Zero : Jurnal Sains, Matematika, dan Terapan
E-ISSN : 2580-5754
P-ISSN : 2580-569X
Vol. 8, No. 2, 2024
DOI: 10.30829/zero.v8i2.21027
pp. 57-64

Graph Coloring With Welch-Powell Algorithm In Determining Lecture Schedule

Tarisa Azrina Tanaya[1], Aghni Syahmarani[2]

[1,2] Department of Mathematics, Universitas Sumatera Utara, Medan, Indonesia

Article Info

Article history:

Received, 08 07 2024
Revised, 13 08 2024
Accepted, 15 12 2024

Keywords:

Graph, Graph Coloring, Scheduling, Welch-Powell Algorithm

ABSTRACT

Arrangement of lecture schedules at the university involves time sharing, lecturer availability, and classroom availability. One algorithm that can be used to make a good schedule without any conflict is the Welch Powell algorithm. In the Bachelor degree of Mathematics FMIPA USU there are 61 courses for odd semester and 44 courses for even semester in 2023/2024. Therefore, graph coloring is needed to organize lecture schedules so that there are no conflict between the time sharing, lecturer availability, and classroom availability. The courses are represented in the form of a graph and afterwards the graph is colored with the Welch-Powell algorithm which produces 12 chromatic numbers or 12 colors so that there are 12 optimum sessions needed for scheduling lectures in odd and even semesters. Based on this result, the same color can be scheduled on the same day and same time with different classes.

Corresponding Author:

Tarisa Azrina Tanaya,
Department of Mathematics,
Universitas Sumatera Utara, Medan, Indonesia
Email: tarisaazrina@gmail.com

1. INTRODUCTION

Many problems have similar properties to graph coloring, so it is interesting to discuss more about the graph coloring aspect. In education, scheduling is very important to optimize the teaching and learning process. This includes the allocation of resources according to their expertise and availability, which is often a challenge in itself [1]. If done manually, course scheduling will take a long time and is prone to problems due to human error. This problem can be overcome by using one of the concepts in graph theory, namely graph coloring [2]. In managing lecture schedules, graph coloring becomes relevant to ensure that all levels of students can attend every course that has been programmed without any schedule collisions between one course and another [3]. Scheduling courses in lectures is one of the challenges faced by universities. The challenge is to allocate resources such as lecturers, students, and classrooms to objects, namely courses, which will be placed at a certain time. This is done by taking into account various constraints with the aim of optimizing the utilization of university facilities in order to meet a set of desired objectives. In general, university scheduling problems are divided into two, namely course scheduling and exam scheduling (Rahadi, 2019). Solving the conflicting schedule problem demands the right scheduling algorithm or method. There are techniques and algorithms that can be used to solve scheduling problems so as to build a suitable schedule, one of which is by using the Welch-Powell algorithm. The Welch-Powell algorithm is a graph coloring technique that uses the Largest Degree Ordering (LDO) method, where graph coloring is done based on the highest degree of its vertices. The main objective of this algorithm is to directly color the graph using as few colors as possible by performing coloring based on the highest to lowest degree and using one color to color the first vertex and use the same color to color vertices that are not neighbors to the first vertex, use the next color to color vertices that have not been colored in the same way as the previous vertex. But keep in mind that the Welch-Powell algorithm may not consistently achieve the minimum number of colors needed to color the entire graph [4].

In previous research, namely research [5], it was found that to place 15 students without placing students from the same study program or UKT in one room, and assuming a maximum capacity of 4 people per room, 6 rooms are needed. This can be attributed to the result of the chromatic number obtained is 4. Then in research [6] also used graph coloring to determine culinary tourism destinations in Bandung City. In this study, restaurants are represented as vertices and food types as edges. The results show that the graph requires 11 colors, which means that there are 11 recommended restaurants that each have at least one type of food in common. [7] conducting research to schedule proposal seminars at the UNIMED Mathematics Study Program using graph coloring produces 10 colors of vertices in the graph. Which means the proposal seminar schedule will be held in 10 different times with students represented as vertices and students who have the same supervisor or examiner will be connected by lines.

Course timetabling in universities involves time-sharing, lecturer availability, and lecture halls. Basically, in preparing the course schedule, it must be adjusted in such a way that there are no schedule clashes so that students can get the desired courses and lecturers who teach without any course schedule clashes. Some of the factors that cause course schedule clashes are students who take many courses, limited lecture time, and limited classrooms used. The result of these constraints is the occurrence of clashes in the course schedule [8].

Due to the limitations in the previously mentioned matters, clashes often occur in the preparation of the schedule for the Bachelor of Mathematics study program at FMIPA USU. So in this study, researchers used the Welch Powell algorithm to determine the odd semester and even semester lecture schedule in the Bachelor of Mathematics study program at FMIPA USU for the 2023/2024 academic year in the hope that these results will later make scheduling that has no clashes and is more satisfying for anyone who uses the results of this study

2. RESEARCH METHOD

This research uses an experimental research design to test the effectiveness and efficiency of the Welch-Powell algorithm in solving lecture scheduling problems. The types of data used include qualitative data and quantitative data. Qualitative data includes the needs and constraints in scheduling, such as lecturer preferences, lecture hall availability, and relationships between courses. Meanwhile, quantitative data includes the number of courses, the number of classes, and the time available.

The data source used in this research is secondary data. Secondary data was obtained from official institutional documents, previous lecture schedules, and academic databases. This data is then analyzed to understand the needs and limitations in scheduling lectures.

The research procedure begins with the collection of all necessary secondary data. Next, the data is used to model a graph representing courses as vertices and scheduling conflicts as edges. The Welch-Powell algorithm is applied by sorting the vertices based on their degree and applying graph coloring. In this research, vertices represent courses and edges represent scheduling conflicts, i.e. courses that have the same lecturer or students attending the same class. The following are the steps that can be followed to use the Welch-Powell algorithm [9]:

1. Order the vertices of G in decreasing degree (this order may not be unique as some vertices may have the same degree).
2. Use one color to color the first vertex (which has the highest degree) and the other vertices (in descending order) that are not neighbors of this first vertex.
3. Start again with the next highest degree vertex in the ordered list that does not yet have a color and repeat the vertex coloring process using the second color.
4. Repeat the addition of colors until all vertices have been colored.

The results of the algorithm are analyzed based on efficiency criteria and adherence to the specified constraints. Validation is done by comparing the generated schedule with an existing schedule or through simulation to ensure there are no conflicts and that the schedule meets the needs of all parties. Finally, a research report is prepared that includes the methods, analysis, results, and recommendations for future implementation

3. RESULT AND ANALYSIS

Data obtained from previous lecture schedules in the Undergraduate Mathematics Study Program FMIPA USU can be seen at the following link:

https://docs.google.com/spreadsheets/d/1E0cof_qVw2Ghh4CLmHvp2VdOYTimkwabeQw37RgcVx8/edit?usp=sharing. With the availability of 9 classrooms, namely unit 4 classrooms FMIPA USU as many as 6 classes, namely: 104.2.2.1, 104.2.2.2, 104.2.2.3, 104.2.2.10, 104.2.2.11, 104.2.2.12 and unit 9 as many as 3 classes, namely: 109.3.2.2, 109.3.2.3, 109.3.2.4, with consideration of MKWU and MKWK schedules and lecturer schedules outside the Bachelor of Mathematics study program are not included. Before the data is converted into graph form, labels are assigned to the data to simplify the representation of each vertex. Vertices to represent courses and edges represent conflicts in the course, namely the presence of the same lecturer. In odd semester, the label is made with 1AA, 1AB, 1AC, ..., 7PJ, 7PK, 7PL with the intention that the first number is a semester clue, the first letter is a class clue, and the second letter is a course clue. The same applies to labels in even semesters.

Because the number of lecturers in the Bachelor of Mathematics study program is still limited and there are the same lecturers who teach different courses or lecturers who teach courses for students in different semesters, there is an adjacency relationship between vertices in one subgraph and vertices in another subgraph. The courses in the Undergraduate Mathematics Study Program FMIPA USU for the odd semester 2023/2024 consist of 61 courses so there are 61 vertices, while the even semester consists of 44 courses so there are 44 vertices. In this process, it is explained that 0 indicates the vertex is not neighboring which means there is no same lecturer in a course and 1 indicates that the vertex has a neighboring relationship between other vertices which means there is the same lecturer in a course. And also because students attend the same class, there is neighborliness between vertices that have the same class, for example, vertex 2AA with 2AB will be 1 because they have the same class.

Representing the lecture schedule of the Bachelor of Mathematics FMIPA USU in the form of a graph consisting of vertices and edges that connect them, so that neighboring and non-neighboring vertices can be seen through the edges that connect them. The following is an image of the odd semester Mathematics schedule which has been represented in graph form in Figure 1 and the even semester Mathematics schedule which has been represented in graph form in Figure 2 below:

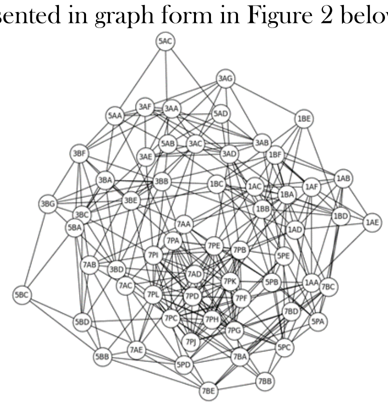

Figure 1. Graph Representation Odd Semester 2023/2024 Mathematics FMIPA USU

Figure 2. Graph Representation Even Semester 2023/2024 Mathematics FMIPA USU

In the figure above is a representation of data into a graph. Where each vertex represents a course and each edge represents a schedule conflict, namely if the course has the same lecturer or students who have the same class. For naming vertices according to the labeling of the data. With numbers indicating the semester, the first letter indicates the class, and the letter indicates the course. In compulsory courses the class is indicated with A and B and for elective courses it is indicated with class P.

Thus there are 61 vertices in Figure 1 and 44 vertices in Figure 2, it can be observed that there are 10 subgraphs of odd semester graph representation as follows:

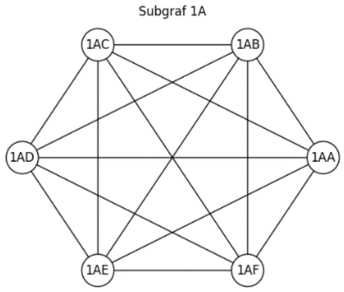

Figure 3. Subgraph S_1

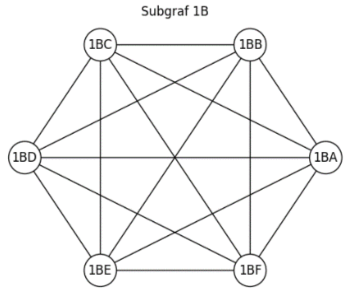

Figure 4. Subgraph S_2

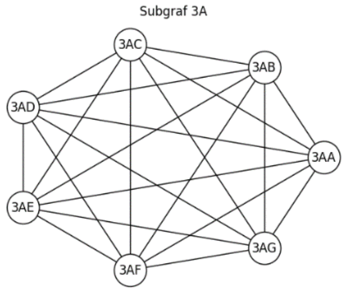

Figure 5. Subgraph S_3

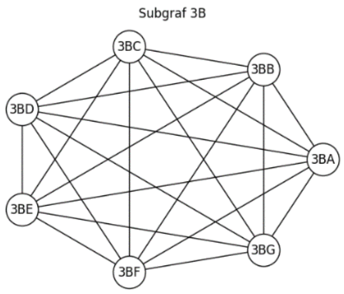

Figure 6. Subgraph S_4

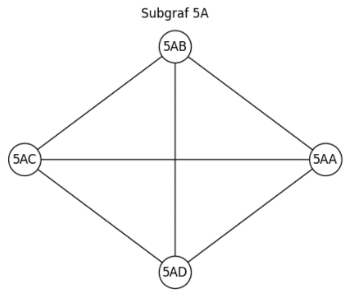

Figure 7. Subgraph S_5

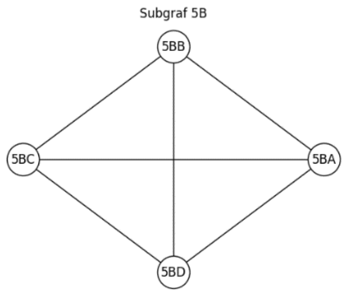

Figure 8. Subgraph S_6

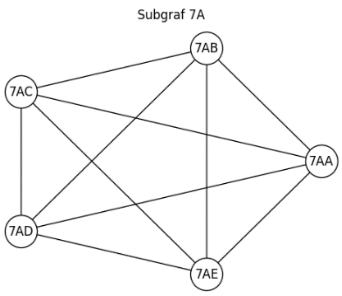

Figure 9. Subgraph S_7

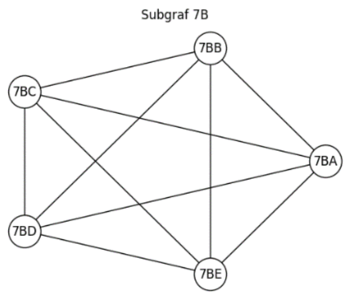

Figure 10. Subgraph S_8

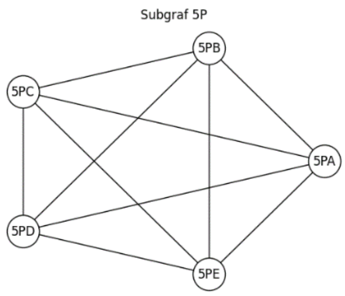

Figure 11. Subgraph S_9

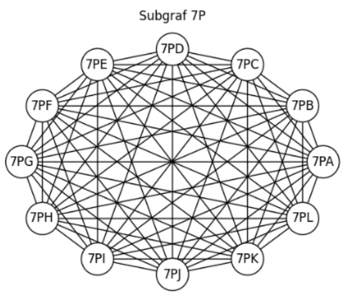

Figure 12. Subgraph S_{10}

All vertices in the subgraph of each semester with the same class are connected because they are attended by the same students. This means that each subgraph is a complete graph. In accordance with the previous vertex labeling, the vertex labels in all subgraphs consist of 1 number which indicates the semester and 2 letters, the first letter indicates the class, and the second letter indicates the course. In the class of compulsory courses represented by the letters A and B and for the class of elective courses represented by

P. Likewise for the even semester subgraph, there are 7 subgraphs from the even semester graph representation as follows:

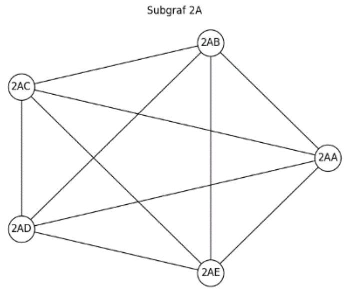

Figure 13. Subgraph T_1

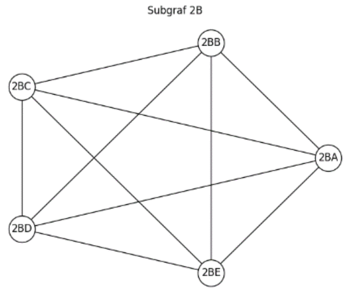

Figure 14. Subgraph T_2

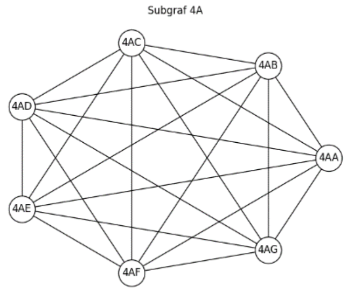

Figure 15. Subgraph T_3

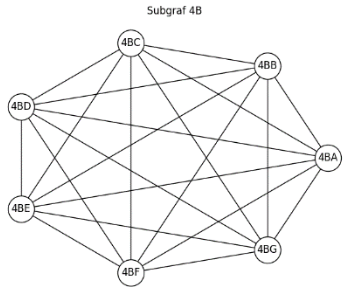

Figure 16. Subgraph T_4

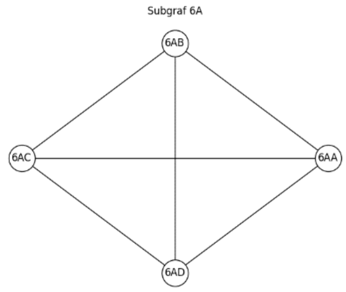

Figure 17. Subgraph T_5

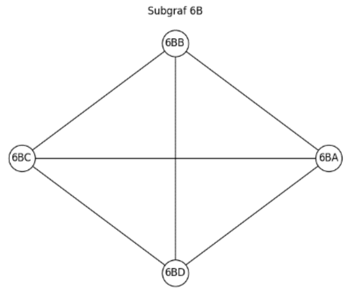

Figure 18. Subgraph T_6

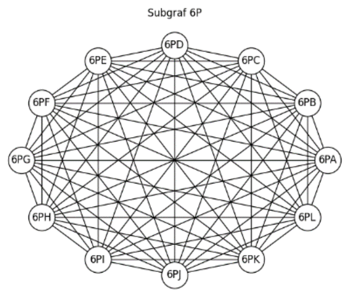

Figure 19. Subgraph T_7

Since all subgraphs are complete graphs, each subgraph has a chromatic number equal to the number of vertices in the subgraph. In chromatic number, a complete graph K_n where n vertices has chromatic number n. Hence, each subgraph has a chromatic number corresponding to the number of vertices. Thus, the minimum limit for the odd semester lecture schedule period in the Mathematics lecture of FMIPA USU in general is

$$max\{\chi(K_6), \chi(K_7), \chi(K_4), \chi(K_5), \chi(K_{12})\} = max\{6,7,4,5,12\} = 12$$

So that in odd semester there are 12 periods per week for the lecture schedule. As for the even semester lecture subgraph, it has 7 subgraphs with a complete graph for each subgraph, so the chromatic number corresponds to the number of vertices. The minimum limit for the even semester lecture schedule period in the FMIPA USU Mathematics lecture in general is

$$max\{\chi(K_5), \chi(K_7), \chi(K_4), \chi(K_{12})\} = max\{5,7,4,12\} = 12$$

So that in even semesters there are 12 scheduling periods per week. Next, the Welch-Powell algorithm is used to color the vertices in the graph. The first step to start coloring vertices with this algorithm is to calculate the degree of the vertex first which is then sorted by the highest to lowest degree. The degrees are sorted based on the highest to lowest degree so that when coloring the vertex is done sequentially from highest to lowest. The first highest vertex will be colored with the first color and likewise for vertices that are not neighbors with the first highest vertex will be colored with the same color. Then the next vertex is colored according to the order of its degree. The most important thing is that connected vertices should not have the same color. So the vertex coloring, viz:

Table 1. Odd Semester Vertex Color Division

No.	Warna	Jumlah	Verteks
1	0	10	7PC, 3AD, 3BE, 1AD, 5PC, 1BC, 5AA, 7AB, 7BC, 5BB
2	1	10	7PE, 1AA, 3BD, 1BB, 3AB, 7AD, 7BE, 5PA, 5BA, 5AD
3	2	10	7PK, 3AC, 3BA, 5PD, 1AB, 7BD, 5AB, 1BE, 7AC, 5BC
4	3	8	7PL, 1BA, 3BB, 1AC, 3AA, 7BA, 5BD, 5AC
5	4	8	7PB, 7AA, 3BC, 1BF, 3AF, 5PE, 7BB, 1AE
6	5	7	7PD, 3BF, 1AF, 3AE, 5PB, 1BD, 7AE
7	6	3	7PF, 3AG, 3BG
8	7	1	7PI
9	8	1	7PA
10	9	1	7PG
11	10	1	7PH
12	11	1	7PJ

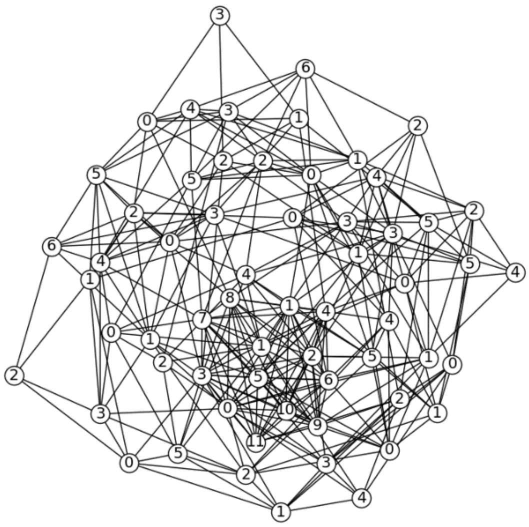

Figure 20. Colored Odd Semester Graph

In Figure 20. above is a graph that has been colored according to the color division in Table 1. So that neighboring vertices do not have the same color. In this graph, colors are represented by numbers so that the first color for the first vertex and vertices that do not neighbor the first vertex are colored with the 0th color, the second color is represented by the number 1, and so on until all vertices are colored provided that no neighboring vertices have the same color. Because the chromatic number is 12, the colors used are 12 colors, which means that scheduling is needed 12 periods a week. Likewise, for the distribution of vertex colors in even semesters as follows:

Table 2. Even Semester Vertex Color Division

No.	Warna	Jumlah	Verteks
1	0	5	6PB, 4AD, 4BD, 6BD, 2AB
2	1	6	6PL, 2BD, 4BE, 4AA, 2AE, 6BB
3	2	7	6PC, 2AD, 4AB, 4BA, 6AC, 2BB, 6BA
4	3	6	6PD, 4AE, 4BC, 2AC, 2BA, 6AA
5	4	6	6PE, 6AD, 2BC, 4AG, 4BB, 6BC
6	5	6	6PG, 2AA, 4BG, 6AB, 2BE, 4AC
7	6	3	6PI, 4AF, 4BF
8	7	1	6PK
9	8	1	6PF
10	9	1	6PH
11	10	1	6PA
12	11	1	6PJ

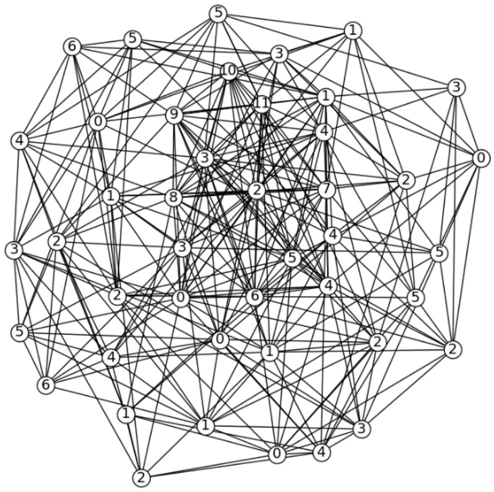

Figure 21. Colored Even Semester Graph

In Figure 21, which has been colored according to the color division in Table 2. The first color in coloring this graph is 0, and so on until the 11th color. And in this graph, the chromatic number is 12, so it takes 12 periods in a week so that there are no conflicting schedules. So that the vertex color grouping is obtained as follows:

Table 3. Color Grouping of Odd Semester Graph Vertices

No.	0	1	2	3	4	5	6	7	8	9	10	11
1	7PC	7PE	7PK	7PL	7PB	7PD	7PF	7PI	7PA	7PG	7PH	7PJ
2	3AD	1AA	3AC	1BA	7AA	3BF	3AG	-	-	-	-	-
3	3BE	3BD	3BA	3BB	3BC	1AF	3BG	-	-	-	-	-
4	1AD	1BB	5PD	1AC	1BF	3AE	-	-	-	-	-	-
5	5PC	3AB	1AB	3AA	3AF	5PB	-	-	-	-	-	-
6	1BC	7AD	7BD	7BA	5PE	1BD	-	-	-	-	-	-
7	5AA	7BE	5AB	5BD	7BB	7AE	-	-	-	-	-	-
8	7AB	5PA	1BE	5AC	1AE	-	-	-	-	-	-	-
9	7BC	5BA	7AC	-	-	-	-	-	-	-	-	-
10	5BB	5AD	5BC	-	-	-	-	-	-	-	-	-

Table 4. Color Grouping of Even Semester Graph Vertices

No.	0	1	2	3	4	5	6	7	8	9	10	11
1	6PB	6PL	6PC	6PD	6PE	6PG	6PI	6PK	6PF	6PH	6PA	6PJ
2	4AD	2BD	2AD	4AE	6AD	2AA	4AF	-	-	-	-	-
3	4BD	4BE	4AB	4BC	2BC	4BG	4BF	-	-	-	-	-
4	6BD	4AA	4BA	2AC	4AG	6AB	-	-	-	-	-	-
5	2AB	2AE	6AC	2BA	4BB	2BE	-	-	-	-	-	-
6	-	6BB	2BB	6AA	6BC	4AC	-	-	-	-	-	-
7	-	-	6BA	-	-	-	-	-	-	-	-	-

Table 4 above indicates that vertices representing courses with the same color can be scheduled at the same time, i.e. on the same day and at the same time using different rooms. For example, courses labeled with vertex labels 6PB, 4AD, 4BD, 6BD, and 2AB may be scheduled at the same time, and similarly to Table 3, courses labeled with vertex labels 7PC, 3AD, 3BE, 1AD, 5PC, 1BC, 5AA, 7AB, 7BC, 5BB may be scheduled at the same time with different classes. It should be noted that the vertex grouping shown is not the only grouping that can be generated by the Welch-Powell algorithm as some vertices have the same degree. With this vertex grouping by Welch-Powell algorithm and using chromatic number produces 12 colors, there are 12 periods of lecture schedule every week.

4. CONCLUSION

Scheduling of courses in the Bachelor of Mathematics study program FMIPA USU has been carried out using graph coloring with the Welch-Powell algorithm. From the data owned, namely vertices representing 61 courses in odd semesters and 44 courses in even semesters and edges representing schedule conflicts, the chromatic number result is 12 colors, which means that the lecture schedule is held with 12 periods per week. From the graph coloring results, there are no conflicting lecturer schedules on teaching time, classrooms to be used, and teaching hours for the courses they teach.

REFERENCES

[1] Y. Rusdiana dan A. Maulani, "Algoritma Welch-Powell Untuk Pewarnaan Graf pada Penjadwalan Perkuliahan," *Sci. Phys. Educ. J.*, vol. 3, no. 1, hal. 37–47, 2019, doi: 10.31539/spej.v3i1.915.
[2] P. W. Sari, L. Yulianti, dan . N., "Penjadwalan Kuliah Dengan Algoritma Welsh-Powell (Studi Kasus: Jurusan Matematika Fmipa Unand)," *J. Mat. UNAND*, vol. 6, no. 1, hal. 134, 2017, doi: 10.25077/jmu.6.1.134-141.2017.
[3] A. M. Nasir, Faisal, dan Dedy Setyawan, "Optimalisasi Penjadwalan Mata Kuliah Menggunakan Teori Pewarnaan Graf," *Prox. J. Penelit. Mat. dan Pendidik. Mat.*, vol. 5, no. 1, hal. 57–69, 2022, doi: 10.30605/proximal.v5i1.1398.
[4] Supiyandi dan M. Eka, "Penerapan Teknik Pewarnaan Graph Pada Penjadwalan Ujian Dengan Algoritma Welch-Powell," *Algoritm. J. Ilmu Mat. dan Komput.*, vol. 3, no. 1, hal. 58–63, 2018, [Daring]. Tersedia pada: http://jurnal.uinsu.ac.id/index.php/algoritma/article/view/3153
[5] S. E. Irwan, "Aplikasi Pewarnaan Graf pada Penempatan Kamar Mahasiswa (Studi Kasus: Asrama Institut Teknologi Sumatera)," *Sainmatika J. Ilm. Mat. dan Ilmu Pengetah. Alam*, vol. 17, no. 1, hal. 17, 2020, doi: 10.31851/sainmatika.v17i1.3137.
[6] E. Siti Nurmala dan Y. Permanasari, "Penerapan Pewarnaan Graf untuk Menentukan Tujuan Wisata Kuliner Kota Bandung," *J. Ris. Mat.*, hal. 149–156, 2023, doi: 10.29313/jrm.v3i2.2834.
[7] N. M. Surbakti, "Implementasi Pewarnaan Graf Menggunakan Metode Algoritma Welch Powell pada Penjadwalan Seminar Proposal Skripsi di Program Studi Matematika Universitas Negeri Medan," *Buana Mat. J. Ilm. Mat. dan Pendidik. Mat.*, vol. 13, no. 2, hal. 137–148, 2023, doi: 10.36456/buanamatematika.v13i2.7261.
[8] D. C. Fransisca dan S. D. Kurniawan, "Welch powell algoritma aplication to identify the conflict of lesson timetable (case study : informatics engineering , stikom yos sudarso Purwokerto)," vol. 1, no. 1, hal. 57–61, 2020.
[9] R. Munir, "Matematika Disktrit," *Inform. Bandung*, hal. 281–308, 2010.