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Analysis Of The Stability Of The Smoking Distribution Model With Educational Factors And Return To Smoking Factors

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ABSTRACT

Cluster analysis analyzes similar elements as different and independent cluster research objects (not interconnected). PT. Indah Logistik Cargo is a company that provides goods and motorcycle shipping services through three channels: land, sea, and air. This research has three objectives. The first is to categorize postage prices based on the type of goods and services provided so that customers can easily see the options available and understand the differences between each option. Thirdly, categorize prices so companies can offer special promotions or discounts for certain services. Fourth, cluster prices so companies can offer special promotions or discounts for certain services. Based on the analysis results, 3 cluster groups were formed: cheap, medium, and expensive. It can be concluded that the group with low postage prices (Rp 16,160 to Rp 622,160) consists of Medan, Bengkulu, Palembang, Padang, and Pekanbaru; the group with medium postage prices (Rp 40,400 to Rp 888,800) is only Banda Aceh; and the group with high postage prices (Rp 62,620 to Rp 1,555,400) consists of Tanjung Pinang and Pangkal Pinang.

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1. INTRODUCTION

Smoking is a serious global health problem and continues to be a challenge in many countries, including Indonesia. Worldwide, it is estimated that there are approximately 1.1 billion smokers, with the majority being men [1]. In Indonesia, smoking prevalence remains high, even among teenagers. According to the Ministry of Health of the Republic of Indonesia [2], around 23.25% of teenagers under the age of 18 are smokers, with a higher proportion of men (19.5%) than women (3.75%).

There are many negative impacts produced by a smoker, both active and passive smokers. When viewed from a health aspect, cigarettes will have an impact on blood circulation, heart, stomach, skin, bones, brain, lungs, mouth and throat, reproduction and fertility, including increasing the risk of tuberculosis (TB) infection. Side effects of smoking are the risk of death for a person, namely cardiovascular disease, chronic obstructive pulmonary disease (COPD), lung cancer, oral cancer, and stroke [3]. Smoking not only harms active smokers but also those who are exposed to cigarette smoke (passive smoking). Cigarette smoke contains more than 7,000 dangerous chemicals, including tar and nicotine, which can damage almost all organs in the body. Health risks include heart disease, lung cancer, stroke, and chronic respiratory disorders. In addition, exposure to cigarette smoke in children can also increase the risk of sudden infant death syndrome (SIDS) [4].

To overcome this problem, a more

effective approach is needed. One way is through mathematical modeling, which can describe the dynamics of the spread of smoking habits in the population. This research uses a mathematical modeling approach, which was modified based on previous research by Syari'ah and Prawoto [5], researchers using a model with a smoking recurrence factor, plus the assumption that novice smokers can immediately stop smoking and individuals who are susceptible to smoking can become active. smokers because they are affected by it. by novice smokers and heavy smokers.

2. RESEARCH METHOD

In this research, the literature study method was used. The literature sources used are reference books, articles, and journals related to smoking behavior and mathematical models of disease spread. Furthermore, the research procedures carried out are as follows:

1) Collect supporting theories in research in the form of books, journals, theses, and articles.
2) Modifying the mathematical model of smoking distribution with educational factors and smoking relapse factors as a reference in the research "Analysis of Smoking Behavior Models with Smoking Relapse Factors" by Syari'ah & Prawoto [5].
3) Determining the equilibrium point of a mathematical model.
   The mathematical model formed on the distribution of smokers is a non-linear differential equation system due to the interaction between the components of the four sub-populations, so it is necessary to find a special solution. The equilibrium point is a state of a system that does not change with time. Equilibrium solutions from x^* on autonomous systems $\frac{dx}{dt} = f(x)$. It is said that the equilibrium point is if it meets the $f(x^*) = 0$ [6]. In dynamic systems, endemic or smoke-free equilibrium points are known and non-endemic or non-smoker-free equilibrium points. The point of equilibrium of smoke-free is a condition where there are no more smokers in a population. The equilibrium point of non-smoker freedom is the condition that smokers are always present in the population [7].
4) Analyzing the type of stability of the equilibrium point of a mathematical model.
   If A is a matrix $n \times n$ and $Av = \lambda v$ for scalar v, then scalar λ called is eigen. To find the eigenvalue of the matrix A, it can be written $Av = \lambda I v$ or equivalent $(A - \lambda I)v = 0$, where the equation has a non-zero solution if and only if $det(A - \lambda I)v = 0$ which is a characteristic equation of A, a scalar that satisfies the equation $det(A - \lambda I)v = 0$ is the eigenvalue of A [8].
   A system of differential equations can be stable at the equilibrium point without any further changes. [9] revealed that the stability of a system can be seen from its eigenvalues, namely,

Table 1. Types of Non-Linear System Stability

Eigenvalue (λ_1, λ_2)		**Type**	**Type of Stability**
$\lambda_1 < \lambda_2 < 0$		Node	Stable Asymptotic
$\lambda_1 > \lambda_2 > 0$		Node	Unstable
$\lambda_1 < 0 < \lambda_2$		Saddle Point	Unstable
$\lambda_1 = \lambda_2 > 0$		Node and Spiral Point	Unstable
$\lambda_1 = \lambda_2 < 0$		Node and Spiral Point	Stable Asymptotic
	$a > 0$	Stable Asimtotik	Unstable
$\lambda_1, \lambda_2 = a \pm bi$	$a < 0$	Spiral Point	Stable Asymptotic
	$a = 0$	Center or Spiral Point	Uncertain

5) Determine the Basic Reproduction Number (R_0).
   Reveals that the basic reproduction number is the parameter value of a model that produces simple criteria for the existence and stability of equilibrium points [10]. The equilibrium point of freedom against disease will be stable when $R_0 < 1$ and the endemic equilibrium point will be stable when $R_0 > 1$. In search of R_0 It is important to distinguish the presence of disease-causing parameters in a population. Suppose ϕ is the rate of new individuals detected adding to the infected class, the rate of new individuals detected adding to the infected class and ψ is the rate of development of diseases of death or recovery that reduces the population of a class. Furthermore, K is defined as the next generation matrix, so that obtained by finding $F = \frac{\partial \phi_i}{\partial u_j}(0, y_0)$ and $V = \frac{\partial \psi_i}{\partial u_j}(0, y_0)$. the expected value

of secondary infection in vulnerable populations is the largest eigen of the K matrix, namely:

$$R_0 = \pi(K) = \pi(FV^{-1})$$

6) Analysis of model simulation results using software Matlab.
7) Carrying out numerical simulations on the system using the Runge-Kutta Order 4, because of its stability in solving complex differential problems with high accuracy.
8) Draw a conclusion.

3. RESULT AND ANALYSIS

3.1 Mathematical Model

This mathematical model consists of 4 compartments, namely potential smoker $P(t)$ vulnerable population, light smoker $L(t)$ population of beginner smokers, severe smoker $S(t)$ population of active smokers, and quit smoker $Q(t)$ population who have quit smoking. Assumptions are formed based on existing facts to make it easier to construct mathematical models, as follows: (1) Individuals susceptible to smoking are individuals who have the potential to smoke at an individual rate of ω. (2) Individuals who are given education are potential individuals who will not have as big an impact on smoking σ. (3) The number of individuals is constant, so the growth rate of recruitment of individuals susceptible to smoking is the same as the rate of natural death μ. (4) The death rate due to smoking is d (5) Beginner smokers and heavy smokers can influence individuals who are susceptible to smoking with a level of influence of β. (6) Beginner smokers have the potential to become heavy smokers with a growth rate of ξ (7) Individuals who smoke heavily can quit smoking with a rate of awareness of δ (8) Beginner smokers can quit smoking with a rate of awareness of ρ (9) Individuals who have quit smoking can return to being active smokers with a relapse rate of γ (10) Individuals who have quit smoking can return to being vulnerable individuals who have been given education α. Based on these assumptions, a compartment diagram can be created as follows:

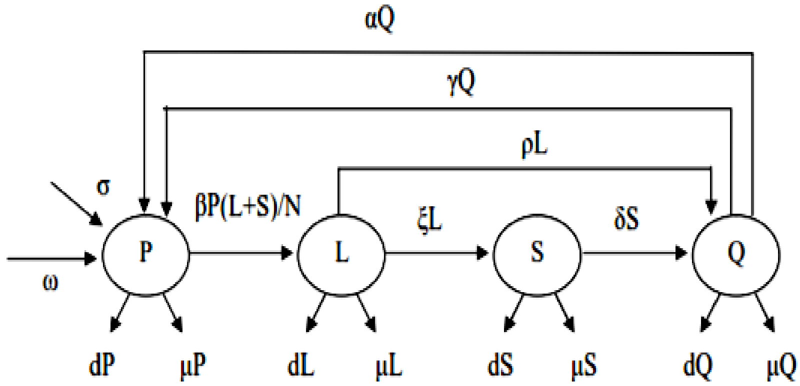

Figure 1. Compartment Diagram

Based on the compartment diagram in Figure 1, a mathematical model can be created in the form of a system of differential equations, namely:

$$\frac{dP(t)}{dt} = \omega + \sigma + \gamma Q + \alpha Q(t) - (d+\mu)P(t) - \frac{\beta P(t)(L(t)+S(t))}{N}$$
$$\frac{dL(t)}{dt} = \frac{\beta P(t)(L(t)+S(t))}{N} - (d+\mu+\xi+\rho)L(t)$$
$$\frac{dS(t)}{dt} = \xi L(t) - (d+\mu+\delta)S(t) \quad (1)$$
$$\frac{dQ(t)}{dt} = \delta S(t) + \rho L(t) - (d+\mu+\gamma+\alpha)Q(t)$$

because N(t) constant, then this system can be simplified: $p = \frac{P}{N}, l = \frac{L}{N}, s = \frac{S}{N}, q = \frac{Q}{N}$. So it is obtained $p + l + s + q = \frac{P}{N} + \frac{L}{N} + \frac{S}{N} + \frac{Q}{N} = 1$. The system of equations (1) is equivalent to:

$$\frac{dp(t)}{dt} = \omega + \sigma + \gamma q + \alpha q(t) - (d+\mu)p(t) - \beta p(t)(l(t)+s(t))$$
$$\frac{dl(t)}{dt} = \beta p(t)(l(t)+s(t)) - (d+\mu+\xi+\rho)l(t)$$
$$\frac{ds(t)}{dt} = \xi l(t) - (d+\mu+\delta)s(t) \quad (2)$$
$$\frac{dq(t)}{dt} = \delta s(t) + \rho l(t) - (d+\mu+\gamma+\alpha)q(t)$$

Table 2. Model Parameters

Parameter	Description
ω	The rate of individuals susceptible to smoking.
σ	Recruitment rate of educated individuals.
β	The rate of movement of vulnerable populations into new smoking populations.
ξ	The rate of movement of the novice smoking population into the heavy smoking population.
ρ	The rate of awareness among new smokers to quit smoking.
δ	The level of awareness among heavy smokers to quit smoking.
γ	Relapse rate of cigarette survivors to smoking again.
α	The rate of movement of individuals who have quit smoking to become vulnerable individuals who have been given education.
d	Death rate caused by smoking.
μ	Natural death rate.

3.2 Equilibrium Point

To determine the equilibrium point in the system of equations (1), you can find it by solving the equation with $\frac{dp}{dt} = 0, \frac{dl}{dt} = 0, \frac{ds}{dt} = 0, \frac{dq}{dt} = 0$, then two equilibrium points. The points that can be searched for are the points smoking-free equilibrium and smoking-free equilibrium point.

1. Smoke-Free Equilibrium Point (X_0)

$$(X_0) = \left(\frac{\omega+\sigma}{(d+\mu)}, 0,0,0\right) \quad (3)$$

2. The Equilibrium Point is not Smoke Free (X_1)

$$(X_1) = (p^*, l^*, s^*, q^*) \quad (4)$$

where

$$p^* = \frac{\omega(d+\mu+\delta)(d+\mu+\gamma+\alpha)+\sigma(d+\mu+\delta)(d+\mu+\gamma+\alpha)+\gamma l^*(\delta\xi+\rho(d+\mu+\delta)+\alpha l^*(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\gamma+\alpha))}{(d+\mu+\gamma+\alpha)(d+\mu)+\beta l^*(d+\mu+\delta+\xi))}$$

$$l^* = \frac{(d+\mu+\delta)(d+\mu+\gamma+\alpha)\big((d+\mu+\xi+\rho)(d+\mu)(d+\mu+\delta)-\omega\beta(d+\mu+\delta+\xi)\big)+\sigma\beta(d+\mu+\delta+\xi))}{\beta(d+\mu+\delta+\xi)(\rho\gamma(d+\mu+\delta)+\gamma\delta\xi-(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu+\gamma+\alpha)}$$

$$s^* = \frac{\xi l^*}{(d+\mu+\delta)}$$

$$q^* = \frac{l^*(\delta\xi+\rho(d+\mu+\delta))}{(d+\mu+\delta)(d+\mu+\gamma+\alpha)}$$

3.3 Basic Reproduction Number (R_0)

The method used to determine the basic reproduction number (R_0) is the *Next Generation Matrix* method. Which consists of matrices F and V. Determination of the basic reproduction number is based on the order of the subpopulation causing the *infection*, so we get:

$$\frac{dl}{dt} = \beta p(l+s) - (d+\mu+\xi+\rho)l$$

$$\frac{ds}{dt} = \xi l - (d+\mu+\delta)s$$

by obtaining a matrix φ and ψ as follows:

$$\varphi = \begin{bmatrix} \beta\rho(l+s) \\ 0 \end{bmatrix} \text{ dan } \psi = \begin{bmatrix} (d+\mu+\xi+\rho)l \\ -\xi l + (d+\mu+\delta)s \end{bmatrix}$$

next, linearization is carried out on φ and ψ as follows:

$$F = \begin{bmatrix} \frac{\partial\varphi_1}{\partial x_1} & \frac{\partial\varphi_2}{\partial x_2} \\ \frac{\partial\varphi_1}{\partial x_1} & \frac{\partial\varphi_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \beta\rho & \beta\rho \\ 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{\partial\psi_1}{\partial x_1} & \frac{\partial\psi_2}{\partial x_2} \\ \frac{\partial\psi_1}{\partial x_1} & \frac{\partial\psi_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} (d+\mu+\xi+\rho) & 0 \\ -\xi & (d+\mu+\delta) \end{bmatrix}$$

so the *next generation matrix* is obtained as follows:

$$K = FV^{-1}$$

$$K = \begin{bmatrix} \frac{\beta\rho}{(d+\mu+\xi+\rho)} + \frac{\beta\rho\xi}{(d+\mu+\xi+\rho)(d+\mu+\delta)} & \frac{\beta\rho}{(d+\mu+\delta)} \\ 0 & 0 \end{bmatrix}$$

next, look for the eigenvalues of the *next generation matrix* $|\lambda I - K| = 0$, and by taking the maximum eigenvalue, it can be obtained:

$$R_0 = \frac{\beta\rho(d+\mu+\delta+\xi)}{(d+\mu)(d+\mu+\xi+\rho)(d+\mu+\delta)}$$

3.4 Stability Analysis

In this section, linearization is carried out on the system of equations to simplify the stability analysis, so that the Jacobian matrix is obtained as follows:

$$J = \begin{bmatrix} -(d+\mu)-\beta(l+s) & -\beta p & -\beta p & \gamma+\alpha \\ \beta(l+s) & \beta p-(d+\mu+\xi+\rho) & \beta p & 0 \\ 0 & \xi & -(d+\mu+\delta) & 0 \\ 0 & \rho & \delta & -(d+\mu+\gamma+\alpha) \end{bmatrix}$$

1. Next it is calculated $|J(X_0)-\lambda I| = 0$ to obtain eigenvalues which, if substituted, are the smoking-free equilibrium point (X_0), 4 eigenvalues can be obtained, namely:
$$\lambda_1 = -d-\mu$$
$$\lambda_2 = -d-\mu-\gamma-\alpha$$
$$\lambda_3 = \frac{1}{2}\left(\frac{\beta\omega+\sigma}{d+\mu} - (2(d+\mu+\delta)+\xi+\rho+\delta\right) + \sqrt{\frac{\beta^2\omega^2+\sigma^2}{d+\mu} + \frac{2\beta\omega+\sigma}{d+\mu}(\xi+\delta-\rho) + 2(-\delta\xi+\rho\xi-2\delta\rho)+\xi^2+(\rho+\delta)^2}$$
$$\lambda_4 = \frac{1}{2}\left(\frac{\beta\omega+\sigma}{d+\mu} - (2(d+\mu+\delta)+\xi+\rho+\delta\right) - \sqrt{\frac{\beta^2\omega^2+\sigma^2}{d+\mu} + \frac{2\beta\omega+\sigma}{d+\mu}(\xi+\delta-\rho) + 2(-\delta\xi+\rho\xi-2\delta\rho)+\xi^2+(\rho+\delta)^2}$$

Based on is a quadratic equation $ax^2+bx+c=0$, which is if the roots of the equation x_1 and x_2 is negative, then $x_1+x_2 = \frac{-b}{a} < 0$ and $x_1.x_2 = \frac{c}{a} > 0$. So, for λ_3 and λ_4 which comes from a quadratic equation $ax^2+bx+c=0$, wich one λ_3 and λ_4 negative value. So $\lambda_3+\lambda_4 = \frac{-b}{a} < 0$ when $\beta < \frac{(2d+2\mu+2\delta+\xi+\rho)(d+\mu)}{\omega+\sigma}$ and $\lambda_3.\lambda_4 = \frac{c}{a} > 0$ when $\beta < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu)}{\rho(d+\mu+\delta+\xi)}$. Jadi, berdasarkan $\lambda_1,\lambda_2,\lambda_3,\lambda_4$, a system at the smoke-free equilibrium point will be asymptotically stable under these conditions $\beta < \frac{(2d+2\mu+2\delta+\xi+\rho)(d+\mu)}{\omega+\sigma}$ dan $\beta < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu)}{\rho(d+\mu+\delta+\xi)}$

2. Next it is calculated $|J(X_1)-\lambda I| = 0$ to obtain eigenvalues which, if substituted into the equilibrium point, are not independent for cigarette users (X_1), 4 eigenvalues can be obtained, namely:
$$\lambda_1 = (-d-\mu) - \left(\frac{(d+\mu+\gamma+\alpha)\big((d+\mu+\xi+\rho)(d+\mu)(d+\mu+\delta)-\omega\beta(d+\mu+\delta+\xi)\big)+\sigma\beta(d+\mu+\delta+\xi)\big)}{\rho\gamma(d+\mu+\delta)+\gamma\delta\xi-(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu+\gamma+\alpha)}\right)$$
$$\lambda_2 = -d-\mu-\gamma-\alpha$$
$$\lambda_3 = \frac{1}{2}\Big(-2d-2\mu-\delta+\xi+\rho-(d+\mu+\delta)(\beta\omega(d+\mu+\delta+\xi)\big(-\rho\gamma+\gamma(2\rho+d+\mu+\xi)\big) + (d+\mu)(d+\mu+\xi+\delta)\gamma\big(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\xi+\rho)(d+\mu)\big)+\sqrt{D}\Big)$$
$$\lambda_4 = -\frac{1}{2}\Big(-2d-2\mu-\delta+\xi+\rho-(d+\mu+\delta)(\beta\omega(d+\mu+\delta+\xi)\big(-\rho\gamma+\gamma(2\rho+d+\mu+\xi)+(d+\mu)(d+\mu+\xi+\delta)\big)-\gamma\big(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\xi+\rho)(d+\mu)\big)+\sqrt{D}\Big)$$

Based on is a quadratic equation $ax^2+bx+c=0$, which is if the roots of the equation x_1 and x_2 is negative, then $x_1+x_2 = \frac{-b}{a} < 0$ and $x_1.x_2 = \frac{c}{a} > 0$. So, for λ_3 and λ_4 which comes from a quadratic equation $ax^2+bx+c=0$, wich one $\lambda_3$and λ_4 negative value. So $\lambda_3+\lambda_4 = \frac{-b}{a} < 0$ and $\lambda_3.\lambda_4 = \frac{c}{a} > 0$ when:

$$\beta < \frac{\gamma(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\xi+\rho)(d+\mu)}{\omega(d+\mu+\delta+\xi)(-\rho\gamma+\gamma(2\rho+d+\mu+\xi)+(d+\mu)(d+\mu+\xi+\delta)}$$

so, based on $\lambda_1,\lambda_2,\lambda_3,\lambda_4$, The system at the equilibrium point without smokers will be asymptotically stable under these conditions $\beta < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu)}{\rho(d+\mu+\delta+\xi)}$, $\gamma < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu+\alpha)}{((d+\mu)^2\delta(d+\mu)\xi(d+\mu))}$ and $\beta < \frac{\gamma(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\xi+\rho)(d+\mu)}{\omega(d+\mu+\delta+\xi)(-\rho\gamma+\gamma(2\rho+d+\mu+\xi)+(d+\mu)(d+\mu+\xi+\delta)}$.

3.5 Simulation

In this section, a numerical simulation is performed to compare the results of analytical calculations with those obtained through numerical methods. The simulation is carried out using Matlab software (version 23b). The initial values for the simulation are: $N = 400$ $p(0) = 0{,}2,$ $l(0) = 0{,}3,$ $s(0) = 0{,}3,$ $q(0) = 0{,}2.$ These parameter values are based on the study by Syari'ah & Prawoto (2022), as shown in the table below: Parameter values are taken based on the journal (Syari'ah & Prawoto 2022) used in the table below:

Table 3. Parameter Values

Parameter	Parameter Values
β	0,021
ξ	0,021
δ	0,000274
ρ	0,03
ω	0,000035
σ	0,2
α	0,09
γ	0,006
d	0,000005
μ	0,0003

The parameters used in the numerical simulation of the smoking distribution model with education factors and smoking recurrence factors are $\beta = 0{,}021,\ \xi = 0{,}021,\ \delta = 0{,}000274,\ \rho = 0{,}03,\ \sigma = 0{,}2$ $\omega = 0{,}000035,\ d = 0{,}000005,\ \mu = 0{,}0003,\ \alpha = 0{,}09,\ \gamma = 0{,}006$

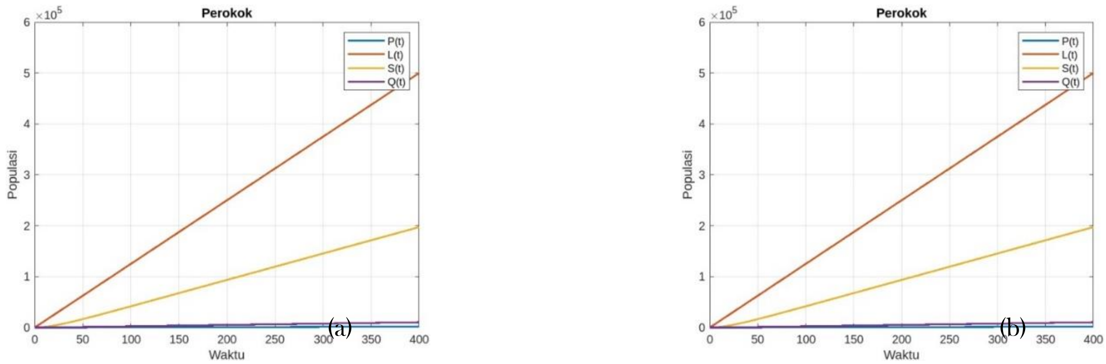

Figure 2. Graph of the Difference between σ, α *and* γ in the Smoking Distribution Model with Education Factors and Smoking Relapse Factors.

Note: In figure (a), if the parameter value $\sigma > \alpha > \gamma$, then the distribution of smokers by education factor and smoking recurrence factor tends not to increase in the subpopulations L (beginner) and Q (recovered), whereas in figure (b), when the parameter value $\sigma = \alpha = \gamma$ experiences changes in the distribution of smokers with educational factors and the smoking recurrence factor in the four populations, it tends to increase with constant P (susceptible) towards the balance point.

3.5.1 Simulation of Equilibrium Point Stability Analysis without Cigarette Users

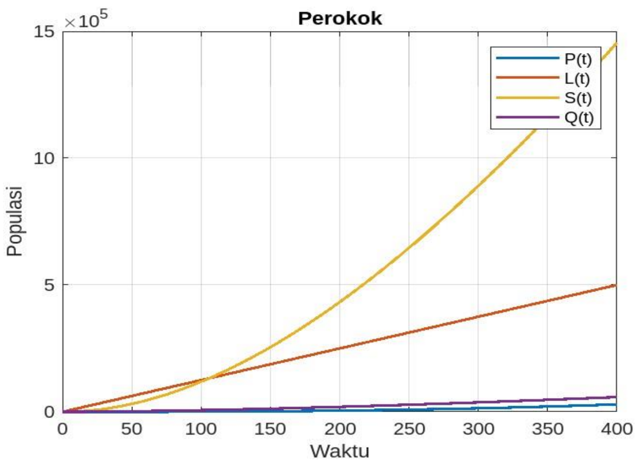

Figure 3. Area of Stability in the Population of Cigarette Users in Smoke-Free Conditions

Based on the results in Figure 3, it shows that the dynamic model in a population of cigarette users with a smoke-free condition will be asymptotically stable at the point $X_0 = \left(\frac{\omega+\sigma}{(d+\mu)}, 0,0,0\right)$ if value $\beta < \frac{(2d+2\mu+2\delta+\xi+\rho)(d+\mu)}{\omega+\sigma}$.

3.5.2 Simulation of Equilibrium Point Stability Analysis without Cigarette Users

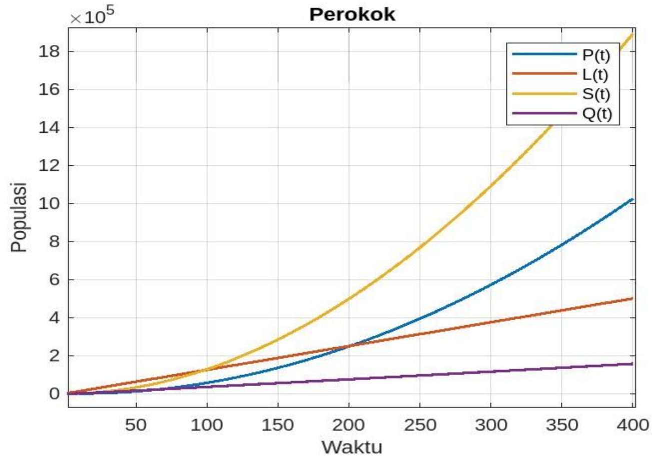

Figure 4. Area of Stability in the Population of Cigarette Users in Non-Smoker Conditions

Based on the results in Figure 4, it shows that the dynamic model in the population of cigarette users with the condition of being non-smokers is with points:

$$X_1 = \left(\begin{array}{c} \frac{\omega(d+\mu+\delta)(d+\mu+\gamma+\alpha)+\sigma(d+\mu+\delta)(d+\mu+\gamma+\alpha)+\gamma l^*(\delta\xi+\rho(d+\mu+\delta)+\alpha l^*(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\gamma+\alpha)}{(d+\mu+\gamma+\alpha)(d+\mu)+\beta l^*(d+\mu+\delta+\xi))}, \\ \frac{(d+\mu+\delta)(d+\mu+\gamma+\alpha)\big((d+\mu+\xi+\rho)(d+\mu)(d+\mu+\delta)-\omega\beta(d+\mu+\delta+\xi)\big)+\sigma\beta(d+\mu+\delta+\xi))}{\beta(d+\mu+\delta+\xi)(\rho\gamma(d+\mu+\delta)+\gamma\delta\xi-(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu+\gamma+\alpha)}, \\ \frac{\xi l^*}{(d+\mu+\delta)}, \frac{l^*(\delta\xi+\rho(d+\mu+\delta))}{(d+\mu+\delta)(d+\mu+\gamma+\alpha)} \end{array} \right)$$

will be asymptotically stable if it meets the conditions with three value limits, namely: $\beta < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu)}{\rho(d+\mu+\delta+\xi)}$, $\gamma < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu+\alpha)}{((d+\mu)^2\delta(d+\mu)\xi(d+\mu))}$, $\beta < \frac{\gamma(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\xi+\rho)(d+\mu)}{\omega(d+\mu+\delta+\xi)(-\rho\gamma+\gamma(2\rho+d+\mu+\xi)+(d+\mu)(d+\mu+\xi+\delta)}$.

4. CONCLUSION

Based on the results of research analyzing the stability of the smoking distribution model with educational factors and smoking recurrence factors, the following conclusions were obtained:

1. Model of distribution of smokers with educational factors and smoking recurrence factors, obtained the following model:

$$\frac{dP(t)}{dt} = \omega + \sigma + \gamma Q + \alpha Q(t) - (d+\mu)P(t) - \frac{\beta P(t)(L(t)+S(t))}{N}$$
$$\frac{dL(t)}{dt} = \frac{\beta P(t)(L(t)+S(t))}{N} - (d+\mu+\xi+\rho)L(t)$$
$$\frac{dS(t)}{dt} = \xi L(t) - (d+\mu+\delta)S(t)$$
$$\frac{dQ(t)}{dt} = \delta S(t) + \rho L(t) - (d+\mu+\gamma+\alpha)Q(t).$$

2. The smoking distribution model has two equilibrium points, $X_0 = \left(\frac{\omega+\sigma}{(d+\mu)}, 0,0,0\right)$ and X_1. The first point, X_0 represents a smoke-free population, which is asymptotically stable if it meets the requirements $\beta < \frac{(2d+2\mu+2\delta+\xi+\rho)(d+\mu)}{\omega+\sigma}$. The second point, X_1 represents a non-smoking population, which is asymptotically stable if it meets the requirements $\beta < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu)}{\rho(d+\mu+\delta+\xi)}$, $\gamma < \frac{(d+\mu+\xi+\rho)(d+\mu+\delta)(d+\mu+\alpha)}{((d+\mu)^2\delta(d+\mu)\xi(d+\mu))}$ dan $\beta < \frac{\gamma(\delta\xi+\rho(d+\mu+\delta)(d+\mu+\xi+\rho)(d+\mu)}{\omega(d+\mu+\delta+\xi)(-\rho\gamma+\gamma(2\rho+d+\mu+\xi)+(d+\mu)(d+\mu+\xi+\delta)}$

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