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The Valuation of European Options with Transaction Costs Using the Barles-Soner Model

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ABSTRACT

This study discusses the pricing of European call options by considering transaction costs using the Barles-Soner model. The method used in this study is an analytical and asymptotic approach based on literature studies. Stock and option price data were obtained from Yahoo Finance, while the risk-free interest rate was taken from the Federal Reserve Bank of St. Louis. Volatility was calculated based on historical stock return data, while transaction costs and risk aversion parameters were determined based on previous studies. The Barles-Soner model introduces non-linear effective volatility, which is estimated using an asymptotic approach to obtain effective volatility. The option price calculated using the Barles-Soner model is \$21,93, higher than the option price using the Black-Scholes model of \$19,78, with a difference of 9,84%. These results confirm that transaction costs have a significant effect on option prices. Therefore, the Barles-Soner model is more comprehensive in calculating stock option prices than the Black-Scholes model.

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1. INTRODUCTION

The modern financial market is growing rapidly, driving efficient capital allocation through various investment instruments. In the financial market, investors tend to seek opportunities that generate optimal returns with minimal risk [1][2]. To maximize investment and reduce risk, derivative products can be used as tools to protect investment portfolios from market volatility [3]. One derivative product that can be used is stock options. Stock options are a type of investment in the derivatives market that gives the contract holder the right to buy (call) or sell (put) shares at an agreed price (strike price) in the future [4]. There are two types of stock options: European stock options and American stock options. European stock options are stock options that can be exercised at maturity, while American stock options can be exercised at any time until maturity [5][6]. In determining the price of stock options, a mathematical model is needed that can produce accurate stock option prices [7]. Currently, there are many models that can be used to determine the price of stock options, one of which is the Black-Scholes model.

The Black-Scholes model was developed by Fischer Black and Myron Scholes in 1973 [8]. This model provides an analytical solution for European stock option pricing under perfect market conditions, with no transaction costs, constant volatility, and a constant risk-free interest rate [9]. This scenario is not in line with the volatility smile phenomenon, where volatility tends to vary according to the strike price. This condition confirms that financial markets are not always perfect [10]. Every transaction in the financial market involves transaction costs, such as brokerage fees and taxes [11]. The presence of transaction costs makes the Black-Scholes model less suitable for representing market conditions. Efforts to overcome market imperfections due to transaction costs have been pioneered by several previous studies. [12] Leland 1985 pioneered this approach by proposing a discrete hedging strategy to accommodate transaction costs. Further developments were made by Hodges and Neuberger 1989 [13], who reviewed transaction costs from a utility maximization perspective, and Boyle & Vorst 1992 [14], who analyzed the impact of transaction costs on the binomial model.

However, if transaction costs are taken into account, this will result in very high costs, as well as affecting hedging strategies and causing differences in the prices of European stock options obtained [15]. These limitations formed the basis for G. Barles and H. M. Soner in 1998 to modify the Black-Scholes model. The Barles-Soner model modifies the model by adding transaction costs. With the addition of transaction costs, the Barles-Soner model produces higher European stock option prices than the Black-Scholes model [16]. Most previous studies are considered inadequate for solving the modified Barles-Soner model. In response, Rafael Company et al. conducted research to help solve the Barles-Soner model using a computational approach [17]. This approach was used to address the nonlinear nature of the model without modifying it. On the other hand, the asymptotic approach developed by Whalley-Wilmott examines the effect of transaction costs on the formation of effective volatility [18]. Although promising, the application of this asymptotic approach to the Barles-Soner model, particularly in determining effective volatility and quantitative comparisons with the Black-Scholes model using the latest data, is still limited.

Based on this research gap, this study focuses on applying the Barles-Soner model combined with the asymptotic approach to European call option price data for technology stocks, as well as conducting a quantitative comparison with the Black-Scholes model. Therefore, the results of this study aim to produce European stock option price estimates that can assist investors and capital market practitioners in designing more accurate pricing and hedging strategies.

2. RESEARCH METHOD

This study uses analytical and asymptotic frameworks to determine the price of European options, taking transaction costs into account. The Black-Scholes model is solved analytically through variable transformation, while the Barles-Soner model is handled using an asymptotic approach to effective volatility. This method was chosen based on the focus of the research, which is to determine the price of European stock options by considering transaction costs using the Barles-Soner model [16]. The literature study method was applied to explore the basic theory, characteristics, and mathematical concepts of European stock option pricing. The literature used included financial mathematics books, journals, and scientific articles relevant to the research topic. The subject of this study is calling options maturing on February 20, 2026, on the shares of a technology company (XXXX) with closing prices taken from November 1, 2024, to November 28, 2025. The selection of XXXX shares was based on their high liquidity, availability of complete option data, and ability to represent the technology sector in the global stock market [19]. All data used in this study is secondary data.

The data was obtained from two main sources, namely Yahoo Finance and the Federal Reserve Bank of St. Louis. Data from Yahoo Finance includes closing prices, call option prices, strike prices, and contract maturity dates [20]. Meanwhile, risk-free interest rate data was obtained from the Federal Reserve Bank of St. Louis website based on tenors corresponding to the option maturity dates [21]. Volatility is calculated based on the standard deviation of historical stock price log-returns. The risk aversion and transaction cost parameters are taken from previous studies [13]. The selection of one option is intended to normalize the calculation, so that the results obtained represent the value and risk per option contract. Complete specifications related to the above data can be seen in Table 1 below:

Table 1. Data used

Variable	Value	Source
Stock price	\$278,85	Yahoo Finance
Risk-free interest rate	3,73%	FRED
Strike price	\$275	Yahoo Finance
Maturity date	0,230137	$\frac{(expiry\ date - valuation\ date)}{365}$
Volatility	0,3144091	$\sqrt{(252 \times (std)^2)}$
Transaction cost	2%	Hodges & Neuberger (1989)
Risk Aversion Parameter	1	Hodges & Neuberger (1989)
Number of options	1	Assumption

The selection of transaction cost parameters and risk aversion levels is fixed, with the aim of maintaining methodological consistency and focusing the analysis on illustrating the influence of transaction costs and risk aversion in the Barles-Soner model. This study uses two main models to determine the price of call options, namely the classic Black-Scholes model and the Barles-Soner model. The Black-Scholes model [8] is the basic model used to determine the price of stock options in a perfect market without transaction costs. Data processing begins with calculating the return and volatility of stocks, which are the main parameters in the Black Scholes model. Daily stock returns are calculated using the following formula:

$$r_t = ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

Where S_t is the closing price on day t and S_{t-1} is the closing price on the previous day. After the return value is obtained, the standard deviation of all return data is calculated to measure the level of dispersion of stock price movements relative to their average. The standard deviation formula is as follows[22]:

$$s = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2} \tag{2}$$

Where r_t is the stock return on day t, $\bar{r}$ is the average of r_t, and n is the number of observation periods. Meanwhile, volatility (σ) is calculated to describe the level of stock price fluctuations during the observation period. The estimation is based on the squared standard deviation of stock returns with time factor k adjustment as formulated [23]:

$$\sigma = \sqrt{ks^2} \tag{3}$$

Where s is the standard deviation of stock returns and k is the number of trading days in a year. If the data used is daily prices, the value of k is generally taken to be 252. Other parameters such as the strike price (K), risk-free interest rate (r), and maturity (T) are determined based on secondary data and research assumptions. The next stage is the construction of a mathematical model, which is the core of this study. The classic Black Scholes partial differential equation is initially written in terms of time t as:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial s} - rV = 0 \tag{4}$$

With the terminal condition $V(S,T) = \max(0, S - E)$ describing the value of the call option at maturity. The Barles-Soner model [16] is an extension of the Black-Scholes model that takes transaction costs into account. The partial differential equation used in this model is:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2\left(1 + \Psi\left(e^{r(T-t)}a^2 S^2 \frac{\partial^2 V}{\partial S^2}\right)\right)S^2\frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial s} - rV = 0 \tag{5}$$

Where $a = \mu\sqrt{\gamma N}$ and Ψ is a volatility correction function defined implicitly. The volatility correction function (Ψ) does not have a closed analytical form. Theoretically, the function is defined through the following nonlinear equation:

$$\Psi'(A) = \frac{\Psi(A) + 1}{2\sqrt{A\Psi(A) - A}} \tag{6}$$

Therefore, this study uses an asymptotic approach [18], as follows:

$$\Psi(A) = \frac{A}{1 + \sqrt{1 + |A|}}, \tag{7}$$

with

$$A = e^{rT}a^2 S^2 \Gamma \tag{8}$$

and

$$\Gamma = \frac{\partial^2 V}{\partial S^2} \tag{9}$$

Where S is the price of the underlying asset and Γ denotes the second-order sensitivity of the option price to the asset price.

In addition to the Whalley-Wilmott approach, there are several other approaches such as the Boyle-Vorst (1992) [14] and Paras-Avellaneda (1997) [24] approaches. However, the Whalley-Wilmott approach was chosen in this study because it is able to represent the nonlinear behavior of the volatility correction function more consistently in accordance with the Barles-Soner model, while also being stable and efficient. With this asymptotic approach, the nonlinearity in the Barles-Soner equation can be handled without solving additional equations. After the volatility correction function is obtained, the stock option price in the Barles-Soner model is calculated by substituting the effective volatility into the Black-Scholes analytical solution. The results of both models will be used for comparison in the price of European call options.

This comparison aims to see how transaction costs affect option values. In addition, the values of the transaction cost and risk aversion parameters are determined. The results of this comparison and analysis are presented in tabular and narrative form to see the relationship between these additional factors and European stock option prices.

3. RESULT AND ANALYSIS

3.1. Black-Scholes Model

Stock price movements are random and non-deterministic, so they cannot be predicted with certainty. However, these random changes are not completely irregular; there are still average patterns (trends) and degrees of uncertainty (volatility). To address these conditions, a stochastic model is used that can describe random variables that change continuously over time [25], so that it can be written as:

$$dS = \mu S dt + \sigma S dW_t \tag{10}$$

Where μ is the rate of return on stocks, σ is the volatility of stock prices, and dW_t is Brownian motion [26][27]. A portfolio consists of one option and $-\Delta$ shares at time t. This means that investors buy one option at a price of V and sell Δ shares at a price of S. Thus, the value of the portfolio is:

$$\Pi = V - \Delta S \tag{11}$$

During the time interval dt, the portfolio value changed by:

$$d\Pi = dV - \Delta dS \tag{12}$$

Since $V = V(S,t)$ depends on time t and stock price S, which is stochastic, using Ito's Lemma [28][29], we obtain:

$$dV = \left(\frac{\partial V}{\partial t} + \mu S \frac{\partial V}{\partial s} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right) dt + \sigma S \frac{\partial V}{\partial s} dW_t \tag{13}$$

Substituting equations (1) and (4) into equation (3), we obtain:

$$d\Pi = \sigma S\left(\frac{\partial V}{\partial S} - \Delta\right) dW_t + \left(\frac{\partial V}{\partial t} + \mu S \frac{\partial V}{\partial s} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - \Delta \mu S\right) dt \tag{14}$$

So that the changes in portfolio value obtained are deterministic. The random component dW_t in equation (5) can be eliminated by selecting a portfolio combination that makes the dW_t coefficient zero, namely:

$$\Delta = \frac{\partial V}{\partial S} \tag{15}$$

Meaning that the number of shares Δ is equal to the value of $\frac{\partial V}{\partial s}$ at time t, so that the change in the portfolio is deterministic, namely:

$$d\Pi = \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} \tag{16}$$

If funds amounting to Π are invested in risk-free assets, for example deposited in a bank, then within the time interval dt the funds will increase by $d\Pi$, namely:

$$d\Pi = r\Pi dt \tag{17}$$

Since the Black-Scholes model is based on the no-arbitrage principle, the two values of $d\Pi$ in equations (7) and (8) must be equal, resulting in:

$$r\Pi dt = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right) dt \tag{18}$$

By substituting the expressions Π and Δ in equations (2) and (6) into equation (9), then dividing both sides by dt to obtain the following Black-Scholes partial differential equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{19}$$

with boundary conditions for call options:

$$C(S,T) = \max\,(S-K,0) \tag{20}$$

Using the variable transformation method, the PDE can be converted into a standard heat equation, yielding the following solution:

$$C(S,T) = SN(d_1) - N(d_2)Ke^{-r(T)} \tag{21}$$

with

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T)}{\sigma\sqrt{T}} \tag{22}$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \tag{23}$$

where $N(x)$ is the standard normal cumulative distribution function[30]:

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\mathrm{x}} e^{-\frac{1}{2}v^2}\, dv \tag{24}$$

3.2. Barles-Soner Model

The determination of stock option prices using the Barles-Soner model is done by taking into account transaction costs and the level of investor risk aversion. With the addition of these two factors, market volatility (σ), which was previously constant, changes to effective volatility (σ_{ef}) which depends on the non-linear equation $\Psi(A)$. Thus, the equation can be expressed as follows:

$$\frac{\partial V}{\partial t} + \frac{1}{2}{\sigma_{ef}}^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial s} - r = 0 \tag{25}$$

where

$${\sigma_{ef}}^2 = \sigma^2\big(1 + \Psi(A)\big) \tag{26}$$

The function $\Psi(A)$ is not available in closed form because it is defined through a nonlinear ODE:

$$\Psi'(A) = \frac{\Psi(A) + 1}{2\sqrt{A\Psi(A) - A'}} \tag{27}$$

As a result of this nonlinearity, the use of an asymptotic approach is encouraged as an solving the Barles-Soner PDE. This situation calls for an alternative approach that can handle the nonlinearity of $\Psi(A)$ without having to solve explicitly.

For this purpose, the asymptotic analysis results of Whalley and Wilmott, who examined the optimal utility model with small transaction costs, are used. They show that the effect of transaction costs on option prices is manifested through a correction that is proportional to $|V_{ss}|^{4/3}$. Because in the Barles-Soner model, the quantity that controls volatility is:

$$A = e^{r(T-t)}a^2S^2\frac{\partial^2 V}{\partial S^2} \tag{28}$$

Therefore, this asymptotic structure indicates that the correction function $\Psi(A)$ should behave as $\Psi(A) \sim |A|^{2/3}$ when A is small, be linear near zero, and grow convex when A is sufficiently large. In other words, an approximate form for $\Psi(A)$ can be constructed that is consistent with both the Barles-Soner model and the Whalley-Wilmott asymptotic structure.

Based on this, a smooth and stable explicit approximation form was adopted as follows:

$$\Psi(A) = \frac{A}{1+\sqrt{1+|A|}} \tag{29}$$

This approximation satisfies three important conditions: (i) it is linear for small A, thus approximating the nonlinear solution, (ii) its value approximates $\Psi(A) \approx A$ for large A, thus aligning with the Barles-Soner model, and (iii) This approach is consistent with the asymptotic behavior of the Whalley-Wilmott formulation. By substituting equation (27) into equation (24), we obtain:

$${\sigma_{ef}}^2 = \sigma^2\left(1+\frac{A}{1+\sqrt{1+|A|}}\right) \tag{30}$$

After obtaining the effective volatility $({\sigma_{ef}}^2)$ from the Barles-Soner equation, the PDE can be calculated explicitly using the same method as the classic Black-Scholes model. Using the effective volatility $({\sigma_{ef}}^2)$ that has been obtained, the following solution is obtained:

$$C(S,T) = SN(d_1) - N(d_2)Ke^{-r(T)} \tag{31}$$

with

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{{\sigma_{ef}}^2}{2}\right)(T)}{\sigma_{ef}\sqrt{T}} \tag{32}$$

and

$$d_2 = d_1 - \sigma_{ef}\sqrt{T}, \tag{33}$$

where $N(x)$ is the standard normal cumulative distribution function:

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}v^2}\,dv \tag{34}$$

3.3. Determination of Stock Option Pricing

The stock price data used in this study was obtained from the website www.finance.yahoo.com. The data used was the call option price on technology company shares (XXXX) with closing prices taken from November 1, 2024 to November 31, 2025, amounting to 269 data points.

From the closing stock data, the last stock price was obtained at $\$278{,}85$. The strike price used was based on research assumptions, which was $\$275$, and the maturity period used was set at $0{,}25$ years or equivalent to three months. The risk interest rate used was obtained based on the Federal Reserve Bank of St. Louis in November 2025, which was $3{,}73\%$.

The first step is to calculate the daily return of stocks based on the logarithmic change in closing prices between days using equation (1). Based on the data processing results, positive and negative return values were obtained, indicating stock price fluctuations during the observation period. The results of the stock return calculations are shown in Table 2.

Table 2. Stock return calculation results

t	Date	S_t	S_{t-1}	r_t
2	1 November 2024	222,91	225,91	-0,013368
3	4 November 2024	222,01	222,91	-0,004045
⋮	⋮	⋮	⋮	⋮
234	25 November 2025	276,97	275,92	0,003798
235	26 November 2025	277,55	276,97	0,002091
236	28 November 2025	278,85	277,55	0,004672

The return values in Table 2 were then used to calculate the average daily return ($\bar{r}$). The calculation results show that the average daily return on shares is 0,00078, which means that in general, the share price increases by 0,078% every day. Meanwhile, based on equation (2), the standard deviation value obtained is 0,019805, which illustrates a fairly high level of daily stock price fluctuation. Furthermore, based on equation (3), the annual volatility obtained is 0,3144091. All basic parameters of this study are summarized in Table 3.

Table 3. Parameter values

No.	Parameter	Symbol	Value
1.	Last stock price	S_0	$278,85
2.	Strike price	K	$275
3.	Maturity date	T	0,230137
4.	Risk-free interest rate	r	0,0373
5.	Average daily return	$\bar{r}$	0,00078
6.	Standard deviation	s	0,0239
7.	Volatility	σ	0,3144091
8.	Transaction costs	μ	0,02
9.	Risk aversion parameter	γ	1
10.	Number of options	N	1

The parameter values in Table 3 are used as the main input in the option pricing stage. The initial stage is carried out by finding the effective volatility (σ_{ef}) value to be used in the Barles-Soner model. Calculation results using the Barles-Soner model produce a call option price value at the initial condition of $21,94. This value is then validated by comparing it with the analytical solution of the classic Black-Scholes model, which is calculated using equation (19). From the analytical calculation, the call option value is obtained to be $19,78. To assess the accuracy of the method used, the relative difference between the analytical results was calculated using the following formula:

$$E_R = \frac{|C_A - C_N|}{C_A} \times 100\% \tag{33}$$

Where C_A is the option value calculated using the Barles-Soner model, and C_N is the option value calculated using the classic Black-Scholes model. A higher E_R value indicates that the Barles-Soner model produces stock option prices that are closer to actual market conditions. A comparison of option prices calculated using the classic Black-Scholes model and the Barles-Soner model is presented in Table 4.

Table 4. Comparison of Black-Scholes and Barles-Soner Option Prices

No	Model	Option Price	Relative Difference (%)
1.	Classic Black-Scholes	$19,78	0
2.	Barles-Soner	$21,94	9,84

The results in Table 4 show a relative difference of 9,84% between option prices calculated using the classic Black-Scholes model and the Barles-Soner model. This difference reflects the impact of transaction costs and risk aversion, which are not accounted for in the Black-Scholes model. From a financial market perspective, the increase in option prices can be interpreted as an adjustment to the hedging risk resulting from market friction.

Higher effective volatility also causes greater price sensitivity, so hedging strategies must be adjusted more frequently.

However, as an asymptotic approach, the Whalley-Wilmott method has limitations, with the accuracy of the results being highly dependent on transaction cost parameters and risk aversion. Thus, the potential for approximation errors may increase when the parameters are outside the asymptotic domain. Therefore, the results obtained in this study should be understood as analytical illustrations. Further evaluation requires comparison with numerical solutions or alternative approaches, which are beyond the scope of this study.

4. CONCLUSION

This study compares the pricing of European stock options using the Black-Scholes Model and the Barles-Soner Model, which take into account transaction costs and risk aversion. This addition changes constant volatility to effective volatility, which is nonlinear in the Barles-Soner Model. Effective volatility is able to capture market dynamics more adaptively than the assumption of constant volatility in the Black-Scholes Model. The asymptotic approach used in this study is able to handle the nonlinearity of the Barles-Soner equation.

By applying this method to a single option contract and specific parameters, it was found that the option price generated by the Barles-Soner model was higher than that generated by the classic Black-Scholes model. This difference was indicated by a relative difference of 9,84%, which shows that transaction costs and risk aversion contribute significantly to the formation of option value.

Although the study focused on a single underlying asset and a single transaction cost and risk aversion parameter, the results obtained provide an initial indication of the magnitude of the impact of market friction on option valuation. Overall, the results of this study show that the Barles-Soner model produces higher European stock option prices by incorporating transaction costs. This model provides a more accurate basis for investors and capital market practitioners in designing pricing and hedging strategies. Further research is recommended to examine the impact of variations in transaction cost and risk aversion parameters in depth, as well as to expand the application of this model to other types of options and financial instruments in order to improve the generalization and validity of the research results.

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