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Modeling and Forecasting Volatility through EGARCH-X and EGARCH-CJ Models

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ABSTRACT

This study compares the performance of EGARCH-X and EGARCH-CJ models in forecasting financial market volatility using daily TOPIX data (2004–2011). Model parameters were estimated using an efficient Bayesian MCMC framework. The results indicate that the EGARCH-CJ model, which decomposes volatility into continuous and jump components, provides a superior in-sample fit. More importantly, in out-of-sample forecasting, the EGARCH-CJ model demonstrates significantly better accuracy for medium- and long-term horizons (e.g., MSE reductions up to 30% at the 5-day horizon, with significant Diebold-Mariano statistics). In contrast, the standard EGARCH model remains more effective for short-term forecasts. These findings underscore the importance of explicitly modeling jump dynamics for medium-term risk management in the Japanese stock market, offering valuable insights for financial modelers and risk managers.

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1. INTRODUCTION

Volatility reflects how much asset returns fluctuate over time and serves as a key indicator of market risk [1], [2]. Conditional heteroskedasticity models, particularly the EGARCH framework, are widely recognized for capturing nonlinear and asymmetric volatility dynamics [3], [4]. Its logarithmic variance specification ensures positivity and models the leverage effect [3], [5]. Recent findings further confirm that EGARCH-type models remain highly accurate and adaptable across various market conditions [6].

Advances in high-frequency data have encouraged extensions of EGARCH. Studies in [7], [8] developed the EGARCH-X model by incorporating an exogenous realized measure (X), such as Realized Volatility (RV), which improves model fit and forecasting performance. Building on this, [9] decomposed X into continuous (C) and jump (J) components, forming the EGARCH-CJ model, which enhanced volatility. However, [9] relied solely on 5-minute RV and a short Maximum Likelihood Estimation (MLE) based evaluation, leaving open questions regarding the model's performance across alternative sampling frequencies and multi-horizon forecasts.

This study addresses these gaps by: (1) comparing EGARCH-X and EGARCH-CJ using four realized measures 1-minute, 5-minute, 10-minute RV, and Realized Kernel (RK); (2) applying the Adaptive Random Walk Metropolis (ARWM) algorithm within a Bayesian Markov Chain Monte Carlo (MCMC) framework to improve estimation efficiency over MLE, as used in [9]; and (3) evaluating volatility and Value-at-Risk (VaR) forecasts across multiple horizons. These contributions advance both the methodological development of EGARCH-type models and their practical relevance for volatility forecasting and financial risk management.

By focusing on the Japanese TOPIX index, which encompasses a broad range of market conditions, including the 2008 Global Financial Crisis, this research offers insights into how the combination of continuous-jump decomposition and data frequency affects volatility forecasting accuracy in a highly liquid market. Beyond its empirical focus, the study also addresses broader regulatory and practical stakes in financial risk management. Post-crisis reforms such as the Basel III accords have placed stronger emphasis on capital adequacy, stress testing, and accurate VaR estimation as core elements of financial stability frameworks [10], [11]. Recent studies have shown that volatility and VaR modeling based on GARCH-type frameworks remain essential for meeting Basel III risk-measurement standards [12]. Moreover, the growing relevance of jump modeling has been underscored by recent global shocks including COVID-19-induced volatility, geopolitical tensions, and rapid asset price swings highlighting the need for models capable of capturing abrupt market movements [13]. In this context, linking EGARCH-X and EGARCH-CJ models to regulatory requirements becomes increasingly important, as improved volatility and VaR forecasts support more accurate capital allocation, strengthen stress-testing procedures, and enhance compliance with Basel III backtesting thresholds. These practical implications make the adoption of jump-enhanced volatility models directly relevant for risk managers and supervisory institutions. These developments underscore the timeliness and significance of this study in developing volatility models that strike a balance between methodological rigor and regulatory applicability.

2. RESEARCH METHOD

2.1 Data and Sample

This study examines the TOPIX data, an index comprising all domestic common stocks listed on the First Section of the Tokyo Stock Exchange [14]. This index was selected because it serves as a key benchmark that reflects the performance of the Japanese market. Its high liquidity is essential for ensuring the accuracy of the RV and RK measures calculated from tick-by-tick data, where the RK is implemented using the Parzen kernel (see [15]) with the optimal bandwidth selection procedure as proposed by [16]. These are essential parts of the EGARCH-X and EGARCH-CJ models.

The sample for this case study includes daily TOPIX values from January 2004 to December 2011, totaling 1962 daily observations. This period was chosen because it encompasses a full range of market conditions, including a time of pre-crisis stability, extreme volatility during the peak of the 2008–2009 Global Financial Crisis, and the subsequent recovery phase. This significant financial event creates a “natural laboratory” to test the models’ ability to capture jump components and volatility asymmetry effectively. However, the crisis period may introduce potential data quality issues such as temporary illiquidity, widened bid ask spreads, or increased noise in high-frequency prices that could affect the precision of realized measures. These conditions do not invalidate the dataset but highlight the importance of using robust estimators like RV and RK to mitigate distortions, particularly during turbulent episodes [17], [18].

The study applies purposive sampling to select the sample, and the index trades actively throughout the entire period. The purposive sampling method suits the analysis because the sample is intentionally chosen based on predefined criteria to meet the research objectives. In practice, the sampling process ensures that the data possess essential characteristics—such as high liquidity and the presence of a crisis period—needed to support a valid comparison between the EGARCH-X and EGARCH-CJ models. Nevertheless, this study focuses on a single index (TOPIX), a single country (Japan), and a single significant crisis period, which may limit the external validity of the findings. Future research is therefore encouraged to conduct cross-market validation by extending the analysis to multiple indices and countries with varying market structures, liquidity conditions, and regulatory environments, in order to test the robustness and broader applicability of the EGARCH-CJ framework.

2.2 Model Specification

This subsection details the econometric framework, including return definitions, realized measures, and the conditional variance equations for the compared models.

2.2.1 Returns and Realized Measures

Let P_t be the financial asset value at time t ($t = 1, \ldots, T$). The daily log-return is defined as in Eq. (1) [19], [20]:

$$R_t = \log P_t - \log P_{t-1}. \tag{1}$$

Given high-frequency intraday returns $\{R_{t,i}\}_{i=1}^{N}$, the RV for day t is computed as shown in Eq. (2):

$$X_t^2 = \sum_{i=1}^{N} R_{t,i}^2. \tag{2}$$

The analysis decomposes RV into continuous and jump components, following the method in [9]. First, the median RV estimator M_t [21] and the jump test statistic Z_t are calculated using Eqs. (3) and (4), respectively:

$$M_t = \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{K}{K-2}\right) \times \sum_{i=2}^{N-1} \text{Med}\left(|R_{t,i-1}|, |R_{t,i}|, |R_{t,i+1}|\right)^2, \quad (3)$$

$$Z_t = \frac{(X_t - B_t)}{X_t\sqrt{(0.25\pi^2+\pi-5)K^{-1}\max(1, Q_t B_t^{-2})}}, \quad (4)$$

where B_t is the realized bipower variation [21] and Q_t is the realized tripower quarticity [22]. Subsequently, the continuous C_t and jump J_t components are defined by Eqs. (5) [9]:

$$C_t = I(Z_t \leq \emptyset_\alpha)X_t + I(Z_t > \emptyset_\alpha)M_t \text{ and } J_t = I(Z_t > \emptyset_\alpha)(X_t - M_t), \quad (5)$$

where $\emptyset_\alpha$ is the α-quantile of the standard Normal distribution ($\alpha = 0.99$ in this study).

2.2.2 Volatility Models

The mean equation for all models is $R_t = \sigma_t \varepsilon_t,\ \varepsilon_t \sim N(0,1)$, where $N(0,1)$ denotes the standard Normal distribution. The conditional variance $h_t = \log \sigma_t^2$ follows these specifications:

a. EGARCH(1,1): The baseline model is given by Eq. (6):

$$h_t = \omega + \alpha_1 \left|\frac{R_{t-1}}{\sigma_{t-1}}\right| + \alpha_2 \frac{R_{t-1}}{\sigma_{t-1}} + \beta h_{t-1}. \quad (6)$$

b. EGARCH-X(1,1): This extension of EGARCH incorporates the lagged log realized measure $\log X_{t-1}$, as shown in Eq. (7):

$$h_t = \omega + \alpha_1 \left|\frac{R_{t-1}}{\sigma_{t-1}}\right| + \alpha_2 \frac{R_{t-1}}{\sigma_{t-1}} + \beta h_{t-1} + \lambda \log X_{t-1}. \quad (7)$$

c. EGARCH-CJ(1,1): This model further decomposes the exogenous signal into continuous and jump parts, specified by Eq. (8):

$$h_t = \omega + \alpha_1 \left|\frac{R_{t-1}}{\sigma_{t-1}}\right| + \alpha_2 \frac{R_{t-1}}{\sigma_{t-1}} + \beta h_{t-1} + \lambda \log C_{t-1} + \gamma \log(J_{t-1} + 1). \quad (8)$$

2.3 Priors and MCMC Settings

A Bayesian MCMC framework estimates all model parameters. The estimation employs the Adaptive Random Walk Metropolis (ARWM) algorithm, which dynamically adjusts the proposal distribution to improve sampling efficiency and accelerate convergence. ARWM dynamically adjusts proposal distributions, reduces chain autocorrelation, and accelerates convergence, outperforming traditional MLE, which often struggles with nonlinearity, asymmetry, and local optima, as demonstrated in [23]. The specification uses weakly informative normal priors for all parameters: $\theta \sim N(0,10)$, where θ represents any model parameter $(\omega, \alpha_1, \alpha_2, \beta, \lambda, \gamma)$, following standard Bayesian practices in volatility modeling (see [23], [24]). This prior centers at zero with a large variance, allowing the data to dominate the posterior. Recent Bayesian GARCH and stochastic process modeling studies commonly use such diffuse Normal priors to balance weak informativeness with computational tractability and convergence stability [25], [26]. Sensitivity analyses presented in Section 3.1 confirm the robustness of results to alternative prior choices (e.g., Beta, Uniform). The procedure runs 35,000 MCMC iterations for each model. The first 5,000 iterations serve as a burn-in period and are discarded. Initial parameter values are set according to Eq. (9):

$$\omega^{(0)} = 0.1, \alpha_1^{(0)} = 0.1, \alpha_2^{(0)} = 0.1, \beta^{(0)} = 0.9, \lambda^{(0)} = 0.5, \gamma^{(0)} = 0.5. \quad (9)$$

Convergence assessment employs both visual and statistical diagnostics. Trace plots provide a visual check, where a dense, stationary fluctuation around the mean indicates that the Markov chain has reached its target distribution [27]. For a quantitative measure, the Integrated Autocorrelation Time (IACT) estimates convergence efficiency using the adaptive truncated periodogram estimator [25], [28].

2.4 Forecasting and Evaluation Metrics

2.4.1 Volatility Forecasting

The out-of-sample forecasting procedure splits the data into a training set (5 January 2004 to 30 December 2008) for model fitting and a test set (5 January 2009 to 30 December 2011) for forecast evaluation. The forecasting procedure generates predictions recursively for horizons $h = 1$ (daily), 5 (weekly), 10 (biweekly), and

20 (monthly). For $h = 1$, the one-step-ahead conditional variance $\hat{\sigma}^2_{t+1}$ is computed directly. For $h > 1$, an iterative expectation-based approach replaces future shocks ε_{t+k} and $|\varepsilon_{t+k}|$ with their unconditional means 0 and $\sqrt{\frac{1}{2}\pi}$, respectively. In the EGARCH-X and EGARCH-CJ models, future values of exogenous variables (X_t, C_t, J_t) are set to their historical means for $h > 1$. The following is an illustration:

Date	$h = 1$	⋯	$h = 5$	⋯	$h = 10$	⋯	$h = 20$
5 Jan. 2009	$\hat{\sigma}^2_{N+1,1}$		$\hat{\sigma}^2_{N+1,5}$		$\hat{\sigma}^2_{N+1,10}$		$\hat{\sigma}^2_{N+1,20}$
6 Jan. 2009	$\hat{\sigma}^2_{N+2,1}$		$\hat{\sigma}^2_{N+2,5}$		$\hat{\sigma}^2_{N+2,10}$		$\hat{\sigma}^2_{N+2,20}$
7 Jan. 2009	$\hat{\sigma}^2_{N+3,1}$		$\hat{\sigma}^2_{N+3,5}$		$\hat{\sigma}^2_{N+3,10}$		$\hat{\sigma}^2_{N+3,20}$
⋮							

Recent studies, such as [6] and [29], have widely adopted Mean Squared Error (MSE) as standard tools for measuring the predictive accuracy of volatility models. When the actual volatility process cannot be directly observed, a suitable exogenous variable serves as a true volatility σ_t^2. The MSE evaluates forecasting accuracy as defined in Eq. (10):

$$\text{MSE} = \frac{1}{N_{\text{test}}}\sum_{t=1}^{N_{\text{test}}}(\sigma_t^2 - \hat{\sigma}_t^2)^2. \tag{10}$$

The Diebold-Mariano (DM) test provides formal statistical comparisons between models [30], as shown in Eq. (11):

$$DM = \frac{\bar{d}}{\sqrt{\hat{S}_d \cdot N_{\text{test}}^{-1}}}, \tag{11}$$

where $d_t = L(e_{1t}) - L(e_{2t})$ represents the loss differential between forecast errors e_{1t} and e_{2t} from models 1 and 2, $\bar{d}$ is the sample mean of d_t, and $\hat{S}_d$ is a consistent estimator of the asymptotic variance of $\bar{d}$. A significant DM statistic indicates a statistically significant difference in forecast accuracy between the two models.

2.4.2 VaR Forecasting and Evaluation

Among various risk indicators, VaR has become one of the most commonly used tools for measuring market risk exposure. According to [31], VaR calculates the maximum expected loss over a set holding period of t days, given a confidence level of $(1-\alpha)$ %. The α-level VaR calculation uses Eq. (12):

$$\text{VaR}_\alpha(t) = N_\alpha \hat{\sigma}_t, \tag{12}$$

where N_α is the α-quantile of standard Normal distribution.

While statistical hypothesis-based approaches test whether VaR violations match the expected frequency, they offer limited insight for model comparison. In contrast, loss-function-based evaluations—specifically the Regulatory Quadratic Loss (RQL) and Failure Rate (FR)—examine both the frequency and severity of violations, enabling regulators and risk managers to identify models that minimize cumulative loss. This study employs both RQL and FR to evaluate VaR forecasts:

a. Eq. (13) defines the RQL, see [32]:

$$\text{RQL} = \begin{cases} 1 + (\text{VaR}_t - R_t)^2 & \text{if } R_t < \text{VaR}_t, \\ 0 & \text{if } R_t \geq \text{VaR}_t. \end{cases} \tag{13}$$

A lower RQL value indicates a better-performing model that produces fewer or less severe losses beyond the predicted threshold.

b. Eq. (14) defines the FR :

$$\text{FR} = \frac{1}{n}\sum_{t=1}^{n} I(R_t < \text{VaR}_t), \tag{14}$$

where $I(\cdot)$ is an indicator function. A well-calibrated model produces an FR close to the nominal significance level α.

2.5 Research Gap and Contributions

Previous studies, such as [9], have demonstrated the potential of decomposing realized volatility into continuous and jump components within GARCH-type frameworks. However, their analysis relied solely on 5-minute RV and was limited to estimation via MLE. This leaves several critical gaps in understanding the model's performance under different data frequencies, estimation methods, and forecasting horizons. This study addresses these limitations and makes the following contributions:

1. Multi-Frequency Realized Measures: We extend the evaluation of EGARCH-X and EGARCH-CJ models by employing four distinct realized measures 1-minute, 5-minute, and 10-minute RV, along with the RK. This allows us to systematically assess how sampling frequency and noise robustness affect model fit and forecast accuracy, an aspect overlooked in prior work.
2. Bayesian Estimation Framework: We implement an efficient Bayesian MCMC estimation procedure using the ARWM algorithm. This approach offers improved estimation efficiency and robustness for complex, nonlinear volatility models compared to traditional MLE, providing more reliable parameter estimates and uncertainty quantification.
3. Comprehensive Multi-Horizon Forecast Evaluation: Moving beyond in-sample fit, we conduct a thorough out-of-sample forecasting exercise across short- to long-term horizons (1, 5, 10, and 20 days). The evaluation encompasses both volatility forecasts (using MSE and DM tests) and risk forecasts (VaR using RQL and FR), offering a holistic view of model performance for both risk modeling and regulatory applications.

These contributions collectively advance the methodological development of jump-enhanced volatility models and provide empirical insights into their practical utility for financial forecasting and risk management in a major equity market.

3. RESULT AND ANALYSIS

3.1 Convergence of the Estimator

A sensitivity analysis confirms the robustness of the Bayesian estimates to prior specification. Table 1 shows that posterior estimates for the EGARCH-CJ parameters remain stable across different prior distributions (Normal, Beta, Uniform), with minimal changes in posterior means, standard deviations (in brackets), and log-likelihoods (LLs). This indicates that the data dominate the posterior, making the results insensitive to reasonable choices of priors. Prior transformations (e.g., $0.5(\theta+1)\sim B(\cdot,\cdot)$) were applied where needed to maintain parameters within the empirical range of −1 to 1.

Table 1. Posterior estimates of the GARCH-CJ model (10-minute) for various priors

	Prior					
	$N(0,1)$	$N(0,10)$	$B(5,6)$	$B(3,4)$	$U(-1,1)$	$U(-10,10)$
	0 (1)	0 (3.16)	−0.09 (0.29)	−0.14 (0.35)	0 (0.33)	0 (33.33)
	Posterior					
ω	0.353 (0.073)	0.352 (0.084)	0.258 (0.054)	0.358 (0.091)	0.335 (0.102)	0.345 (0.071)
α_1	0.016 (0.045)	0.020 (0.041)	0.081 (0.028)	0.007 (0.086)	0.020 (0.045)	0.015 (0.044)
α_2	−0.193 (0.026)	−0.193 (0.027)	−0.207 (0.027)	−0.205 (0.027)	−0.192 (0.027)	−0.195 (0.026)
β	0.634 (0.043)	0.633 (0.049)	0.612 (0.040)	0.610 (0.058)	0.645 (0.055)	0.633 (0.048)
λ	0.573 (0.067)	0.576 (0.084)	0.555 (0.064)	0.590 (0.074)	0.553 (0.102)	0.570 (0.075)
γ	0.132 (0.163)	0.131 (0.131)	0.285 (0.097)	0.199 (0.259)	0.135 (0.190)	0.155 (0.171)
LL	−3104.85	−3105.38	−3106.84	−3106.59	−3105.26	−3104.85

For instance, Figure 1 displays the trace plots of the estimated values for the EGARCH-CJ model using 10-minute RV data. Each Markov chain visually appears to have converged/stabilized, as its graph fluctuates around the mean (red line). These diagnostics confirm that the estimation procedure generates stable parameter values and that both the prior distribution and the data's log-likelihood function are selected appropriately.

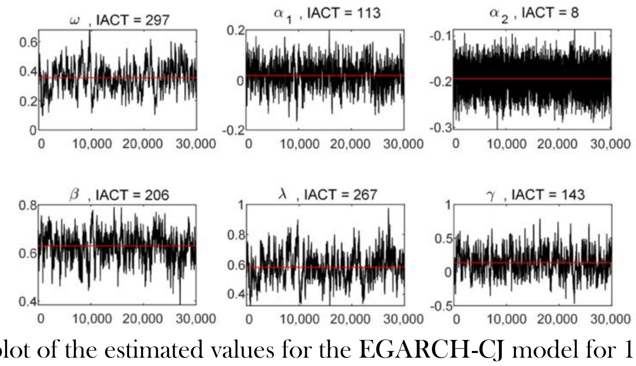

Figure 1. Trace plot of the estimated values for the EGARCH-CJ model for 10- minute RV data

Convergence is assessed using the IACT and Effective Sample Size (ESS). The ESS, calculated as retained draws divided by IACT, indicated the number of effectively independent samples. While conservative thresholds (ESS ≥ 200–400) are often recommended [26], [33], [34], our design produces an ESS of approximately 100 given the estimated IACT. This outcome meets a minimal benchmark but falls short of stricter targets, a noted limitation for transparency.

3.2 In sample Parameter Estimation and Model Diagnostics

The in-sample analysis uses the complete observation period from 5 January 2004 to 30 December 2011 for model estimation and diagnostics. Due to space limitations, this study presents estimation results only for data involving 10-minute RVs, as shown in Table 2. SD denotes standard deviation, and * indicates parameter significance from a value of 0 based on a 95% Highest Posterior Density interval (see [24] for the estimation algorithm). In the EGARCH-X specification, the exogenous realized measure contributes meaningfully to volatility dynamics. In the EGARCH-CJ model, the continuous and jump components are both statistically significant, indicating that each plays an important role in explaining conditional variance. The parameter estimation results for all cases of realized measures indicate that the addition of exogenous variables does not affect changes in the asymmetric parameter α_2. However, the negligible impact of exogenous variables on asymmetry parameter across all specifications contrasts with theories emphasizing the role of external shocks in amplifying leverage effects (e.g., [5]). This divergence may reflect TOPIX's unique response to crises or limitations in the realized measures' ability to capture asymmetric shock transmission.

Table 2. Parameter estimates for models applied to 10-minute RV data

Statistics	Parameter					
	ω	α_1	α_2	β	λ	γ
EGARCH						
Mean	−0.168*	0.236*	−0.123*	0.955*	-	-
SD	0.023	0.030	0.016	0.008	-	-
EGARCH-X						
Mean	0.318*	0.022	−0.195*	0.628*	0.605*	-
SD	0.091	0.050	0.026	0.062	0.111	-
EGARCH-CJ						
Mean	0.352*	0.020	−0.193*	0.633*	0.576*	0.131*
SD	0.084	0.041	0.027	0.049	0.084	0.131

Note: Asterisk * represents significance from a value of 0

To select the model with the best data fit, this study uses four criteria: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Adjusted BIC (ABIC), and Consistent AIC (CAIC) (see [35] for the formulas of these four criteria). The selection is based on criterion dominance because different criteria often support other models. For each criterion, a smaller value indicates a better-fitting model. Table 3 presents the criterion values for the EGARCH-X and EGARCH-CJ models across all realized measure cases. In all cases, AIC selects the EGARCH-CJ model as the best model. However, based on the dominance of the four criteria, the EGARCH-CJ model outperforms only for the 10-minute RV measure.

Table 3. Information criteria for EGARCH-X and EGARCH-CJ across realized measures

Model	Criteria	RV1	RV5	RV10	RK
EGARCH-X	AIC	6258.1	6226.8	6227.7	6220.2
	ABIC	6270.2	6238.8	6239.8	6232.3
	BIC	6286.0	6254.7	6255.6	6248.1
	CAIC	6291.0	6259.7	6260.6	6253.1
EGARCH-CJ	AIC	6254.0	6224.9	6221.9	6219.7
	ABIC	6274.4	6239.3	6236.4	6234.1
	BIC	6293.5	6258.4	6255.4	6253.1
	CAIC	6299.5	6264.4	6261.4	6259.1

Note: The bold values indicate the minimum values of the error metric

The EGARCH-CJ model provides superior in-sample fit over EGARCH-X when using 10-minute RV, as indicated by lower AIC, BIC, ABIC, and CAIC values. This aligns with evidence that decomposing volatility into jump and continuous components improves model fit [13], [36]. The finding that the 10-minute RV outperforms 5-minute RV for Japan's TOPIX suggests market-specific optimal sampling, where this coarser interval may better separate persistent volatility from jumps while reducing microstructure noise [37], highlighting the noise-resolution trade-off in realized measure selection.

3.3 Evaluation of Volatility Forecasting

Table 4 summarizes the out-of-sample volatility forecasting evaluation across all models. For short-term forecasts (h = 1), the standard EGARCH model without exogenous inputs gives the lowest or similar MSE values across most realized measures. This suggests that recent historical volatility adequately captures short-term market dynamics without needing additional realized variables. In contrast, for medium- to long-term horizons (h = 5, 10, and 20), the EGARCH-CJ model typically produces smaller MSE values than both the EGARCH and EGARCH-X models. This benefit is clear across almost all realized measures.

Using the simpler model as the benchmark, the DM tests in Table 5 show that for short-term forecasts (h = 1), the statistics are negative or insignificant, confirming that the standard EGARCH benchmark is superior. However, for medium- and long-term horizons, the tests reveal a consistent and significant superiority of the EGARCH-CJ model, as indicated by large positive t-statistics (e.g., 7.91, 16.52, 17.08 for 1-min RV), confirming that the EGARCH-CJ model's forecasting advantage is not only empirical but also statistically robust. More importantly, the magnitude and consistency of these positive statistics across multiple realized measures and horizons demonstrate that the improvement is systematic rather than sample-specific, providing strong formal evidence that incorporating continuous and jump components materially enhances multi-step volatility forecasting accuracy.

Table 4. Out-of-sample volatility forecasting evaluated by the MSE measure

X	Model	h = 1	h = 5	h = 10	h = 20
1-min RV	EGARCH	4.38	3.79	2.86	2.31
	EGARCH-X	6.43	2.03	1.16	2.23
	EGARCH-CJ	6.81	1.83	1.98	2.04
5-min RV	EGARCH	4.18	4.21	3.34	2.87
	EGARCH-X	4.74	2.68	2.83	2.88
	EGARCH-CJ	4.18	2.58	2.74	2.81
10-min RV	EGARCH	5.42	6.48	5.64	5.28
	EGARCH-X	6.06	5.07	5.25	5.31
	EGARCH-CJ	6.34	4.99	5.17	5.23
RK	EGARCH	4.67	5.09	4.23	3.81
	EGARCH-X	4.96	3.61	3.77	3.81
	EGARCH-CJ	7.40	3.37	3.49	3.52
$RV^{(5)}$	EGARCH	7.69	5.23	3.91	2.99
	EGARCH-X	6.86	3.29	2.83	2.83
	EGARCH-CJ	4.09	2.56	2.73	2.79

Note: The bold values indicate the minimum values of the error metric

Table 5. Out-of-sample evaluation of the volatility forecasts: DM test

X	Benchmark Model	Alternative EGARCH Model							
		h = 1		h = 5		h = 10		h = 20	
		X	CJ	X	CJ	X	CJ	X	CJ
1-min	EGARCH	−1.07	−1.06	1.40	1.40	1.49	1.57	1.13	1.89
RV	EGARCH-X		−1.06		7.91*		16.52*		17.08*
5-min	EGARCH	−1.01	−0.98	1.34	1.34	1.33	1.34	0.70	0.83
RV	EGARCH-X		1.07		3.83*		8.80*		9.93*
10-min	EGARCH	−1.05	−1.02	1.31	1.32	1.25	1.30	0.60	0.84
RV	EGARCH-X		1.25		2.35*		7.69*		8.02*
RK	EGARCH	−1.00	−1.06	1.32	1.37	1.30	1.49	0.76	1.86
	EGARCH-X		−1.06		8.97*		9.45*		9.81*
$RV^{(5)}$	EGARCH	−1.06	−1.03	1.41	1.37	1.41	1.35	0.89	0.57
	EGARCH-X		−1.01		−3.17*		−14.4*		−17.2*

Note: t-statistics for volatility forecasts; $|t| > 1.96$ indicates 5% significance, and * denotes significance

A comparison across different models and exogenous data frequencies also indicates that the best daily volatility forecasts come from the standard EGARCH model using the 5-minute RV measure. Meanwhile, the most accurate forecasts for longer horizons come from the EGARCH-CJ model, which utilizes the 1-minute RV measure. This indicates that high-frequency realized measures enhance the model's responsiveness in multi-step forecasting. In contrast, moderate-frequency realized inputs, such as the 5-minute RV, provide greater stability and valuable insights for short-term daily volatility dynamics.

The degradation of 1-minute RV in short-term forecasts, contrasting with its long-term improvement, signals microstructure noise contamination from effects like bid-ask bounce [38]. This noise fades over longer horizons as persistent volatility dominates. Pre-averaging $RV^{(5)}$—applied here via the 5-minute RV measure—mitigates this by smoothing high-frequency data [38]. Table 4 shows $RV^{(5)}$ yields more stable and accurate forecasts, especially at $h = 1$, confirming that noise-robust techniques balance high-frequency detail with noise reduction.

The findings in this study match those in [9]. They showed that adding jump components to realized-based volatility models significantly improves forecasting accuracy, especially over medium-term periods. Similar to the results in [9], the EGARCH-CJ model in this study does well by clearly modeling the continuous and jump parts. This captures both smooth market trends and sudden changes from extreme events. This consistency, now reinforced by rigorous DM testing, supports the idea that considering breaks in volatility processes strengthens the reliability and consistency of multi-step forecasts, particularly when using high-frequency realized measures.

In summary, the EGARCH model is best for short-term forecasts, especially when using moderate-frequency realized inputs like the 5-minute RV. The EGARCH-CJ specification gives the most accurate volatility predictions for medium and longer terms, particularly when backed by high-frequency (1-minute) realized measures—a conclusion firmly supported by both lower MSE values and statistically significant DM test results. This result shows a trade-off between immediate forecast accuracy and long-term stability, underscoring that including continuous and jump components is key to improving predictive performance during persistent volatility patterns.

3.4 Evaluation of VaR Forecasting

Based on the results in Table 6, the EGARCH-CJ model usually shows lower RQL values for realized volatility measures using return sampling intervals of 1-minute, 5-minutes, and 10-minutes, especially at short and long horizons ($h = 1$, 10, and 20). This means that modeling continuous and jump components helps the model better capture sudden changes in volatility in the short term and maintain forecasting accuracy over longer periods. However, when using the RK measure, the standard EGARCH model shows the lowest RQL values, particularly at longer horizons ($h = 10$ and 20). This suggests that the RK estimator's inherent strength against market microstructure noise and its ability to handle discontinuities reduce the benefits of modeling jumps explicitly. Therefore, while the EGARCH-CJ model is highly effective for realized measures affected by microstructure noise, the EGARCH model performs better with smoother, noise-resistant inputs, such as RK.

Table 6. Out-of-sample VaR forecasting evaluated by RQL measure with $\alpha = 5\%$

X	Model	$h = 1$	$h = 5$	$h = 10$	$h = 20$
1-min RV	EGARCH	30.55	70.85	28.05	62.56
	EGARCH-X	29.87	39.82	30.16	70.08
	EGARCH-CJ	26.05	37.83	29.50	62.09
5-min RV	EGARCH	30.51	70.72	27.58	62.51
	EGARCH-X	29.18	59.09	31.69	71.05
	EGARCH-CJ	28.02	60.46	30.34	70.09
10-min RV	EGARCH	30.47	70.53	28.02	62.50
	EGARCH-X	27.94	56.95	31.67	70.83
	EGARCH-CJ	27.01	60.96	30.79	70.29
RK	EGARCH	30.54	70.82	28.04	62.53
	EGARCH-X	25.94	54.92	31.68	70.99
	EGARCH-CJ	25.51	53.50	29.22	65.21
$RV^{(5)}$	EGARCH	30.60	71.10	28.08	62.65
	EGARCH-X	29.52	61.52	29.07	58.32
	EGARCH-CJ	25.20	53.33	31.54	70.45

Note: The bold values indicate the minimum values of the loss metric

Across the realized measures, the EGARCH-CJ model with the RK input delivers the strongest overall forecasting performance at $h = 1$. At longer horizons ($h = 5$ and $h = 20$), the EGARCH-CJ model using the 1-minute RV produces the best results. At the intermediate horizon ($h = 10$), the standard EGARCH model combined with the RK input yields the lowest RQL value. These findings indicate that high-frequency realized measures enhance EGARCH-CJ's responsiveness to short- and long-horizon risk, whereas the RK measure complements the simpler EGARCH framework for medium-term forecasts where volatility dynamics are smoother and less influenced by sudden jumps.

Table 7 presents the out-of-sample VaR forecasting results assessed using the FR criterion at the 5% confidence level. The FR measures how well the proportion of VaR violations matches the nominal rate. Values nearest to 5% indicate the best calibration performance. Across all forecast horizons, the standard EGARCH model consistently achieves FR values closest to the nominal 5%. This suggests strong calibration and reliability. Specifically, for the 10-minute RV and RK, EGARCH demonstrates the best alignment at h = 1, 5, and 20, with FR values ranging from 4.77% to 5.82%. This shows the model's stability in maintaining a consistent exceedance frequency across both short and long horizons.

Failure Rate (FR) results show varying optimal calibration across models and realized measures. For instance, Table 7 indicates that EGARCH and EGARCH-X tie for the 1-minute RV at h = 1, while EGARCH performs best at h = 5 and EGARCH-X leads at h = 10 and h = 20. With 5-minute RV, EGARCH-X is best at h = 1, and EGARCH-CJ provides the best alignment at h = 5, 10, 20. Overall, mid-frequency measures (e.g., 5-minute RV) offer the most balanced calibration, whereas high-frequency data (1-minute RV) combined with exogenous terms benefits longer horizons.

Table 7. Out-of-sample VaR forecasting evaluated by FR measure

X	Model	h = 1	h = 5	h = 10	h = 20
1-min RV	EGARCH	4.77%	4.91%	4.50%	5.32%
	EGARCH-X	4.77%	4.77%	4.91%	4.91%
	EGARCH-CJ	6.14%	5.32%	5.59%	5.46%
5-min RV	EGARCH	4.77%	4.91%	4.64%	5.32%
	EGARCH-X	5.18%	4.37%	4.64%	4.64%
	EGARCH-CJ	5.45%	5.05%	4.91%	4.91%
10-min RV	EGARCH	4.77%	4.91%	4.50%	5.32%
	EGARCH-X	5.59%	4.23%	4.64%	4.64%
	EGARCH-CJ	5.87%	4.77%	4.64%	4.64%
RK	EGARCH	4.77%	4.91%	4.50%	5.32%
	EGARCH-X	6.00%	4.77%	4.64%	4.64%
	EGARCH-CJ	6.14%	5.46%	5.59%	5.59%
$RV^{(5)}$	EGARCH	4.78%	4.92%	4.51%	5.33%
	EGARCH-X	4.92%	4.64%	4.92%	4.92%
	EGARCH-CJ	6.42%	4.92%	4.64%	4.78%

Note: The bold values indicate the closest values to the 5% VaR-level

As shown in Table 6, no single model universally dominates VaR forecasting. The standard EGARCH model with RK often excels at short horizons (h = 1) and occasionally at longer ones, while the EGARCH-CJ model generally provides superior tail-risk forecasts (lower RQL) at medium and long horizons (h = 5, 10, 20), especially with 1-minute RV. This horizon-dependent performance underscores that the advantage of explicit jump modeling is contingent on both the forecast horizon and the chosen realized measure. Therefore, EGARCH-CJ is most valuable for medium- to long-term risk forecasting in jump-prone settings, whereas simpler EGARCH specifications paired with robust measures like RK remain competitive for short-term and calibration-focused applications.

These forecasting results carry direct relevance for regulatory risk management. Under frameworks such as Basel III, accurate VaR estimation is critical for determining capital requirements and passing backtesting standards. The observed superiority of the EGARCH-CJ model in reducing tail losses (lower RQL) suggests its potential for improving economic capital allocation, while the consistent calibration of the standard EGARCH model—particularly with RK and mid-frequency RV—supports its reliability in meeting strict regulatory coverage thresholds. Further regulatory implications of these findings are elaborated in the Discussion.

Optimal model selection depends on forecast horizon, performance metric, and realized measure. Table 8 summarizes the best model for each key objective—volatility accuracy (MSE), tail-risk (RQL), and regulatory calibration (5% FR)—across short, medium, and long horizons, highlighting the horizon-dependent trade-off between EGARCH and EGARCH-CJ.

Table 8. Summary of optimal model selection by forecast horizon

Horizon	Best for volatility	DM test significant?	Best for tail risk (RQL)	Best for regulatory (5% FR)	Key insight
Short (h = 1)	EGARCH (5-min RV)	No	EGARCH-CJ (1-min RV/RK)	EGARCH (10-min RV/RK)	EGARCH excels for short-term volatility; EGARCH-CJ better for immediate tail risk

Medium (h = 5, 10)	EGARCH-CJ (all RVs)	Yes	EGARCH-CJ (most RVs)	EGARCH (RK)	EGARCH-CJ dominates forecasting
Long (h = 20)	EGARCH-CJ (all RVs)	Yes	EGARCH-CJ (1-min RV) or EGARCH (RK)	EGARCH (RK)	EGARCH-CJ best for long-term predictions, but EGARCH with RK remains robust for regulatory VaR

3.5 Discussion

The empirical results reveal a nuanced landscape where model superiority is conditional on forecast horizon, the choice of realized measure, and the specific risk management objective. This discussion synthesizes these findings into four key thematic insights, interprets their implications, and highlights the study's contributions relative to the existing literature.

3.5.1 Horizon-Dependent Model Performance

The empirical results demonstrate that the performance of the EGARCH-CJ model is conditionally superior, with its advantages being particularly dependent on the forecast horizon. The standard EGARCH model consistently offers the most accurate short-term volatility forecasts and the most reliable VaR calibration (closest to the 5% failure rate). In contrast, the EGARCH-CJ model provides the best volatility forecasts and lower tail losses (RQL) at medium- and long-term horizons. The Diebold-Mariano test results in Table 5 statistically validate this pattern, showing the EGARCH-CJ model's significant superiority at medium and long horizons while confirming no significant advantage at the one-day horizon.

3.5.2 The Critical Role of Realized Measures and Sampling Frequency

Model performance is critically moderated by the choice and intraday frequency of the realized volatility measure. The observed degradation of 1-minute RV in short-term forecasts contrasting with its long-horizon benefits signals contamination by microstructure noise such as bid-ask bounce. This noise diminishes over longer horizons where persistent volatility dominates. Robust estimators like the RK or coarser sampling (e.g., 10-minute RV) mitigate this noise and often enable the standard EGARCH model to perform competitively, particularly for VaR calibration where stable inputs are valued. This evidence underscores a market-specific noise-resolution trade-off, where the optimal sampling interval (10-minute for TOPIX) balances high-frequency information with noise resilience. This granular analysis across multiple frequencies extends prior work that relied on a single sampling scheme.

3.5.3 Implications for Risk Management and Basel Regulation

These findings carry direct consequences for financial risk management, particularly within post-crisis regulatory frameworks like Basel III that emphasize accurate VaR estimation and backtesting. The EGARCH-CJ model's strength in reducing tail losses (lower RQL) makes it valuable for internal economic capital allocation and stress testing, where capturing extreme movements is paramount. However, the standard EGARCH model, especially when paired with robust measures like RK, provides more consistent calibration (FR closest to the 5% nominal level), a critical feature for regulatory compliance and capital reporting. This divergence suggests institutions might strategically employ a dual-model approach: EGARCH-CJ for internal risk modeling and the standard EGARCH for regulatory submissions, thereby addressing both economic and regulatory objectives effectively.

3.5.4 Limitations and Avenues for Future Research

From a methodological standpoint, stable estimation results and low IACT values confirm the robustness of the ARWM algorithm, consistent with previous findings [23]. The minimal impact of exogenous shocks on the asymmetry parameter suggests that market-wide dynamics during stressful periods may overshadow leverage effects captured by external variables. These models exhibit limitations in calmer market environments where jump components contribute little additional information, and performance can weaken when volatility dynamics are dominated by smooth, low-amplitude fluctuations. Future research should extend this framework to diverse markets and investigate macroeconomic drivers of jump components [39].

3.5.5 Alignment with Recent Advances

These findings align with recent advances in volatility forecasting. Studies have shown that incorporating realized volatility into CAViaR-type models improves VaR forecasting [40], and that decomposing continuous and jump components enhances multi-step forecast performance [39]. Our DM test results provide formal statistical confirmation of this enhancement for EGARCH-type models. Mid-frequency realized measures enable stable calibration [41], and realized-based GARCH frameworks remain competitive when data frequency and horizon align well [6].

The evidence indicates a clear trade-off: EGARCH-CJ provides better forecasting for medium-to-long horizons and jump-driven markets, whereas EGARCH remains superior for short-term forecasting and regulatory calibration. Model selection should be guided by specific risk management objectives—whether prioritizing tail-risk reduction, nominal coverage accuracy, or leveraging robust realized measures.

4. CONCLUSION

This study evaluates the performance of EGARCH-X and EGARCH-CJ models in forecasting volatility for the Japanese TOPIX index using multiple realized measures within a Bayesian MCMC framework. The findings reveal a clear, horizon-dependent trade-off: while the standard EGARCH model remains optimal for short-term forecasting and regulatory VaR calibration, the EGARCH-CJ model provides statistically superior accuracy for medium- and long-term horizons by explicitly capturing jump-driven risk. The choice of realized measure—particularly the trade-off between high-frequency information and microstructure noise—critically influences model performance.

Practically, the results support a dual-model approach in financial risk management: EGARCH-CJ is recommended for internal capital allocation and stress testing where tail-risk accuracy is paramount, whereas the standard EGARCH model, especially with robust measures like the RK, is better suited for regulatory reporting under frameworks such as Basel III. Future research should extend this framework to a Realized EGARCH-CJ specification, validate findings across diverse markets and asset classes, and investigate the macroeconomic drivers of jump components to enhance model interpretability and forecasting robustness.

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