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Simplex vs. Revised Simplex for Budget Allocation in Madrasah Teacher and Staff Development Programs under MoRA Kudus, Indonesia

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ABSTRACT

This study compared the Simplex and Revised Simplex Methods for allocating the 2025 budget for the madrasah teacher and staff development programs under the Madrasah Education Division of the Ministry of Religious Affairs (MoRA) in Kudus, Indonesia. A quantitative case study design was employed to formulate a weighted linear programming model that identifies the most beneficial combination of programs under fixed budget constraints, with an emphasis on procedural comparison rather than policy impact. Both methods converged to the same optimal solution after four iterations, indicating equivalent computational performance for the small-scale model. The optimal allocation resulted in implementable program frequencies and achieved a maximum total weighted benefit of 129,133.6, primarily by prioritizing cost-free, high-benefit training programs while maintaining compliance with budget limits. Methodologically, the study demonstrates a transferable and reproducible linear programming framework that contributes to applied mathematics by illustrating how Simplex-based techniques can be adapted as transparent decision-support tools for small-scale public-sector budgeting problems. Although the Revised Simplex Method offers theoretical advantages for larger models, empirical differences in this case were minimal due to the model's scale. The study was limited to a single fiscal year and did not include sensitivity analysis, which should be addressed in future research.

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1. INTRODUCTION

Linear programming is a core branch of applied mathematics that focuses on optimization problems, particularly those involving the maximization or minimization of a linear objective function subject to linear constraints [1]. It provides a systematic framework for determining the most efficient allocation of limited resources among competing activities. Linear programming has been widely applied in fields such as production, finance, transportation, and public-sector management [2].

Among the various optimization techniques, the Simplex Method, originally developed by George Dantzig in 1947, remains one of the most fundamental algorithms in linear programming [3]. Recent studies reaffirm that the Simplex algorithm explores the polyhedral feasible region by iteratively moving from one extreme point to another along the edges of the polyhedron to reach the optimal solution [4]. The process requires precise numerical reasoning and accuracy in each iteration, which reflects the ability to interpret and apply mathematical concepts appropriately in problem-solving situations [5].

Although effective for small and medium-scale problems, the Simplex Method requires full tableau recalculation at each iteration, which can increase the computational burden as the number of decision variables and constraints grows. To address this limitation, the Revised Simplex Method was developed to update only essential components of the basis matrix, thereby reducing redundant computations while preserving solution accuracy [6].

While these algorithmic advances have been well documented, their practical adoption of linear programming in public budgeting contexts remains limited. Despite the proven success of linear programming in industrial and financial sectors, its application in educational budgeting has received comparatively little attention. Most studies have focused on optimizing production schedules, transportation routes, or investment portfolios, while few have explored how mathematical optimization could support strategic decision-making in educational institutions [7]. Beyond these domains, mathematical optimization techniques have also been applied to public-sector budgeting and resource allocation problems. For example, goal programming has been used to formulate budget planning and optimize financial allocations in regional government budgeting frameworks, demonstrating how mathematical models can support structured budget decisions for competing public expenditure categories [8]. Linear programming has been used to model and solve general resource allocation problems where limited resources must be distributed optimally under constraints, highlighting its theoretical and practical relevance for constrained budgeting contexts [9]. Recent research has also applied linear programming models specifically to budget allocation problems with complex constraints in public financial management systems, showing potential improvements in resource utilization and allocation efficiency [10]. However, applications of such optimization techniques to educational budgeting, particularly within religious or public institutions, remain limited.

In Indonesia, madrasahs, Islamic educational institutions overseen by the Ministry of Religious Affairs (MoRA), play a crucial role in providing both general and religious education. Within MoRA, the Madrasah Education Division is responsible for improving teacher quality and institutional capacity across Islamic schools [11]. Azizi, Fauzi, and Gunawan reported that the MoRA of Banten Province implemented a strategic plan aligned with Law No. 14 of 2005 on Teachers and Lecturers, emphasizing teacher certification programs covering pedagogical, social, professional, and personal competencies. These initiatives were supported through collaboration with madrasah principals, Subject Teacher Consultations (MGMP), and Teacher Performance Groups (KKG), alongside periodic formative, summative, and progress-based evaluations. However, their implementation frequently encountered financial constraints [12], even though budget allocation plays a significant role in supporting and sustaining the quality of the educational process in madrasahs [13]. At the regional level, such constraints are often more pronounced. Interviews with the Head of the Madrasah Education Division in Kudus Regency indicate that a substantial portion of the budget is absorbed by salaries and routine operational expenditures, leaving limited funding for teacher and staff development programs. As a result, capacity-building efforts rely largely on teacher motivation, school-initiated workshops, and national platforms such as MOOC Pintar rather than structured, budget-supported initiatives. Given increasing accountability demands in public-sector spending and fixed budget ceilings, the adoption of systematic, data-driven budget allocation approaches has become increasingly urgent to avoid inefficiencies and misallocation of resources.

Several recent studies have employed linear programming techniques to optimize resource allocation across various sectors. Safitri et al. employed the Revised Simplex Method within a goal programming framework to maximize profit in a home-industry setting in Padang Panjang, demonstrating the method’s efficiency in multi-objective decision contexts [14]. Syifa et al. utilized the Simplex Method to determine optimal production strategies for a manufacturing company [15]. Similarly, Sudiantini et al. applied the Simplex Method supported by the POM-QM software to small and medium enterprises (SMEs), resulting in optimization calculations that identified strategies for achieving maximum profit and a more efficient allocation of limited resources [16]. Recent Indonesian studies have likewise applied the Simplex Method to optimize profit in culinary SMEs such as homemade traditional cakes, confirming its effectiveness for production planning under raw material and capacity constraints [17]. A 2025 systematic literature review on Indonesian SMEs confirms that the Simplex Method is consistently effective for optimizing production and profit under multiple resource constraints [18]. More recently, Shrivastava and Dwivedi solved multi-objective transportation problems using the Revised Simplex Method. The study demonstrated that the Revised Simplex Method, integrated with the Weight Sum approach, efficiently generated optimal solutions for multi-objective transportation models under varying weight conditions [19]. Collectively, these studies confirm the effectiveness of linear programming techniques, particularly the Simplex and Revised Simplex Methods, in improving efficiency and optimizing resource utilization. Nevertheless, their applications have largely focused on industrial, transportation, and financial domains, with limited attention to educational or governmental budgeting systems.

While optimization-based studies rely on quantitative models, research on madrasah education has predominantly employed qualitative and descriptive approaches focusing on policy implementation, teacher development, and institutional management. Previous studies consistently report that teacher competence development in madrasahs is pursued through certification programs, collaborative professional forums, and school-based initiatives, yet these efforts are frequently constrained by limited funding, managerial weaknesses, low teacher motivation, and inadequate institutional support [12], [20]–[23]. More specifically, professional development activities include self-development, scientific publications, and innovative work integrated into school planning [20], while teacher professionalism has been linked to improved instructional practices and student outcomes [21]. However, weaknesses in leadership management, the absence of structured development plans, limited facilities, and challenges in information technology use continue to hinder program effectiveness [22], [23]. Overall, this body of literature demonstrates that research on madrasah education has largely emphasized descriptive evaluations of teacher development and program implementation, with minimal integration of quantitative or optimization-based approaches into financial decision-making. This stands in contrast to the growing body of Indonesian research that systematically reviews and applies the Simplex Method in SMEs and other sectors, indicating substantial untapped potential for similar quantitative approaches in madrasah education budgeting [18].

These gaps highlight several unresolved issues. First, optimization-based approaches to educational budgeting, particularly within religious government institutions, remain underexplored, as most existing studies emphasize descriptive analyses of policy implementation and professional development rather than quantitative decision-support models. Second, direct procedural comparisons between the Simplex and Revised Simplex Methods in applied case settings are still limited. Third, to the best of the authors' knowledge, no prior study has applied mathematical optimization to budget management within the Madrasah Education Division of Kudus Regency. Accordingly, this study addresses these gaps by providing an applied, procedural comparison of the Simplex and Revised Simplex Methods in the context of budget allocation under the Ministry of Religious Affairs.

Accordingly, this study conducts a procedural and educational comparison of the Simplex and Revised Simplex Methods using three effectiveness indicators. First, computational efficiency, defined in this study as procedural efficiency reflected by the number of iterations required to reach the optimal solution. Second, solution accuracy, which refers to the extent to which the obtained solution satisfies all model constraints. Third, implementation feasibility, which evaluates whether the resulting allocations are realistic and yield integer-based, practically applicable results representing feasible program distributions.

Therefore, this study compares the Simplex and Revised Simplex Methods as decision-support tools for the 2025 budget allocation and selection of teacher and staff development programs under the Madrasah Education Division of MoRA Kudus Regency. This study focuses on a procedural and educational demonstration of the Simplex and Revised Simplex Methods, rather than on the actual impact of budget allocation on teacher performance or program outcomes. Accordingly, the study aims to (1) examine procedural differences in computational efficiency, and (2) identify the most beneficial combination of teacher and administrative staff development programs within limited financial resources. The novelty of this research lies in applying mathematical optimization techniques to educational budgeting within a religious government institution. Through this contribution, the study advances both applied mathematics and educational management by promoting transparent, data-driven, and efficient budget allocation strategies within MoRA's budgeting system.

2. RESEARCH METHOD

This study employed a quantitative case study design that applied both the Simplex Method and the Revised Simplex Method to the same linear programming model, demonstrating their procedural differences within a real budgeting context. The comparison focused on the algorithms' iterative structures rather than on policy impact, actual expenditure, or educational outcomes. Therefore, the study does not claim empirical effectiveness in improving teacher performance or program quality.

The research was conducted in a single institutional setting, namely the Madrasah Education Division of the Ministry of Religious Affairs (MoRA) in Kudus Regency, Indonesia, which is responsible for planning and managing teacher and administrative staff development programs. The model was developed for the 2025 fiscal planning cycle, as it represents the most recent complete budget year with accessible and internally validated data. As is typical for case-study-based optimization research, the findings are context-specific. However, the modeling framework, variable definitions, and computational procedures are replicable and may be adapted to other regions or budget planning scenarios.

The study analyzed a single planning scenario characterized by a fixed budget ceiling, a single set of program frequency bounds, and a single configuration of priority weights, reflecting the actual planning conditions within the Madrasah Education Division. Three development programs with zero direct financial cost were fixed at their respective upper frequency bounds as a modeling assumption. This assumption reflects the institutional policy that these programs can be implemented without budgetary expenditure and are therefore always maximized in practice. Fixing these variables reduced unnecessary degrees of freedom in the model and allowed the analysis to focus on budget-constrained programs without affecting feasibility or optimality. No sensitivity or parametric analysis was conducted; the stability of the optimal solution under alternative budget ceilings, program bounds, or priority weight configurations is therefore identified as a direction for future research.

The study population included all educational programs administered by the Madrasah Education Division that required budget allocation. The sample consisted of teacher and administrative staff development programs, selected through purposive sampling based on three criteria: (1) the programs directly contribute to teacher or staff development, (2) they require measurable financial resources, and (3) their performance indicators (such as number of training sessions, participants, and measurable outcomes) can be expressed quantitatively and analyzed through a linear programming approach. Purposive sampling was chosen for this study because it allowed the researcher to deliberately select programs that are most relevant and information-rich for addressing the research objectives. According to Memon et al. [24], purposive sampling in quantitative research enabled the selection of cases based on predefined criteria aligned with the study's aims, thus enhancing the relevance and analytical precision of the sample. This approach is consistent with the updated sampling guidelines proposed by Zickar and Keith [25], who emphasized the importance of purposeful and context-appropriate sample selection to improve data quality and research validity.

Secondary data consisted of an internal 2025 planning summary provided by the Head of the Madrasah Education Division, including program types, budget ceilings, costs, frequency bounds, estimated participants, and institutional priority scores (Table 1). An interview with the Head of the Madrasah Education Division was conducted solely to clarify budgeting practices and the interpretation of institutional priorities. These priorities are determined internally based on regulatory directives, program feasibility, and past performance evaluations. All quantitative parameters used in the model were cross-checked against official budget planning records and internal administrative documents to ensure data accuracy. The researcher did not construct or modify the priority scores. Then, weighted benefits per program implementation (w_i) computed as:

$$w_i = a_i \times p_i \tag{1}$$

where a_i denotes the institutional priority weight and p_i denotes the estimated number of participants.

Methodological and model validity were ensured through expert validation by a single academic with specialization in applied mathematics. The expert reviewed the formulation of the objective function and constraints, the transformation of the model into standard form, the use of artificial variables and the Big-M approach, and the correctness of the computational procedures for both the Simplex and Revised Simplex Methods. A single expert was deemed sufficient because the validation focused on deterministic mathematical correctness and algorithmic logic rather than on subjective judgment or empirical interpretation, which is consistent with common practice in applied optimization studies. This approach is in line with Subhaktiyasa [26], who emphasized the importance of domain-expert verification to ensure methodological accuracy and consistency in quantitative studies.

Then, a comparative analysis between the Simplex and Revised Simplex Methods was conducted using three indicators. First, procedural computational efficiency, defined as the number of iterations required to reach the optimal solution, and the computational structure involved in each iteration [27]. This indicator reflects procedural efficiency, acknowledging that both methods typically follow identical pivot logic. Second, solution accuracy, which measures how well the obtained solution satisfies all model constraints and achieves the intended optimization objectives [19]. Third, implementation feasibility, which assessed how realistic and applicable the resulting solution is, particularly whether the optimization model yields implementable integer results that reflect feasible program allocations. Last indicator aligned with Adouani [28], who emphasized that integer-based optimization models provide realistic and practically applicable solutions in allocation and scheduling contexts. These indicators were synthesized based on the operational characteristics of linear programming models and their relevance to public budgeting contexts, as highlighted in recent optimization and budgeting literature. Together, they provided a comprehensive basis for determining the relative effectiveness of the Simplex and Revised Simplex methods in optimizing budget allocation. Elapsed computation time per iteration was recorded using Microsoft Excel on a standard hardware setup, providing a practical reference for small-scale models. Because the differences were negligible for the present model, these measurements are reported descriptively rather than as comparative performance metrics.

To solve the optimization model, this study applied two linear programming methods and used an artificial variable initialization when required. The Simplex Method was implemented in its classical pivot–tableau form, starting from a basic feasible solution, the algorithm iteratively selected entering and leaving variables and performed pivot operations until no improving pivot remained. In each iteration, the pivot operation updated the tableau to reflect the new basis. This updating process was carried out using Gauss–Jordan row operations, where the computation consisted of transforming the pivot row and then applying row-replacement operations to all remaining rows, including the objective row (z) [29].

a. Pivot row:
   a) Replace the leaving variable in the basic column with the entering variable.
   b) New pivot row = current pivot row ÷ pivot element
b. All other rows, including z
   New row = (current row) – (its pivot column coefficient) × (new pivot row)

The Revised Simplex method was a systematic procedure for implementing the steps of the Simplex method using a smaller array, thus saving storage space [30]. Algorithm for the Revised Simplex Method [29]:

a. Step 0. Construct a starting basic feasible solution, and let $\mathbf{B}$ and $\mathbf{C}_\mathrm{B}$ be its associated basis and objective coefficients vector, respectively.
b. Step 1. Compute the $\mathbf{B}^{-1}$, inverse of the basis $\mathbf{B}$, by using an appropriate inversion method.
c. Step 2. For each nonbasic vector $\mathbf{P}_\mathrm{j}$, compute $\mathrm{z_j - c_j} = \mathbf{C}_\mathrm{B}\mathbf{B}^{-1}\mathbf{P}_\mathrm{j} - \mathrm{c_j}$. If $\mathrm{z_j - c_j} \geq 0$ in maximization (≤ 0 in minimization) for all nonbasic vectors, stop; the optimal solution is $\mathbf{X}_\mathbf{B} = \mathbf{B}^{-1}\mathbf{b}, \mathrm{z} = \mathbf{C}_\mathrm{B}\mathbf{X}_\mathrm{B}$. Otherwise, determine the entering vector $\mathbf{P}_\mathrm{j}$ having the most negative (positive) $\mathrm{z_j - c_j}$ in case of maximization (minimization) among all nonbasic vectors.
d. Step 3. Compute $\mathbf{B}^{-1}\mathbf{P}_\mathrm{j}$. If all the elements of $\mathbf{B}^{-1}\mathbf{P}_\mathrm{j}$ are negative or zero, stop; the solution is unbounded. Otherwise, use the ratio test to determine the leaving vector $\mathrm{P_i}$.
e. Step 4. Form the next basis by replacing the leaving vector $\mathbf{P}_\mathrm{i}$ with the entering vector $\mathbf{P}_\mathrm{j}$ on the current basis $\mathbf{B}$. Go to step 1 to start a new iteration.

Because some constraints in the model are expressed in "≥" or "=" form, an artificial-variable technique was needed to obtain an initial feasible basis: the Big-M approach was used here by adding artificial variables with sufficiently large penalty coefficients in the objective function so that they are driven out of the basis as the optimization proceeds [31]. A moderate penalty value of $\mathrm{M} = 100$ was adopted, consistent with standard treatments of small-scale linear programming problems, to ensure numerical stability while enforcing the elimination of artificial variables during optimization [32]. This choice is appropriate for the present model, which is small in scale and intended to illustrate the procedural mechanics of the Simplex and Revised Simplex Methods.

Data analysis was conducted manually, with Microsoft Excel used to facilitate tabular operations, arithmetic calculations, and the recording of iteration results for both the Simplex and Revised Simplex Methods. This approach enabled transparent documentation of each computational step, supporting methodological clarity and reproducibility, in line with principles of transparent reporting [33]. Although manual computation is not intended to represent large-scale or real-time optimization environments, it is appropriate for the study's objective of demonstrating and comparing the procedural mechanics of the two methods in a small, well-defined case. Descriptive analysis was employed to present the optimization results, while methodological comparison was used to evaluate differences in procedural steps, iteration structure, and feasibility outcomes.

3. RESULT AND ANALYSIS

3.1 Result

This section presents the computational results obtained using both the Simplex and Revised Simplex Methods, followed by a comparative discussion of their efficiency, accuracy, and feasibility. The Ministry of Religious Affairs (MoRA) Office in Kudus Regency allocated IDR 27,000,000 for programs aimed at improving the quality of teachers and administrative staff. Table 1 summarizes the primary work programs included in the optimization model.

Table 1. Work Programs to Improve the Quality of Teachers and Administrative Staff

Program	Program Description	Cost per Program	Freq Limit per Year	Participant	Priority Weight (%)	Weighted Benefit per Freq
1	Empowerment of the teacher working-group (KKG/MGMP)	0	6-12	4,664	100	4,664
2	Coordination meetings for madrasah principals	17,000,000	2-3	376	100	376
3	Capacity-building for educational operators	10,000,000	1-2	376	95	357.2
4	Teacher training through the MOOC Pintar of the MoRA	0	Min 2	4,664	75	3,498
5	Administrative staff training through the MOOC Pintar of the MoRA	0	Min 2	774	60	464.4

Source: Head of the Madrasah Education Division, MoRA Kudus Regency

To optimize budget allocation, both the Simplex and Revised Simplex Methods were applied using the same linear programming model. The objective function maximized the total weighted number of beneficiaries (teachers and staff), subject to budget and frequency constraints. The decision variables, representing the frequency of each program, were defined as x_i, where $i = 1,2,3,4,5$.

The linear programming model was formulated as follows. The objective function is given in (2), and the constraints are given in (3) - (10). Hence, the objective function was to maximize:

$$z = 4664x_1 + 376x_2 + 357.2x_3 + 3498x_4 + 464.4x_5 \quad (2)$$

Subject to:

$$6 \leq x_1 \leq 12 \quad (3)$$

$$2 \leq x_2 \leq 3 \quad (4)$$

$$1 \leq x_3 \leq 2 \quad (5)$$

$$x_4 \geq 2 \quad (6)$$

$$x_5 \geq 2 \quad (7)$$

$$c_2x_2 + c_3x_3 \leq 27000000 \quad (8)$$

$$c_2x_2 \leq 17000000 \quad (9)$$

$$c_3x_3 \leq 10000000 \quad (10)$$

where $x_i \geq 0$ and c_i represents the unit cost of the program i, where $i = 1,2,3,4,5$.

The coefficients in the objective function represent program-weighted benefits per implementation, computed as the product of the priority weight and the number of potential participants. Programs 1, 4, and 5 incur no additional local implementation costs. Consequently, their decision variables are not constrained by the budget limitation and are therefore set at their maximum feasible values to maximize the total objective value.

Programs 4 and 5 utilize the national MOOC Pintar platform, which does not require additional local funding. The minimum frequency of two sessions per year reflects MoRA's requirement for teachers and staff to participate in at least one training per semester. Based on actual activity records that 12 MOOC sessions were held in 2023 and 24 in 2024, resulting in an average of 18 sessions annually. Therefore, the feasible frequency range for both programs was set at 2–18 sessions, representing realistic operational capacity. While treating MOOC programs as cost-free is consistent with their funding structure, it may also influence the model toward allocating higher frequencies to these activities. Similarly, the priority weights were adopted directly from the division's internal planning process; although this preserves ecological validity, it still reflects subjective managerial judgment. These assumptions constitute practical simplifications but may affect the resulting allocation pattern.

Although the initial formulation included five decision variables, three variables were analytically fixed at their feasible bounds due to zero-cost structures or inactive constraints. Under these conditions, the fixed variables do not affect the optimality conditions of the remaining variables, and their reduced costs do not enter the basis during optimization. As a result, the effective optimization problem is reduced to two active decision variables without loss of generality, and the reduced model captures the essential budget–benefit trade-off faced by the institution.

The parameters c_2 and c_3 represent the unit costs of Program 2 and Program 3, respectively. Because these costs depend on implementation frequency (x_2 and x_3), the optimal frequencies must first be determined. After the optimal values of x_2 and x_3 are obtained, c_2 and c_3 are computed by dividing each program's total cost by its corresponding optimal frequency.

Before iterative computation, the reduced model was transformed into standard form by introducing surplus, slack, and artificial variables to convert all inequality constraints into equalities, thereby enabling computation using both the Simplex and Revised Simplex Methods. Specifically, variables x_6, x_9 were introduced as surplus variables, x_8, x_{11} as slack variables, and x_7, x_{10} as artificial variables. The resulting standard form is presented in Equations (11) – (14):

$$\text{Max } z = 376x_2 + 357.2x_3 + 127{,}291.2 + 0.x_6 - 100x_7 + 0.x_8 + 0.x_9 - 100x_{10} + 0.x_{11} \quad (11)$$

The constant term $127{,}291.2$ represents the aggregated weighted benefits of Programs 1, 4, and 5 evaluated at their maximum feasible frequencies. Because these programs are fixed at their bounds due to zero-cost structures, the constant does not affect the optimality conditions and is therefore excluded during the iterative Simplex computation and added back after the optimal solution is obtained. Accordingly, the objective function used in the iterative process is:

$$\text{Max } z^* = 376x_2 + 357.2x_3 + 0.x_6 - 100x_7 + 0.x_8 + 0.x_9 - 100x_{10} + 0.x_{11} \quad (12)$$

Subject to:

$$2 \leq x_2 \leq 3 \Rightarrow x_2 \geq 2 \text{ and } x_2 \leq 3 \Rightarrow x_2 - x_6 + x_7 = 2 \text{ and } x_2 + x_8 = 3 \quad (13)$$

$$1 \leq x_3 \leq 2 \Rightarrow x_3 \geq 1 \text{ and } x_3 \leq 2 \Rightarrow x_3 - x_9 + x_{10} = 1 \text{ and } x_3 + x_{11} = 2 \quad (14)$$

where $x_i \geq 0, i = 2, 3,6,7,8,9,10,11$.

Both the Simplex and Revised Simplex Methods converged to the same optimal solution after four iterations. The optimal reduced objective value was $z^* = \frac{9212}{5}$, yielding a total optimal objective value of $z = 129{,}133.6$ after adding the constant term derived from the zero-cost programs. The corresponding optimal solution was $x_2 = 3$ and $x_3 = 2$.

Although both methods reached identical optimal allocations, their intermediate basis transitions differed across iterations. A comparative summary of the entering and leaving variables at each iteration is presented in

Table 2. Detailed simplex tableaux and pivot operations for the Simplex Method are provided in Appendix A, while complete computational steps for the Revised Simplex Method are documented in Appendix B.

Table 2. Iteration Steps and Basic Variable Transitions

Method	Iteration			
	$0 \rightarrow 1$	$1 \rightarrow 2$	$2 \rightarrow 3$	$3 \rightarrow 4$ (Final)
Simplex	ev x_2	ev x_6	ev x_3	ev x_9
	lv x_7	lv x_8	lv x_{10}	lv x_{11}
Revised Simplex	ev x_2	ev x_3	ev x_6	ev x_9
	lv x_7	lv x_{10}	lv x_8	lv x_{11}

Source: Researcher's Computation

Accordingly, a summary of the optimization results is presented in Table 3.

Table 3. Optimal Program Frequencies and Weighted Benefits

1	Empowerment of Teacher Working-Group (KKG/MGMP)	12	55,968
2	Coordination Meeting for Madrasah Principals	3	1,128
3	Capacity Building for Educational Operators	2	714.4
4	Teacher MOOC Pintar Training	18	62,964
5	Staff MOOC Pintar Training	18	8,359.2
	Total Weighted Benefit		129,133.6

Source: Computational Results of the Simplex and Revised Simplex Methods

The costs of implementing Programs 2 and 3 were capped at IDR 17,000,000 and IDR 10,000,000, respectively. Under the optimal solution ($x_2 = 3$ and $x_3 = 2$), the corresponding unit costs were IDR 5,666,667 per implementation for Program 2 and IDR 5,000,000 per implementation for Program 3.

Both the Simplex and Revised Simplex Methods produced identical optimal results. To ensure the validity of these findings, the study employed expert validation: a single linear programming specialist, selected based on formal qualifications and prior experience in reviewing optimization models, reviewed the model formulation, algorithmic steps (including pivot operations and basis updates), and manual computation results through a structured document review and step-by-step verification. The expert confirmed that the iteration logic and optimal solution were mathematically sound and appropriate for the budgeting context. Because only one expert was involved, inter-rater reliability could not be established; thus, the validation strengthens internal correctness but does not constitute multi-expert consensus.

Although the optimization model provided a mathematically valid allocation, the results were not directly compared with the division's actual budget realization or measured impacts on teacher outcomes due to the lack of accessible performance data. Consequently, the analysis shows theoretical efficiency but does not establish empirical improvements in educational quality. The findings should therefore be interpreted as model-based recommendations rather than direct evidence of program impact.

3.2 Analysis

The results demonstrate that both the Simplex and Revised Simplex Methods successfully optimized the allocation of a limited IDR 27,000,000 budget for teacher and administrative staff development within the Madrasah Education Division of MoRA Kudus. The resulting optimal frequencies for the work programs were: 12 sessions for teacher working-group empowerment (Program 1), 3 coordination meetings for madrasah principals (Program 2), 2 capacity-building trainings for educational operators (Program 3), and 18 MOOC-based trainings each for teachers and administrative staff (Programs 4 and 5). This combination produced the maximum weighted benefit of 129,133.6. The feasibility of this solution confirms that the constraint set is internally consistent and that the reduced two-variable model correctly captures the essential budget–benefit trade-off faced by the institution.

The comparative evaluation of the Simplex and Revised Simplex Methods was conducted using three indicators: procedural computational efficiency, solution accuracy, and implementation feasibility. First, with respect to procedural computational efficiency, both methods converged to the optimal solution in four iterations. Measured purely by iteration count, no difference was observed between the two algorithms for this small-scale problem. The outcome aligns with Sulaiman and Muhammed [6], who reported that both Simplex and Revised Simplex are valid and can be generally applied to solve linear programming problems, often producing identical optimal solutions.

However, beyond iteration count, the two methods differ in their computational structure. In the classical Simplex Method, each iteration requires full tableau updates, involving row operations across all constraints and variables, with computational effort proportional to the tableau size. In contrast, the Revised Simplex Method operates on a reduced representation by updating the inverse of the basis matrix and computing reduced costs through matrix–vector products. Consequently, each iteration of the Revised Simplex Method requires fewer arithmetic operations on the order of basis updates and vector multiplications rather than full tableau row operations and lower memory usage. While this structural advantage does not produce a measurable difference in the present small model, it becomes increasingly significant as the number of decision variables and constraints grows. Therefore, the identical iteration counts observed here do not contradict the theoretical efficiency advantage of the Revised Simplex Method for larger-scale optimization problems.

Second, solution accuracy. Both algorithms produced the same optimal solution that satisfies all model constraints, including: (1) budget ceiling (IDR 27,000,000), (2) program-specific cost limits, (3) minimum–maximum frequency bounds, and (4) non-negativity conditions. The coincidence of optimal objective values and basis structures indicates that the model formulation, constraint transformation, and algorithmic procedures were mathematically consistent. This confirms that both algorithms reliably identify the same optimal extreme point of the feasible polyhedron under the given assumptions.

Third, implementation feasibility. The resulting frequencies are realistic and implementable, reflecting both operational capacity and historical activity patterns of the Madrasah Education Division. For example, three coordination meetings for principals and two operator trainings per year correspond to actual practice and remain feasible within staff workload, program availability, and budget constraints. Integer values also ensure that each recommended activity represents a real program that can be executed, which is essential for budgeting applications. This aligns with Adouani [28], who emphasized that integer-based linear programming models provide realistic and actionable solutions to scheduling and allocation problems. The finding further supports the conclusions of Kanu, Ozurumba, and Emerole [34], who highlighted that linear programming models are effective tools for practical decision-making in public-sector organizations, particularly because they generate implementable allocation strategies that reflect real operational limitations.

Despite these strengths, several limitations should be acknowledged. First, the model was built on a single scenario one budget ceiling, one set of bounds, and one set of priority weights without sensitivity analysis. Consequently, the robustness of the optimal solution under alternative budget levels or weighting schemes remains unexamined. Second, the linear programming model relies on standard assumptions of linearity, parameter certainty, and continuous decision variables, and it does not explicitly address integrality constraints, rounding effects, or robustness to parameter uncertainty. Third, the study relied on data from one fiscal year and one local office, which limits external validity and restricts generalization to other districts or multi-year budgeting contexts. Fourth, the priority weights used in the optimization model were based solely on managerial judgment and were not derived through structured decision-making methods; this may introduce subjective bias. Fifth, only one expert participated in the model validation process, preventing triangulation or inter-rater reliability assessment. Sixth, the model did not compare the optimized allocation to the division's actual budget realization or to measured improvements in teacher and staff performance, which means that the results demonstrate mathematical optimality but not empirical impact.

Overall, the optimization results confirm that linear programming techniques, both Simplex and Revised Simplex, can serve as effective decision-support tools in public-sector budgeting, especially within educational contexts. Under the constraints and weighting structure defined in the model, both methods successfully identify an allocation that maximizes the weighted benefit of teacher and staff development programs within a fixed budget. The results, therefore, demonstrate the mathematical optimality of the allocation under the stated assumptions, rather than empirical evidence of realized value or program impact. Methodologically, this study contributes by illustrating how linear programming techniques can be applied to educational budgeting problems, an area previously dominated by qualitative or normative planning. In practice, the proposed model may serve as a decision-support framework for government agencies, including the Ministry of Religious Affairs. Future research may extend this framework by incorporating mixed-integer programming to explicitly enforce integrality, multi-objective optimization to balance efficiency with equity considerations, and sensitivity or robustness analysis to assess solution stability under alternative budget scenarios.

4. CONCLUSION

This study demonstrates that both the Simplex and Revised Simplex Methods yield the same mathematically optimal solution for the proposed linear programming model of budget allocation for teacher and administrative staff development within the Madrasah Education Division of the Ministry of Religious Affairs (MoRA) Kudus. Both algorithms produced the same optimal solution, achieving the maximum weighted benefit of 129,133.6 while satisfying all policy, budgetary, and feasibility constraints. For this small-scale problem, the two algorithms exhibit equivalent performance in terms of convergence and solution accuracy. However, the Revised Simplex Method retains a theoretical computational advantage due to its reduced matrix-based structure. These results should be interpreted as methodological findings that demonstrate mathematical optimality under the stated assumptions rather than evidence of realized institutional or educational impact.

The model's limitations include reliance on a single fiscal year, one set of priority weights, no sensitivity analysis, validation by only one expert, and no comparison with actual budget realization or program outcomes. Thus, the results represent mathematically optimal recommendations rather than demonstrated improvements in educational quality or organizational performance. Overall, the study confirms that linear programming can serve as a practical and transparent decision-support tool for public-sector budgeting in educational contexts. Future studies should incorporate sensitivity analysis, multi-year and multi-institutional datasets, structured weighting methods, and mathematically richer formulations such as integer or mixed-integer programming, multi-objective optimization, and robust or stochastic linear programming to address parameter uncertainty and enhance policy relevance. The use of computational tools such as Python, Excel Solver, or LINGO may further support larger-scale simulations and extended computational experiments.

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