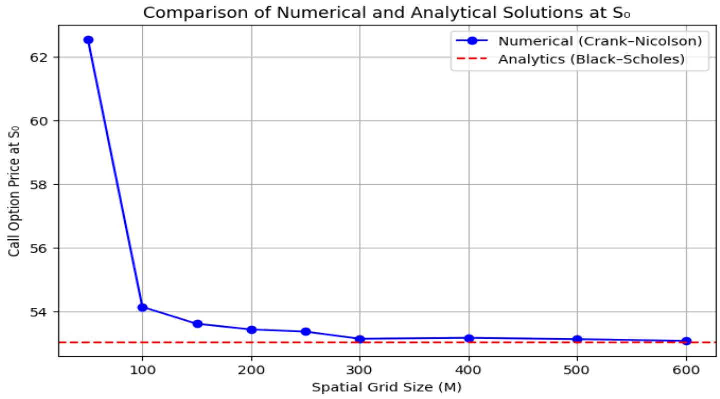

Figure 2. Comparison Between Numerical Crank–Nicolson Solutions and Analytical Black–Scholes Prices for European Call Options

At relatively small values of M, the numerical estimates tends to produce option prices that are higher than those obtained from the analytical solution. As the grid spacing becomes finer, the numerical solution gradually approaches and eventually almost coincides with the analytical solution. This behavior reflects the increased numerical accuracy due to more detailed discretization, and supports the convergence results shown quantitatively in Table 3.

3.4 Relative Error Behavior and Spatial Distribution

Figure 3 shows the relationship between relative error and the number of spatial grid points. It can be seen that the relative error decreases monotonically as the grid resolution increases, confirming the numerical stability and convergence characteristics of the Crank–Nicolson method in solving the Black–Scholes model.

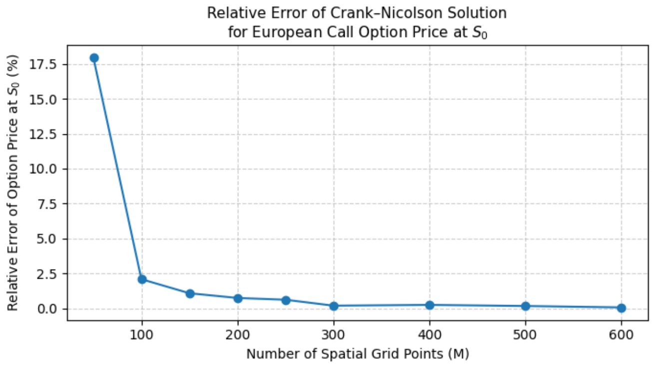

Figure 3. Relative Error of the Crank–Nicolson Numerical Solution as a function of the spatial grid size M

At sufficiently fine grid resolutions, the relative error is below 1%, indicating that the numerical solution obtained is very close to the analytical solution. This stable numerical performance is consistent with the characteristics of the market analyzed in this case, where the annual volatility of 29.37% reflects conditions without sharp price fluctuations or excessive structural instability.

In this context, the Crank–Nicolson method demonstrates superiority because its unconditional stability allows for the use of relatively flexible time steps without triggering the numerical instability commonly found in explicit schemes. Conversely, in market conditions with much more extreme price changes, the fully implicit method is often more appropriate, despite the consequences of greater numerical diffusivity and higher computational costs. Therefore, the characteristics of the market analyzed make the Crank–Nicolson method an appropriate compromise between stability, accuracy, and relative computational cost.

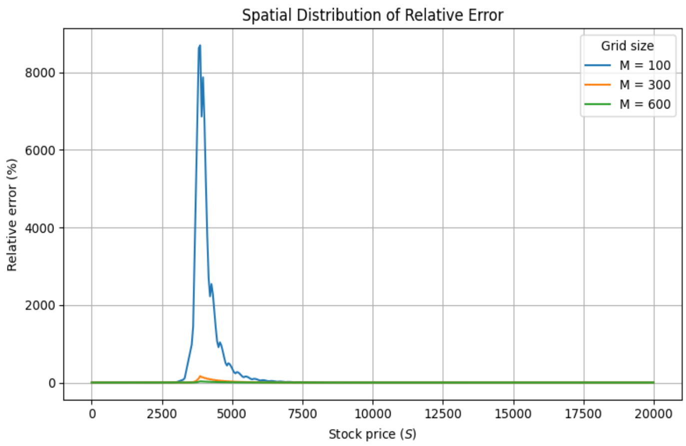

Figure 4. Spatial Distribution of Relative Errors with Stock Price for Various Spatial Grid Sizes

Figure 4 shows the distribution of errors relative to the underlying stock price. The largest errors are concentrated around the at-the-money region ($S \approx K$) , which is related to local discontinuities in the option payoff function. Outside this region, in the in-the-money and out-of-the-money conditions, the relative errors tend to be smaller and more stable, and show no signs of numerical instability.

It should be noted that all numerical calculations in this study are deterministic once the model parameters are set. The annual volatility estimates are obtained from historical data, which inherently contain statistical variability. Therefore, this study does not perform confidence interval estimation or advanced statistical sensitivity analysis, as the main focus is the evaluation of the numerical performance of the Crank–Nicolson method against the Black–Scholes analytical solution as a mathematical reference. Market option price-based evaluation, computational efficiency measurement, and broader quantitative comparisons with alternative numerical methods could be the direction of future research.

As a supplement to the analysis of stability and numerical convergence, Table 4 presents a conceptual comparison between the Crank–Nicolson method and other finite difference schemes commonly used in option pricing.

Table 4. Conceptual Comparison of the Final Difference Method for Determining Option Prices

Scheme	Stability	Accuracy	Cost (Relative)
Explicit	Conditional	1st/2nd	Low
Implicit	Unconditional	1st	High
Crank–Nicolson	Unconditional	2nd	Medium

Table 4 shows that the Crank–Nicolson method provides a practical compromise between numerical stability and computational cost. Compared to explicit schemes that are sensitive to time step constraints and fully implicit schemes that are more computationally expensive, Crank–Nicolson offers a suitable balance for numerical evaluation in markets with moderate volatility. Basic execution time measurements using the timeit function in Python, with the same spatial and temporal grid resolution, show that the computation time of the Crank–Nicolson scheme lies between that of the explicit and fully implicit schemes, supporting the classification of "medium" relative computational cost as shown in Table 4.

The relative errors obtained in this research are consistent with the results reported in the numerical literature on option pricing using the Crank–Nicolson final difference method. Previous numerical studies, such as [10] and [12], report that at sufficiently fine spatial and temporal grid resolutions, the Crank–Nicolson scheme generally produces relative errors below 1% when verified against the Black–Scholes analytical solution. Thus, the error levels obtained in this study are within the range of accuracy consistent with the numerical benchmarks commonly reported in the literature.

3.5 Sensitivity Analysis to Volatility

Sensitivity analysis was performed by varying the volatility value around its estimated value, while other model parameters were kept constant. All calculations were performed at a sufficiently fine grid resolution to minimize the effect of discretization errors.

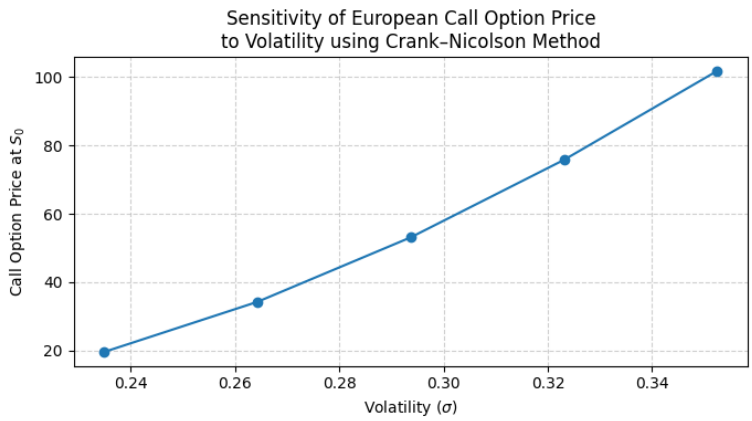

Figure 5. Sensitivity of European Call Option Prices to Variations in Volatility Parameters

Figure 5 shows that the price of European call options increases monotonically as volatility increases. This pattern is consistent with the theoretical properties of the Black–Scholes model, in which volatility represents the degree of uncertainty in the movement of the underlying asset price and contributes directly to an increase in the value of the option.

From a numerical perspective, these results show that the Crank–Nicolson scheme is capable of capturing the solution's response to variations in volatility parameters in a stable and consistent manner, without exhibiting numerical oscillations or computational instability within the range of variations considered. From a practical perspective, this monotonic relationship allows the Crank–Nicolson method to be used as a reliable computational tool for scenario analysis and basic sensitivity testing in derivative evaluation.

It should be noted that this sensitivity analysis was conducted within the framework of the Black–Scholes model with the assumption of constant volatility and without considering the dynamics of stochastic volatility. Therefore, the results obtained are intended as a controlled numerical evaluation, not as a full representation of real market conditions. Further development may include the application of more complex volatility models or a comparison of numerical sensitivity between methods to expand the practical relevance of these findings.

3.6 Temporal Convergence

To complete the numerical convergence evaluation, a time convergence analysis was performed by changing the time step size, while the spatial resolution was maintained at a sufficiently fine level. This approach aims to assess the consistency of numerical solutions with respect to controlled time discretization.

In this test, the spatial resolution was chosen to be sufficiently fine so that spatial errors could be relatively ignored, and the variation in numerical solutions primarily reflected the influence of time discretization. Although the Crank–Nicolson scheme theoretically has second-order accuracy with respect to the time step size, this study does not aim to estimate the order of convergence quantitatively, but rather to verify the stability and consistency of the numerical solution with respect to temporal refinement.

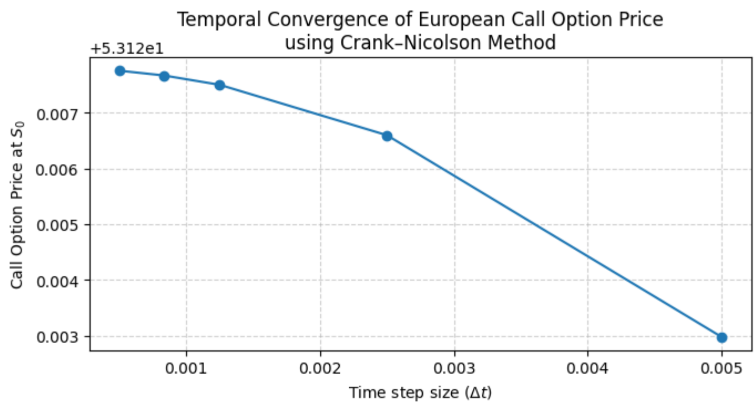

Figure 6. Temporal Convergence of European Call Option Prices Using the Crank–Nicolson Method

Figure 6 shows that the numerical option prices tend to converge to stable values as the time step is refined. This behavior indicates that the Crank–Nicolson scheme produces solutions that are consistent with temporal smoothing and shows no indication of numerical instability within the range of time steps considered.

Although this research does not perform a detailed quantitative comparison of execution times for all grid variations, basic execution time measurements using Python's *timeit* function have been performed as a reference for classifying the relative computational costs between numerical schemes.

4. CONCLUSION

This research evaluates the performance of the Crank–Nicolson finite difference method in determining the price of European call options based on the Black–Scholes model using Indonesian stock market data. Numerical results show that the Crank–Nicolson method produces stable and convergent solutions as the grid resolution increases, and approximates the Black–Scholes analytical solution used as a validation reference. The relative error decreases systematically and is below 1% on a sufficiently fine grid, which indicates the consistency and reliability of the numerical approach applied.

Spatial and temporal convergence analysis confirms that the Crank–Nicolson scheme shows no indication of numerical instability within the range of parameters and step sizes considered. The relative error distribution also exhibits a pattern consistent with the theoretical characteristics of the option payoff function, with the largest errors concentrated around the at-the-money region. These findings support the suitability of the Crank–Nicolson method as a stable numerical approach within the framework of the classical Black–Scholes model.

In the context of the Indonesian stock market, which during the observation period was characterized by an annual volatility of around 29.37%, representing moderate volatility conditions without extreme spikes, the Crank–Nicolson method shows a good balance between numerical stability and accuracy. Compared to explicit schemes, which are limited by time-step stability requirements, and fully implicit schemes, which tend to be more diffusive and computationally expensive, Crank–Nicolson offers a practical compromise suitable for numerical evaluation under such market conditions. With these characteristics, this method can be used as a stable and easily implemented initial numerical baseline for option pricing and basic sensitivity analysis.

From a practical perspective, the findings of this research equip practitioners in emerging financial markets, such as derivatives analysts and risk managers, with computational tools that can be efficiently implemented in common computing environments, including Python-based modeling or numerical spreadsheets. In practice, the Crank–Nicolson method can be used for initial decision-making and basic scenario evaluation before applying more complex models, especially in market conditions with moderate volatility. This study has limitations because it only considers the Black–Scholes model with constant volatility assumptions and has not conducted a detailed quantitative comparison of computational efficiency between numerical methods.

Therefore, further research could be directed toward systematic measurement of execution time, more in-depth analysis of convergence sequences, application of more realistic volatility models, and quantitative comparisons with alternative numerical methods to expand the relevance and practical validity of this study's results.

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