**Zero : Jurnal Sains, Matematika, dan Terapan**
E-ISSN : 2580-5754; P-ISSN : 2580-569X
Volume 9, Number 3, 2025
DOI: 10.30829/zero.v9i3.26477
Page: 921 - 931

# Comparative Performance of Spatial Robust Small Area Estimation Methods: A Simulation Study

[1] Baiq Dian Arianingsih
Study Program of Statistics and Data Science, IPB University, Bogor, Indonesia

[2] Kusman Sadik
Study Program of Statistics and Data Science, IPB University, Bogor, Indonesia

[3] Indahwati
Study Program of Statistics and Data Science, IPB University, Bogor, Indonesia

**Article Info**

***Article history:***

Accepted 26 December 2025

***Keywords:***

Outliers;
Simulation;
Small Area Estimation;
Spatial Dependence;
Spatial Robust Empirical Best Linear Unbiased Prediction.

**ABSTRACT**

Estimating parameters for small areas often faces limitations due to insufficient sample sizes, resulting in low-precision estimates. The Small Area Estimation (SAE) approach is used to address this problem by utilizing auxiliary variables to improve estimation efficiency. This study evaluates four SAE methods, namely EBLUP, REBLUP, SEBLUP, and SREBLUP, through a simulation study based on a nested error model across 18 scenarios that combine two area sizes (16 and 64 areas), levels of outlier contamination in the error component, and degrees of spatial correlation in the area-level random effects. Each scenario is replicated 50 times. Model performance is evaluated using Relative Bias (RB) and Relative Root Mean Square Error (RRMSE). The results show that non-robust methods are sensitive to outliers, whereas robust methods produce more stable estimates. The SREBLUP method demonstrates the best performance under low to moderate spatial correlation. In addition, an ANOVA test is conducted to identify factors that significantly affect the response.



***Corresponding Author:***

Kusman Sadik,
Statistics and Data Science Program Study,
School of Data Science, Mathematics, and Informatics, IPB University, Bogor, Indonesia
Email: kusmans@apps.ipb.ac.id

## 1. INTRODUCTION

Survey data are commonly available at higher administrative levels, such as national or provincial, but are often unreliable or unavailable for smaller regions due to limited sample sizes. Direct estimation at the district or village level thus produces estimates with low precision [1]. Indirect estimation methods such as Small Area Estimation (SAE) have been developed to address this issue. SAE improves estimation accuracy in small domains by incorporating auxiliary variables correlated with the study variable [2].

Small area estimation models are typically divided into area-level models, such as the Fay–Herriot and unit-level models [3]. Traditional SAE assumes independent random effects [4], but neighbouring areas often exhibit spatial dependence in practice. To capture this, spatial effects can be introduced into the model. The incorporation of spatial correlation in SAE was first proposed by [5] cited in [6], and later refined by [7] through the Spatial Empirical Best Linear Unbiased Prediction (SEBLUP) method, which outperforms the conventional Empirical Best Linear Unbiased Prediction (EBLUP) when spatial relationships exist.

Although these models perform well under normality assumptions, they are sensitive to outliers [8]. The presence of outliers in the data can lead to inaccurate estimates however, outliers often contain important information and therefore cannot simply be discarded. Outliers can be addressed using a statistical approach known as robust statistics, which is designed to handle outliers without the need to remove them. [9] first addressed this issue using a robust M-quantile approach, followed by [8], who proposed the Robust EBLUP (REBLUP). More recently, [9], [10] extended this framework into the Spatial Robust EBLUP (SREBLUP), accommodating spatial correlation and outlier robustness. Many studies in Indonesia have applied Small Area Estimation using the EBLUP, REBLUP, and SEBLUP methods [11], [12] [13], [14], [15].

It is essential to employ methods that can accommodate the presence of outliers and spatial dependence. Outliers are often found in expenditure or income data, which are frequently used in poverty estimation. Such data may also exhibit spatial dependence, as neighbouring regions tend to share similar socioeconomic patterns due to geographical proximity. To obtain reliable estimates in the presence of outliers and spatial correlation, methods such as SAE with robust and spatial approaches are highly needed.

Previous studies, particularly those employing the SREBLUP method, are relatively scarce and generally consider only a single number of areas. This study extends prior research by comparing two different area structures, namely a smaller number of areas (16) and a larger number of areas (64). In addition, the outlier scenario is generated only in the error component, while the random effects are assumed to follow a normal distribution with incorporated spatial impact. Outliers were introduced using a contaminated normal mechanism by mixing standard normal errors with high-variance errors. This mechanism generates vertical outliers in the response variable without affecting the explanatory variables.

This study aims to evaluate and compare the performance of four SAE methods EBLUP, REBLUP, SEBLUP, and SREBLUP, through a simulation study. The simulation design explicitly considers key factors, including two different numbers of small areas (16 and 64 areas), varying levels of outlier contamination in the error component, and different degrees of spatial correlation incorporated in the area-level random effects. Model performance is assessed using Relative Bias (RB) and Relative Root Mean Square Error (RRMSE). By examining the results across these regimes, this study provides methodological insight into the relative advantages of robust and spatially robust approaches and offers practical guidance for selecting between REBLUP and SREBLUP under different data conditions. While also evaluating higher-order interactions among these factors using ART ANOVA. The simulation study in this research aims to evaluate the performance of the models under various conditions and to serve as a guide for selecting the most appropriate method for similar data conditions.

## 2. RESEARCH METHOD

### 2.1 Small Area Estimation

According to the availability level of auxiliary variables, [3] classify SAE models into two types: area level models and unit level models.

**Area Level Model**

The area level model is applied when auxiliary variables are only available at the area level, making unit level modelling infeasible. Let $\boldsymbol{x_i} = (z_{1i}, \dots, z_{pi})^T$ be a vector of auxiliary variables for area $i$, and $y_i$ the parameter to be estimated, which is assumed to have a linear relationship with $\boldsymbol{x_i}$. The model is expressed as:

$$y_i = \boldsymbol{x}_i^T\boldsymbol{\beta} + b_i v_i, i = 1, \dots, m \quad (1)$$

where $b_i$ is a positive constant, $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)^T$is a $p \times 1$vector of fixed parameters, and $v_i$ represents random effects that are assumed to be independently and identically distributed as $v_i \sim N(0, \sigma_v^2)$. The term $m$ denotes the total number of small areas.

The area level SAE model was introduced by [16] to estimate average income in small areas. Their study assumed normally distributed income. Let $\hat{y}_i = \hat{\bar{Y}}_i$ denote the direct estimator, then:

$$\hat{y}_i = \theta_i + e_i, i = 1, \dots, m \quad (2)$$

where $e_i$ is the sampling error, assumed to be normally distributed and mutually independent. By combining equations (1) and (2), the Fay–Herriot (FH) model is obtained:

$$\hat{y}_i = \boldsymbol{x}_i^T\boldsymbol{\beta} + b_i v_i + e_i, i = 1, \dots, m \quad (3)$$

**Unit-Level Model**

The unit-level model is used when auxiliary variables are available at the observation (unit) level. Let $y_{ij}$ denote the response variable for area $i$ and unit $j$, and $\boldsymbol{X}_{ij} = (x_{ij1}, \dots, x_{ijp})^T$the corresponding auxiliary variable vector. The model can be expressed as:

$$y_{ij} = \boldsymbol{X}_{ij}^T\boldsymbol{\beta} + v_i + e_{ij}, j = 1, \dots, N_i;\ i = 1, \dots, m \quad (4)$$

Here, $v_i$ represents the area-specific random effect, assumed to follow an independent and identically distributed normal distribution $v_i \sim N(0, \sigma_v^2)$, and $e_{ij} \sim N(0, \sigma_e^2)$ is the individual error term, independent of $v_i$.

### 2.2 Robust Empirical Best Linear Unbiased Prediction

The model-based small area estimation approach using unit-level data adopts the framework proposed by [17]. This model is a linear mixed model with area specific random effect, expressed as follows.

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{Zv} + \boldsymbol{e} \tag{5}$$

where $\boldsymbol{\beta}$ is a $p \times 1$ vector of regression coefficients, $\boldsymbol{v} \sim N(0, \boldsymbol{G})$ is a $m \times 1$ vector of area-specific random effects, and $\boldsymbol{e} \sim N(0, \boldsymbol{R} = \sigma_e^2 \boldsymbol{I}_N)$ is a $N \times 1$ vector of sampling errors. $\boldsymbol{I}_N$ denotes an identity matrix of dimension $N$. The covariance matrices $\boldsymbol{G}$ and $\boldsymbol{R}$ depend on the variance parameter vector $\boldsymbol{\theta} = (\sigma_v^2, \sigma_u^2)$.

When the area effects are assumed to be independent, the covariance matrix of the random effects simplifies to $\boldsymbol{G} = \sigma_v^2 \boldsymbol{I}_m$. Since the random effects $\boldsymbol{v}$ and the sampling errors $\boldsymbol{e}$ are independent, the covariance matrix of $y$ is defined as $\boldsymbol{V} = \boldsymbol{R} + \boldsymbol{ZGZ}^T$. Therefore, the Empirical Best Linear Unbiased Prediction (EBLUP) of the small area mean $\bar{y}_i$ can be expressed as follows.

$$\hat{\bar{y}}_i^{EBLUP} = N_i^{-1} \left\{ \sum_{j \in s_i} y_{ij} + \sum_{j \in r_i} \hat{y}_{ij} \right\} = N_i^{-1} \left\{ \sum_{j \in s_i} y_{ij} + \sum_{j \in r_i} \left( \boldsymbol{x}_{ij}^T \hat{\boldsymbol{\beta}} + \hat{v}_i \right) \right\} \tag{6}$$

with $\hat{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}}(\hat{\boldsymbol{\theta}})$ being the empirical best linear unbiased estimator (EBLUE) of $\boldsymbol{\beta}$, and $\hat{\boldsymbol{v}}(\hat{\boldsymbol{\theta}})$ being the empirical best linear unbiased predictor (EBLUP) of $\boldsymbol{v}$ [18]. The vector $\hat{\boldsymbol{\theta}}$ is estimated using the maximum likelihood estimator (ML) or the restricted maximum likelihood estimator (REML) for $\boldsymbol{\theta} = (\sigma_v^2, \sigma_u^2)$ [19]. The variable $y_{ij}$ represents the response value $y$ observed for the $j$-th unit in the $i$-th small area, while $\hat{y}_{ij}$ denotes the estimated response value $y$ for the $j$-th unit that is not included in the sample within the $i$-th small area.

The Robust Empirical Best Linear Unbiased Prediction (REBLUP) method was first introduced by [8]. They developed a robust version of the traditional Empirical Best Linear Unbiased Prediction (EBLUP) that is resistant to the influence of outliers. Robust estimators of $\beta$ and $v$ are obtained by applying an influence function $\psi(\cdot)$ to the residuals in the likelihood-based estimating equations. The REBLUP estimator for the small area means $\bar{y}_i$ in area $i$ is defined as:

$$\hat{\bar{y}}_i^{REBLUP} = N_i^{-1} \left\{ \sum_{j \in s_i} y_{ij} + \sum_{j \in r_i} \left( \boldsymbol{x}_{ij}^T \hat{\boldsymbol{\beta}}^{\psi} + \hat{v}_{\boldsymbol{i}}^{\psi} \right) \right\} \tag{7}$$

[2] applied an influence function $\psi$ to the residuals in the maximum likelihood estimating equations and obtained the robust estimators $\hat{\boldsymbol{\beta}}^{\psi}$ and $\hat{\boldsymbol{v}}^{\psi}$ through an iterative algorithm.

### 2.3 Spatial Robust Empirical Best Linear Unbiased Prediction

The spatial SAE model incorporates spatial effects into the area-specific random effects using a Simultaneously Autoregressive (SAR) process [7]

$$\boldsymbol{v} = \rho \mathbf{W} \boldsymbol{v} + \boldsymbol{u}, u \sim N(0, \sigma_u^2 I) \tag{8}$$

where $\rho$ is the spatial autoregressive coefficient, $\boldsymbol{W}$ is the spatial weight matrix representing area neighbourhoods, and $u$is the vector of independent area-specific random errors. This can be rewritten as:

$$\boldsymbol{v} = (\mathbf{I} - \rho \mathbf{W})^{-1} \boldsymbol{u} \tag{9}$$

yielding the SAR based SAE model:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{Z}(\mathbf{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{u} + \boldsymbol{e} \tag{10}$$

Variance components estimation: $\sigma_u^2$ and $\rho$are estimated under normality assumptions using ML or REML. The model for the Spatial Empirical Best Linear Unbiased Prediction (SEBLUP) can be formulated as

$$\hat{\bar{y}}_i^{SEBLUP} = x_i \hat{\boldsymbol{\beta}} + \boldsymbol{b}_i^T \hat{\boldsymbol{G}} \boldsymbol{Z}^T \hat{\boldsymbol{V}}^{-1} \left( \boldsymbol{y} - \boldsymbol{X} \hat{\boldsymbol{\beta}} \right) \tag{11}$$

with $\hat{\boldsymbol{G}} = \sigma_u^2 [(\mathbf{I} - \rho \mathbf{W})((\mathbf{I} - \rho \mathbf{W}^T)]^{-1}$and $\mathbf{V} = \mathbf{R} + \mathbf{ZGZ}^{\mathrm{T}} = diag(\sigma_e^2) + \mathbf{Z}\sigma_u^2[(\mathbf{I} - \rho \mathbf{W})(\mathbf{I} - \rho \mathbf{W}^T)]^{-1} \mathbf{Z}^{\mathrm{T}}$. The vector $\boldsymbol{b}_i^T$indicates the $i$-th area.

Following the approach of [2], a robustification procedure was applied to the maximum likelihood (ML) equations for estimating $\beta$, $\theta$, and $\rho$ (under the spatial correlation assumption), resulting in

$$\alpha(\boldsymbol{\beta}) = \boldsymbol{X}^T \boldsymbol{V}^{-1} \boldsymbol{U}^{\frac{1}{2}} \psi(\boldsymbol{r}) = 0 \tag{12}$$

$$\Phi(\theta_l) = \psi^T(\boldsymbol{r})\boldsymbol{U}^{\frac{1}{2}}\boldsymbol{V}^{-1}\frac{\partial \boldsymbol{V}}{\partial \theta_l}\boldsymbol{V}^{-1}\boldsymbol{U}^{\frac{1}{2}}\psi(\boldsymbol{r}) - tr\left(\boldsymbol{V}^{-1}\frac{\partial \boldsymbol{V}}{\partial \theta_l}\boldsymbol{K}\right) = 0 \quad (13)$$

$$\Omega(\rho) = \psi^T(\boldsymbol{r})\boldsymbol{U}^{\frac{1}{2}}\boldsymbol{V}^{-1}\frac{\partial \boldsymbol{V}}{\partial \rho}\boldsymbol{V}^{-1}\boldsymbol{U}^{\frac{1}{2}}\psi(\boldsymbol{r}) - tr\left(\boldsymbol{V}^{-1}\frac{\partial \boldsymbol{V}}{\partial \rho}\boldsymbol{K}\right) = 0 \quad (14)$$

where $\boldsymbol{r} = \boldsymbol{U}^{-\frac{1}{2}}(\boldsymbol{y} - \boldsymbol{X\beta})$ and $\boldsymbol{U}$ is a diagonal matrix with diagonal elements equal to those of the diagonal of $\boldsymbol{V}$. $\boldsymbol{K}$ is also a diagonal matrix defined as $\boldsymbol{K} = E(\psi_b^2(t))\boldsymbol{I}_n$, where $t$ follows a standard normal distribution. The estimation of spatial random effects under robustness is performed using the Newton–Raphson algorithm, as introduced by [20], to solve the following equation:

$$Z^T R^{-\frac{1}{2}}\psi(R^{-\frac{1}{2}}(y - X\beta - \boldsymbol{Z}^T\boldsymbol{R}^{-\frac{1}{2}}\psi\left(\boldsymbol{R}^{-\frac{1}{2}}(\boldsymbol{y} - \boldsymbol{X\beta} - \boldsymbol{Zv})\right) - \boldsymbol{G}^{-\frac{1}{2}}\psi\left(\boldsymbol{G}^{-\frac{1}{2}}\boldsymbol{v}\right) = 0 \quad (15)$$

To mitigate the influence of outliers, robust estimation of the small area mean is obtained using a robust maximum likelihood approach for parameters $\beta$, $\theta$, and $\rho$. Consequently, the Spatial Robust Empirical Best Linear Unbiased Prediction (SREBLUP) model for the mean of area $i$ is formulated as:

$$\begin{aligned}\hat{\bar{y}}_i^{SREBLUP} &= N_i^{-1}\left\{\sum_{j\in s_i} y_j + \sum_{j\in r_i}(\boldsymbol{x}_j^T\widehat{\boldsymbol{\beta}}^{\psi,sp} + \hat{v}_i^{\psi,sp})\right\} \\ &= N_i^{-1}\left(n_i\bar{y}_{si} + (N_i - n_i)\left(\bar{\boldsymbol{x}}_{ri}^T\widehat{\boldsymbol{\beta}}^{(\psi,sp)} + \hat{v}_i^{(\psi,sp)}\right)\right) \quad (16)\end{aligned}$$

The superscript $\psi$ denotes dependence on the influence function, while the superscript $sp$ indicates that the parameters depend on the spatial autocorrelation parameter $\rho$ [10]. Further details on the SREBLUP model can be found in [9].

### 2.4 Simulatio Design

This study employed simulated data modified from the work of [9]. The simulation was conducted to evaluate the performance of several models under different scenarios. The experimental design was adapted from previous studies with several modifications. Specifically, it was assumed that no outliers were present in the random effects ($v_i$), while outliers were introduced only in the individual error component ($e_{ij}$).

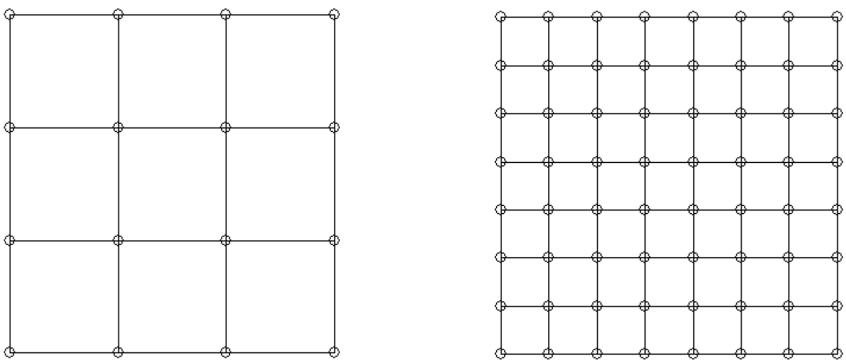
A synthetic population consisting of $N = 1600$ and $N = 6400$ units was generated and divided into $m = 16$ small areas, each containing $N_i = 100$ units and a sample size of $n_i = 5$ for every area $i = 1, \ldots, m$. The data were generated according to the following nested-error regression model [21], [22], [23] :

$$y_{ij} = 100 + 5x_j + v_i + e_{ij},$$

where $v_i \sim N(0, G)$ and $e_{ij} \sim (1 - \gamma_e)N(0,4) + \gamma_e N(10,25)$. The covariance matrix $G$ is defined as

$$\boldsymbol{G} = \sigma_v^2[(\boldsymbol{I} - \rho\boldsymbol{W})(I - \rho\boldsymbol{W}^T)]^{-1},$$

using $\sigma_v^2 = 3$, where $\rho$ represents the spatial autocorrelation parameter and $\mathbf{W}$ is the spatial weight matrix. Figure 1 shows the underlying neighborhood structure of the areas, which is the rook structure. In this structure, areas are considered neighbors if they share a common side.



(a) (b)

**Figure 1.** Neighborhood structure for 16 areas (a) and 64 areas (b)

The spatial correlation coefficient $\rho$ was varied to represent three conditions: low ($\rho = 0.1$), moderate ($\rho = 0.5$), and high ($\rho = 0.85$). The auxiliary variable $x$ was generated from a lognormal distribution with a mean of

$\mu_x = 1$ and a standard deviation of $\sigma_x = 0.5$. The contamination proportion $\gamma_e$ controlled the level of outliers, set at 1%, 5%, and 15%, yielding 18 simulation scenarios. Each scenario was replicated 50 times. In total, three spatial correlation levels, three outlier proportions, and two area settings were investigated.

The specific scenarios were as follows:

a. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}01, \rho = 0{,}1$), where $e_{ij} \sim 0{,}99 \cdot N(0,4) + 0{,}01 \cdot N(10,25)$
b. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}01, \rho = 0{,}5$), where $e_{ij} \sim 0{,}99 \cdot N(0,4) + 0{,}01 \cdot N(10,25)$
c. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}01, \rho = 0{,}85$), where $e_{ij} \sim 0{,}99 \cdot N(0,4) + 0{,}01 \cdot N(10,25)$
d. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}05, \rho = 0{,}1$), where $e_{ij} \sim 0{,}95 \cdot N(0,4) + 0{,}05 \cdot N(10,25)$
e. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}05, \rho = 0{,}5$), where $e_{ij} \sim 0{,}95 \cdot N(0,4) + 0{,}05 \cdot N(10,25)$
f. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}05, \rho = 0{,}85$), where $e_{ij} \sim 0{,}95 \cdot N(0,4) + 0{,}05 \cdot N(10,25)$
g. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}15, \rho = 0{,}1$), where $e_{ij} \sim 0{,}85 \cdot N(0,4) + 0{,}15 \cdot N(10,25)$
h. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}15, \rho = 0{,}5$), where $e_{ij} \sim 0{,}85 \cdot N(0,4) + 0{,}15 \cdot N(10,25)$
i. Outliers were introduced in the sampling error, and spatial correlation was present ($\gamma_e = 0{,}15, \rho = 0{,}85$), where $e_{ij} \sim 0{,}85 \cdot N(0,4) + 0{,}15 \cdot N(10,25)$

**Table 1.** Simulation study scenario

| $m$ | total sample size | sample size per area | % Outlier | $\rho$ |
|---|---|---|---|---|
| 16 | 100 | 5 | 1 | 0,1 |
| | 100 | 5 | | 0,5 |
| | 100 | 5 | | 0,85 |
| | 100 | 5 | 5 | 0,1 |
| | 100 | 5 | | 0,5 |
| | 100 | 5 | | 0,85 |
| | 100 | 5 | 15 | 0,1 |
| | 100 | 5 | | 0,5 |
| | 100 | 5 | | 0,85 |
| 64 | 100 | 5 | 1 | 0,1 |
| | 100 | 5 | | 0,5 |
| | 100 | 5 | | 0,85 |
| | 100 | 5 | 5 | 0,1 |
| | 100 | 5 | | 0,5 |
| | 100 | 5 | | 0,85 |
| | 100 | 5 | 15 | 0,1 |
| | 100 | 5 | | 0,5 |
| | 100 | 5 | | 0,85 |

For each replication, the mean of the response variable was estimated for every area using four estimation methods: EBLUP, SEBLUP, REBLUP, and SREBLUP. The performance of these estimators was evaluated using the Relative Bias (RB) and the Relative Root Mean Square Error (RRMSE), defined as follows:

$$RB(\hat{\bar{y}}_i) = \frac{1}{R}\sum_{r=1}^{R}\frac{\hat{\bar{y}}_i - \bar{y}_i}{\bar{y}_i}$$

$$RRMSE(\hat{\bar{y}}_i) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\left(\frac{\hat{\bar{y}}_i - \bar{y}_i}{\bar{y}_i}\right)^2}$$

where $\hat{\bar{y}}_i$ denotes the estimator of the true mean $\bar{y}_i$ for area $i$, and $R = 50$ is the number of Monte Carlo replications. The number of Monte Carlo replications was set to $R = 50$, which provides a reasonable balance between estimation stability and computational feasibility. RRMSE and bias estimates tend to stabilize at this replication level, whereas using a larger number of replications would substantially increase the computational burden, given the large number of scenarios considered.

All simulations were conducted using the R statistical software. The estimation procedures were implemented using user-defined functions for robust and spatial small area estimation developed by [9]. To ensure reproducibility, a fixed random seed (047) was used throughout the simulation study.

## 3. RESULT AND ANALYSIS

### 3.1 Parameter Estimation Method

Two different estimation approaches were applied for model parameter estimation, depending on the small area estimation method used. For the EBLUP method, parameters were estimated using the Restricted Maximum Likelihood (REML) approach[3]. REML was selected because it generally yielded unbiased estimates in linear mixed models. This finding was consistent with [3], who stated that REML estimates variance components and therefore produces unbiased variance estimators.

In contrast, for the REBLUP, SEBLUP, and SREBLUP methods, parameter estimation was carried out using the Maximum Likelihood (ML) approach. The use of ML was motivated by its compatibility with estimation procedures involving influence functions, such as the Huber function, which are designed to mitigate the effect of outliers and handle spatial structure complexities [24]. The robustification process was performed directly at the likelihood estimation stage, which would have been challenging under REML due to the separation between fixed and random effects. Hence, ML was considered the most suitable method for implementing these estimators.

Table 2 presents the parameter estimates for scenario 9 (m = 16, %p = 15, ρ = 0,85) out of the 18 scenarios analyzed, using four small area estimation methods: EBLUP, REBLUP, SEBLUP, and SREBLUP. The estimated parameters include the regression coefficients ($\beta_0$ and $\beta_1$), the area effect variance ($\sigma_v^2$), the error variance ($\sigma_e^2$), and the spatial correlation (ρ).

**Table 2.** Comparison of parameter estimates for Scenario 9
($m = 16$, %outlier = 15%, ρ = 0,85)

| Parameter | True Value | EBLUP | REBLUP | SEBLUP | SREBLUP |
|---|---|---|---|---|---|
| $\beta_0$ | 100 | 102,09 | 101,19 | 102,08 | 101,18 |
| $\beta_1$ | 5 | 4,84 | 4,87 | 4,84 | 4,88 |
| $\sigma_v^2$ | 3 | 4,23 | 4,30 | 2,86 | 3,41 |
| $\sigma_e^2$ | 19,9 | 30,72 | 18,36 | 30,08 | 18,24 |
| $\rho$ | 0,85 | | | 0,28 | 0,37 |

Based on the results presented in Table 2, it can be seen that the estimates of parameters $\beta_0$ and $\beta_1$from the four methods are relatively close to the actual values. The SREBLUP method produces the estimates closest to the actual values, namely 101,18 for $\beta_0$ and 4,88 for $\beta_1$, followed by the REBLUP method. This indicates that all four methods are capable of estimating the regression parameters well, although there are slight differences in precision among the methods.

Furthermore, for the variance of the area random effect ($\sigma_v^2$) with an actual value of 3, the SEBLUP method provides the closest estimate, which is 2,86, followed by the SREBLUP method with a value of 3,42, while the other methods tend to overestimate the variance. This may occur because SEBLUP and SREBLUP specifically incorporate spatial information. Spatial information helps the model distinguish between pure area variance and variation that arises due to spatial adjacency. For the error variance ($\sigma_e^2$) with an actual value of 19,9, the EBLUP and SEBLUP methods produce estimates that deviate considerably from the actual value. This indicates a potential difficulty of these models in accurately estimating the individual error variance under this scenario. Conversely, robust methods such as REBLUP and SREBLUP provide estimates that are closer to the actual value.

### 3.2 Performance of Small Area Estimation

Table 3 presents the average values of RRMSE and RB for the four methods with 16 areas. As shown in Table 3, the overall performance of the four methods was relatively similar, as indicated by the RRMSE and RB values that did not differ substantially from one another. Each method exhibited specific advantages depending on the combination of factors. The robust methods outperformed the non-robust ones under data conditions with 1% outliers, as reflected by lower RRMSE and RB values. The REBLUP method performed better than the other approaches, including those with spatial components, when the spatial correlation was low (ρ = 0,1). This occurred because the regular robust method could still manage low spatial correlation, allowing REBLUP to handle such conditions effectively.

**Table 3.** Average RRMSE and RB values for the EBLUP, REBLUP, SEBLUP, and SREBLUP models across 16 areas in the simulation study

| | Scenario | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** |
| | | | | | **RRMSE (%)** | | | | |
| EBLUP | 0,129 | 0,136 | 1,020 | 1,901 | 1,947 | 2,011 | 1,591 | 1,612 | 1,707 |
| REBLUP | 0,113 | 0,095 | 0,440 | 1,227 | 1,273 | 1,380 | 1,174 | 1,207 | 1,363 |
| SEBLUP | 0,115 | 0,140 | 1,018 | 1,811 | 1,865 | 1,948 | 1,508 | 1,531 | 1,645 |
| SREBLUP | 0,113 | 0,101 | 0,305 | 1,227 | 1,229 | 1,170 | 1,169 | 1,209 | 1,285 |
| | | | | | **RB (%)** | | | | |
| EBLUP | 1,013 | 1,015 | 1,020 | 1,317 | 1,319 | 1,323 | 0,835 | 0,836 | 0,844 |
| REBLUP | 0,406 | 0,417 | 0,440 | 0,592 | 0,608 | 0,629 | 0,130 | 0,156 | 0,201 |
| SEBLUP | 1,013 | 1,015 | 1,018 | 1,314 | 1,317 | 1,320 | 0,834 | 0,835 | 0,840 |
| SREBLUP | 0,403 | 0,398 | 0,305 | 0,593 | 0,581 | 0,521 | 0,126 | 0,150 | 0,158 |

As the level of spatial correlation increased, the RRMSE values of EBLUP and SEBLUP tended to rise. In contrast, REBLUP and SREBLUP remained consistent, showing lower RRMSE and RB values than the non-robust methods. When the spatial correlation became high, SREBLUP showed a slight decline in performance. This was likely due to the trade-off between the robust and spatial components within the model. Nevertheless, the RB values of SREBLUP remained the smallest among all methods, demonstrating its ability to maintain unbiased estimates.

The differences among methods became more pronounced when the proportion of outliers increased to 5%. The RRMSE values of EBLUP and SEBLUP increased significantly, particularly under high spatial correlation ($\rho$ = 0,85). This pattern indicated that moderate levels of contamination reduced the performance of non-robust models, especially in the presence of strong spatial dependence. In contrast, SREBLUP was able to resist the influence of outliers effectively. Both REBLUP and SREBLUP produced lower RRMSE values than the other methods, suggesting that robust approaches were more reliable under higher contamination levels, as they mitigated the influence of outliers on the estimation results. These findings were consistent with those reported by [8], who stated that classical methods often failed to produce accurate estimates when data contained extreme values.

When the proportion of outliers further increased to 15%, the RRMSE differences among models became smaller. Although the gap between methods was narrower than that observed under moderate contamination, SREBLUP consistently achieved lower RRMSE values under both low and high spatial correlations. The most notable difference appeared in the RB values, where non-robust methods exhibited relatively high bias, while robust methods successfully suppressed the bias to near zero. This pattern was observed across all levels of spatial correlation. Therefore, robust methods, particularly SREBLUP, remained superior due to their ability to maintain low bias even under severe contamination. These results aligned with the findings of [8], which demonstrated that SREBLUP maintained estimation stability and accuracy under extreme conditions characterized by high outlier proportions and substantial spatial correlation among areas. Regarding spatial correlation, its influence was found to be significant for method performance. High spatial correlation indicated strong interdependence among neighbouring areas, meaning that information from one area could substantially contribute to estimating adjacent areas.

**Table 4.** Average RRMSE and RB values for the EBLUP, REBLUP, SEBLUP, and SREBLUP models across 64 areas in the simulation study

| | Scenario | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **S10** | **S11** | **S12** | **S13** | **S14** | **S15** | **S16** | **S17** | **S18** |
| | | | | | **RRMSE (%)** | | | | |
| EBLUP | 1,505 | 1,512 | 1,541 | 1,963 | 2,022 | 2,012 | 1,601 | 1,661 | 1,682 |
| REBLUP | 0,914 | 0,934 | 1,002 | 1,219 | 1,250 | 1,335 | 1,145 | 1,190 | 1,287 |
| SEBLUP | 1,511 | 1,481 | 1,481 | 1,873 | 1,978 | 2,188 | 1,530 | 1,633 | 1,842 |
| SREBLUP | 0,916 | 0,909 | 0,958 | 1,217 | 1,182 | 1,349 | 1,150 | 1,167 | 1,338 |
| | | | | | **RB (%)** | | | | |
| EBLUP | 1,016 | 1,016 | 1,018 | 1,347 | 1,351 | 1,359 | 0,777 | 0,780 | 0,789 |
| REBLUP | 0,433 | 0,440 | 0,459 | 0,603 | 0,620 | 0,657 | 0,040 | 0,060 | 0,105 |
| SEBLUP | 1,016 | 1,015 | 1,012 | 1,345 | 1,349 | 1,364 | 0,776 | 0,780 | 0,793 |
| SREBLUP | 0,430 | 0,416 | 0,430 | 0,596 | 0,567 | 0,583 | 0,038 | 0,031 | 0,049 |

Table 4 presents the average RRMSE and RB values of the four estimation methods for 64 areas. The simulation results indicate that the RRMSE values are generally comparable to those obtained in the 16-area case, while the RB values remain relatively stable across both area configurations. In both settings, the SREBLUP method consistently achieves the lowest RB values, approaching zero.

The performance of all estimators is strongly influenced by the level of contamination and the degree of spatial correlation. Under low contamination, the EBLUP and SEBLUP methods tend to produce relatively higher RRMSE and bias values, whereas the robust methods demonstrate superior performance with smaller RRMSE and RB. Among the robust approaches, SREBLUP slightly outperforms REBLUP, particularly under moderate to high spatial correlation, reflecting the advantage of explicitly incorporating spatial dependence as correlation increases.

When contamination increased to 5%, the differences among methods became more evident. The non-robust methods, EBLUP and SEBLUP, were more affected by outliers, resulting in higher RRMSE and RB values. In contrast, the robust methods REBLUP and SREBLUP exhibited clear advantages in handling extreme observations. Among them, SREBLUP performed best under low to moderate spatial correlation, with lower RRMSE and RB compared to REBLUP. However, at high spatial correlation, the performance of SREBLUP slightly declined. This result suggests that spatially connected outliers may propagate their influence across neighbouring areas, thereby reducing estimation stability.

At a high contamination level of 15%, the performance gap between robust and non-robust methods became even more pronounced. EBLUP and SEBLUP experienced further increases in RRMSE and RB compared to previous scenarios, likely due to the violation of their normality assumption caused by extreme outliers. Although SREBLUP maintained relatively good RRMSE performance, it was no longer superior to REBLUP under high spatial correlation. Nevertheless, SREBLUP still exhibited the smallest bias, indicating that it remained the least biased estimator among the four.

A comparison between the 16 area and 64 area settings revealed an interesting pattern in RRMSE and RB dynamics. For the 16-area case, RRMSE values tended to be slightly higher across all methods, likely because a smaller number of areas led to greater between-area variability and less stable estimates. In contrast, with 64 areas, the larger total sample size per area resulted in more stable and consistent estimations. This stability was also reflected in RB values: areas with smaller sample sizes exhibited higher bias, whereas in the 64-area case, RB values approached zero, indicating less biased and more consistent estimates.

The patterns observed in this study are largely consistent with the findings of [9] and [10]. In particular, the strong robustness of SREBLUP against outliers, as reflected by consistently low bias across contamination levels, confirms earlier evidence that robustification through influence functions effectively mitigates the impact of extreme observations.

Moreover, our results extend their findings by explicitly examining the interaction between robustness and spatial dependence. While [9] reported stable performance of SREBLUP under contamination, the present study shows that under high spatial correlation, the propagation of spatially clustered outliers may slightly reduce efficiency, especially in terms of RRMSE. This highlights an important trade-off between robustness and spatial smoothing that was not explicitly addressed in previous studies.

Overall, the results suggest that increasing the number of areas tends to reduce both RRMSE and RB, particularly for robust methods. Therefore, in small area estimation contexts involving data with outliers or strong spatial conditions, the REBLUP and SREBLUP methods are recommended due to their robustness and stability across varying conditions. Furthermore, these findings reinforce the conclusions of previous studies regarding the advantages of robust small area estimators, while also providing new insights into their behavior under varying levels of spatial correlation and different numbers of areas.

### 3.3 ART ANOVA

The comparative analysis of factor effects was conducted to identify which factors significantly influenced the response. ANOVA testing helped determine whether there were significant differences among the various groups being compared. The ANOVA test had several assumptions, such as independence, normality, and homogeneity of error variances, which needed to be satisfied. When these assumptions were violated, an alternative method, the Aligned Rank Transform (ART) ANOVA, was applied. Conversely, no further tests were conducted when the ART ANOVA results were not significant. The ART ANOVA results in this study, based on the RRMSE values, were presented in Table 5

**Table 5**. Results of the ART ANOVA based on RRMSE values

| | df | F-statistic | *p-value* |
|---|---|---|---|
| Method | 3 | 1653,89922 | < 2,22e-16 *** |
| Area | 1 | 1,68138 | 0,19484815 |
| % Outlier | 2 | 1017,94862 | < 2,22e-16 *** |
| Rho | 2 | 101,20939 | < 2,22e-16 *** |
| Method*area | 3 | 9,70918 | 0,51034833 |

| | df | F-statistic | *p-value* |
|---|---|---|---|
| Method*outlier | 6 | 45,28144 | <2,22e-16 *** |
| area*outlier | 2 | 46,80655 | <2,22e-16 *** |
| Method*rho | 6 | 4,65566 | 0,00010178 *** |
| area*rho | 2 | 0,80818 | 0,44577059 |
| Outlier*rho | 4 | 7,08884 | 1,1239e-05 *** |
| Method*area*outlier | 6 | 3,23503 | 0,00360106** |
| Method*area*rho | 6 | 1,72103 | 0,11201486 |
| Method*outlier*rho | 12 | 2,86373 | 0,00062341 *** |
| Area*outlier*rho | 4 | 6,99296 | 1,3419e-05 *** |
| Method*outlier*area*rho | 12 | 2,54700 | 0,00238560** |

Beyond the statistical significance reported in Table 5, the ART ANOVA results reveal several important practical implications for small area estimation. The results indicate that prediction accuracy is primarily driven by the estimation method, the proportion of outliers, and the degree of spatial correlation. These factors jointly determine the relative performance of the competing SAE approaches. In particular, the significant interaction between method and outlier proportion confirms that robust methods respond differently to contamination compared to non-robust methods. Likewise, the interaction between method and spatial correlation highlights that incorporating spatial dependence alters estimator performance as spatial dependence strengthens.

Several higher-order interactions are also significant, suggesting that the effectiveness of each method depends on the combined data conditions rather than on any single factor alone. While the number of areas does not exhibit a strong main effect, its interactions with outlier contamination and spatial correlation indicate that area configuration influences estimation accuracy indirectly. Overall, these findings emphasize that method selection in small area estimation should be guided by both the presence of outliers and the strength of spatial dependence, rather than relying on a single modelling assumption.

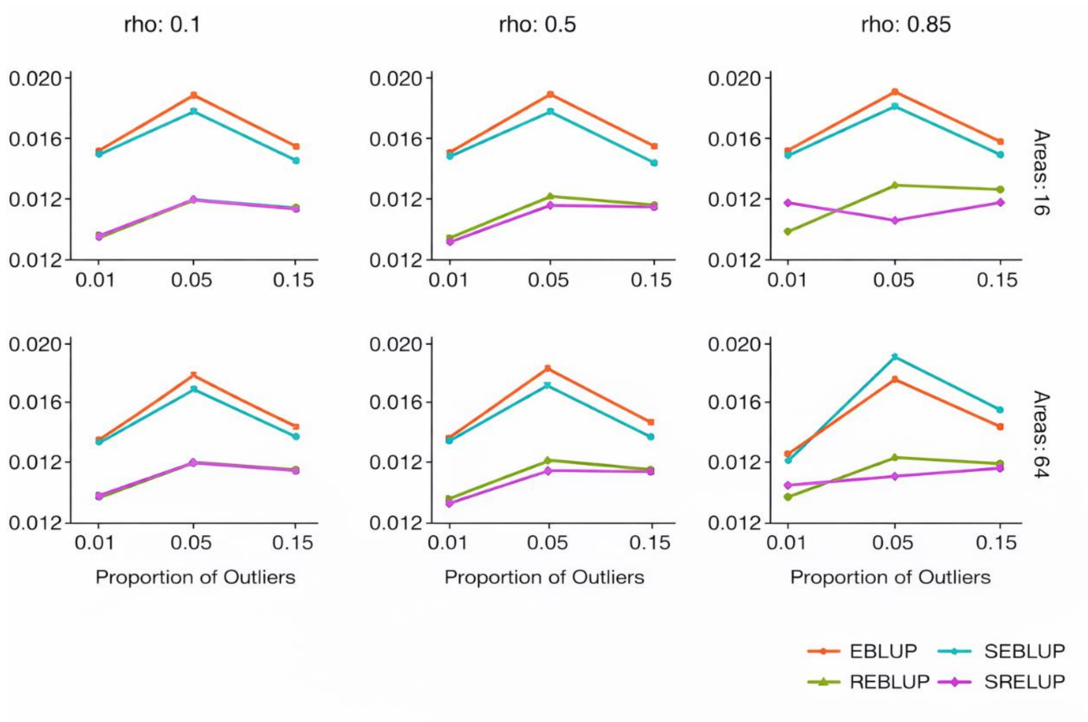



**Figure 2.** Interaction plot among area, method, percentage of outliers, and spatial correlation

Figure 2 illustrates the interaction among four key factors: estimation method, percentage of outliers, number of areas, and spatial correlation. The plot displays RRMSE values across increasing levels of outlier contamination, with separate panels corresponding to different area configurations and spatial correlation levels. The horizontal axis represents the percentage of outliers (1%, 5%, and 15%), while the vertical axis shows the RRMSE. Different coloured curves correspond to the four estimation methods (EBLUP, REBLUP, SEBLUP, and SREBLUP). Rows indicate the number of areas (16 and 64), and columns represent the level of spatial correlation (ρ = 0.1, 0.5, and 0.85).

Under low contamination, all methods yield relatively small RRMSE values; however, the non-robust methods (EBLUP and SEBLUP) already begin to exhibit greater instability across both area settings. As the percentage of outliers increases to moderate and high levels, these methods experience sharp increases in RRMSE, confirming their sensitivity to contamination. In contrast, the robust methods (REBLUP and SREBLUP) remain considerably more stable across all scenarios.

SREBLUP shows clear advantages over REBLUP under moderate to high spatial correlation, where incorporating spatial dependence enables more effective smoothing and reduces the influence of extreme observations. Nevertheless, when spatial correlation becomes very high, the RRMSE of SREBLUP increases slightly, likely due to the propagation of spatially clustered outliers through neighbouring areas.

The number of areas also affects estimation stability. The 16-area design exhibits larger fluctuations in RRMSE, reflecting greater between-area variability and limited information for borrowing strength. In contrast, the 64-area configuration provides a richer spatial structure, leading to more stable and consistent estimates across all methods. These findings are consistent with [9], which highlights the benefits of combining spatial information with robust estimation to improve prediction accuracy.

From a practical perspective, the results suggest that under low contamination and weak spatial correlation, REBLUP is generally sufficient. However, under moderate to high contamination or increasing spatial dependence, SREBLUP emerges as the most reliable method. Overall, increasing the percentage of outliers consistently increases RRMSE, underscoring the importance of robust and spatially robust estimators in contaminated data settings.

## 4. CONCLUSION

The simulation results demonstrate that estimation accuracy in small area estimation is strongly influenced by the presence of outliers, the choice of estimation method, and the strength of spatial correlation. The ART ANOVA analysis confirms significant interactions between these factors, indicating that method performance cannot be assessed independently of data conditions. In particular, non-robust methods exhibit rapidly increasing RRMSE as contamination grows, while robust approaches maintain greater stability, especially when supported by an increasing number of areas.

From a practical perspective, the findings provide clear guidance for method selection. EBLUP is appropriate only in settings with negligible contamination and weak spatial dependence. REBLUP performs reliably under low to moderate levels of outliers when spatial correlation is weak. In contrast, SREBLUP consistently delivers the most stable and accurate predictions under moderate to high contamination and increasing spatial correlation, benefiting from both robustness and spatial borrowing of strength. These recommendations are especially relevant for real data applications such as poverty mapping, small area income or expenditure estimation, and regional welfare analysis, where outliers and spatial dependence commonly arise. The simulation insights suggest that adopting robust and spatially explicit estimators can substantially reduce bias and improve reliability in such applications.

## 5. REFERENCES


[1] C. Baldermann, N. Salvati, and T. Schmid, "Robust small area estimation under spatial non-stationarity," International Statistical Review, vol. 86, no. 1, pp. 136–159, Apr. 2018, doi: 10.1111/insr.12245.

[2] S. K. 'Sinha and J. N. K. 'Rao, "Robust small area estimation," Can J Statistics, vol. 37, pp. 381–399, 2009.

[3] J. N. K. Rao and I. Molina, "Small area estimation". Hoboken, NJ: Wiley.

[4] A. Petrucci, M. Pratesi, and N. Salvati, "Geographic information in small area estimation: small area models and spatially correlated random area effects," 2005.

[5] N. A. C. Cressie, "Statistics for spatial data". New York: Wiley, 1993.

[6] J. N. K. Rao, "Small Area Estimation," New York: Wiley , 2003. doi: DOI:10.1002/0471722189.

[7] N. Salvati, "Small Area Estimation by Spatial Mo dels: the Spatial Empirical Best Linear Unbiased Prediction (Spatial EBLUP)." 2004. Working Paper, University of Pisa.

[8] N. Wichitaksorn, S. T. B. Choy, and R. Gerlach, "A generalized class of skew distributions and associated robust quantile regression models," Canadian Journal of Statistics, p. n/a-n/a, Sep. 2014, doi: 10.1002/cjs.

[9] T. Schmid, N. Tzavidis, R. Münnich, and R. Chambers, "Outlier robust small-area estimation under spatial correlation," Scandinavian Journal of Statistics, vol. 43, no. 3, pp. 806–826, 2016, doi: 10.1111/sjos.12205.

[10] T. Schmid and R. T. Münnich, "Spatial robust small area estimation," Statistical Papers, vol. 55, no. 3, pp. 653–670, 2014, doi: 10.1007/s00362-013-0517-y.

[11] G. Fauziah, A. Kurnia, and A. Djuraidah, "The empirical best linear unbiased prediction and the emperical best predictor unit-level approaches in estimating per capita expenditure at the subdistrict level," Scientific Journal of Informatics, vol. 12, no. 2, pp. 283–294, Jun. 2025, doi: 10.15294/sji.v12i2.25037.

[12] N. H. Pusponegoro and R. N. Rachmawati, "Spatial empirical best linear unbiased prediction in small area estimation of poverty," in Procedia Computer Science, Elsevier B.V., 2018, pp. 712–718. doi: 10.1016/j.procs.2018.08.214.

[13] N. Rakhsyanda, K. Sadik, and I. Indahwati, "Simulation study of robust geographically weighted empirical best linear unbiased predictor on small area estimation," Indonesian Journal of Statistics and Its Applications, vol. 5, no. 1, pp. 50–60, Mar. 2021, doi: 10.29244/ijsa.v5i1p50-60.

[14] A. Apriliansyah and I. Y. Wulansari, "Application of Spatial Empirical Best Linear Unbiased Prediction (SEBLUP) of Open Unemployment Rate on Sub-District Level Estimation in Banten Province", icdsos, vol. 2021, no. 1, pp. 905–913, Jan. 2022.

[15] A. Salma, K. Sadik, and K. A. Notodiputro, "Small area estimation of per capita expenditures using robust empirical best linear unbiased prediction (REBLUP)," in AIP Conference Proceedings, American Institute of Physics Inc., Mar. 2017. doi: 10.1063/1.4979443.

[16] R. E. Fay, R. A. Herriot, and R. E. F. Iii, "Estimates of Income for Small Places: An Application of James-Stein Procedures to Census Estimates of Income for Small Places: An Application of James-Stein Procedures to Census Data," 1979.

[17] G. E. Battese, R. M. Harter, and W. A. Fuller, "An error-components model for prediction of county crop areas using survey and satellite data," J Am Stat Assoc, vol. 83, no. 401, p. 28, Mar. 1988, doi: 10.2307/2288915.

[18] C. R. 'Henderson, "Best linear unbiased estimation and prediction under a selection model," Biometrics, vol. 31, no. 2, pp. 423–47, 1975.

[19] D. A. 'Harville, "Maximum likelihood approaches to variance component estimation and to related problems," J Am Stat Assoc, vol. 72, no. 358, pp. 320–338, 1977.

[20] W. H. Fellner, "Robust estimation of variance components," Technometrics, 28, 51-60. https://doi.org/10.1080/00401706.1986.104880971986.

[21] H. Li, Y. Liu, and R. Zhang, "Small area estimation under transformed nested-error regression models," Statistical Papers, vol. 60, no. 4, pp. 1397–1418, Aug. 2019, doi: 10.1007/s00362-017-0879-7.

[22] P. A. Parker, R. Janicki, and S. H. Holan, "A comprehensive overview of unit-level modeling of survey data for small area estimation under informative sampling," J Surv Stat Methodol, vol. 11, no. 4, pp. 829–857, Sep. 2023, doi: 10.1093/jssam/smad020.

[23] L. 'Mori and M. R. 'Ferrante, "Small area estimation of household economic indicators under unit-level generalized additive models for location, scale and shape," J Surv Stat Methodol, vol. 13, no. 1, pp. 160–196, 2025.