Zero: Jurnal Sains, Matematika, dan Terapan
E-ISSN: 2580-5754
P-ISSN: 2580-569X
Vol. 8, No. 1, June 2024
DOI: 10.30829/zero.v8i1.19947
pp. 01-07

Algebras of Interaction and Cooperation

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Article Info

Article history:

Received 04 28, 2024
Revised 05 30, 2024
Accepted 06 30, 2024

Keywords:

Ctivity System, Cooperative Game, Decision System, Evolution, Fourier Transform, Galois Transform, Interaction System, Measurement, Quantum Game.

ABSTRACT

Systems of cooperation and interaction are usually studied in the context of real or complex vector spaces. Additional insight, however, is gained when such systems are represented in vector spaces with multiplicative structures, i.e., in algebras. Algebras, on the other hand, are conveniently viewed as polynomial algebras. In particular, basic interpretations of natural numbers yield natural polynomial algebras and offer a new unifying view on cooperation and interaction. For example, the concept of Galois transforms and zero-dividends of cooperative games is introduced as a nonlinear analogue of the classical Harsanyi dividends. Moreover, the polynomial model unifies various versions of Fourier transforms. Tensor products of polynomial spaces establish a unifying model with quantum theory and allow to study classical cooperative games as interaction activities in a quantum-theoretic context.

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1. INTRODUCTION

What is a game? While ”playing” is often understood as an activity involving one or more human players, we take here the general game-theoretic approach of [1] and think of an underlying system that is characterized by the states it can assume. The states typically depend on certain actions that players may or may not take in order to achieve other states. We do not necessarily require players to be human beings with human interests and feelings etc. and thus might refer to the players also simply as agents. The agents my act, interact or cooperate according to the rules within specified contexts.

A mathematical analysis of games, of course, requires mathematical models for the underlying systems and their states. States have to be described as mathematical objects. Similarly, observations on systems should be modeled accordingly as mathematical functions on the collection of states. The present paper concentrates on these aspects. In particular, we refer to quantum games if the underlying systems fit or extend standard mathematical models of physical quantum systems.

Questions about optimal strategies for the achievement of certain goals or about the existence of Nash equilibria etc. are disregarded. Given appropriate models, such questions lead to mathematical optimization problems that can be studied in their own right.

In all of mathematical application analysis, and in game theory in particular, linear models have been proven to be of utmost importance. These models could be formulated and studied as abstract structures. The key in our analysis is the representation of relevant system parameters by polynomials as this point of view ties together many otherwise seemingly different models. Our polynomials are not polynomial functions in variables xi at the outset, but formal polynomials in indeterminates xi. Appropriate interpretations of and substitutions into the indeterminates then reveal fundamental aspects of various game-theoretic models.

2. RESEARCH METHODS

The algebraic structures in our analysis are suggested naturally when one repre- sents system states as polynomials rather than vectors. The idea of expressing mathematical system characteristics via polynomials has a long tradition in gen- eral algebra (see, e.g., [2]). In fact, classical algebra arose from the wish to solve polynomial equations. The study of cooperative games in terms of an associated polynomial function in n variables was initiated in [3]. Polynomial functions, however, typically imply just one particular algebraic structure: the addition and multiplication rules of scalar-valued functions.

A substantially improved modeling flexibility is gained withh formal poly- nomials rather than polynomial functions. In this case, one deals with indeterminates instead of variables. Depending on the interpretation of the indeterminates, one is then led to various natural algebraic structures.

The present approach is based on the three fundamental aspects of natural numbers: cardinalities of finite sets, binary representation of finite sets and representation of information in terms of (0,1) sequences. Each of these aspects implies its own multiplication rule for formal polynomials.

The polynomial model is then applied to cooperative games and activity sys- tems. Section 6 links polynomials to linear transforms and, in particular, to Fourier transforms. Moreover, the new concept of Galois transforms is shown to appear naturally and yields nonlinear transforms of cooperative games. Interaction games are treated with the emphasis on the role of a valuation on a game-theoretic system as a particular state-dependent Heisenberg type measure on the system. A discussion with a perspective on future work concludes the presentation.

Relation to Earlier Work

Often, game theory is regarded as a scientific discipline in its own right. Moreover, cooperative games and strategic games with non-cooperative players are treated separately. However, as questions about the computation of strategies became more and more of interest, the many connections of game theory with other mathematical fields (e.g., mathematical optimization) became prominent. Computational questions have furthermore led to the emergence of game-theoretic re- search in computer science. Moreover, a seemingly new area of game theory has been initiated where the games are supposed to be played according to the physical laws of quantum mechanics.

As it turns out, a comprehensive approach to mathematical game theory is possible which ties together various areas of applied mathematics and includes the different aspects above as special cases5. In particular, the relationship between game-theoretic cooperation and quantum mechanics has been recognized. In this sense, the present work is a continuation of the mathematical game theory research program begun in [4, 5]. As in the classical foundations of mathematical optimization6, our mathematical model is essentially linear. However, it is observed that quadratic (and thus geometric, but nonlinear) measurement mod- els arise naturally from the projection of linear operators onto lower dimensional spaces.

While [6] explores some first aspects, the present analysis pursues more generally the realization that the employment of polynomial algebras instead of pure abstract vector spaces as modeling tools offers a distinctly more refined mathematical analysis and, furthermore, relates game theory to classical algebra. No previous mathematical game-theoretical model seems to have taken this route before.

3. RESULTS AND ANALYSIS

3.1 Mathematical preliminaries

Let $\mathbb{N} = \{0,1,2,\dots,n,\dots\}$denote the set of natural numbers[7]. $\mathbb{R}$ is the field of real numbers and C the field of complex numbers, *i.e.*, numbers of the form

$$z = a + ib \text{ where } a, b, \in \mathbb{R} \text{ and } i^2 = -1.$$

The complex number $z = a + ib$ admits a representation in the form of de Moivre:

$$z = re^{it} = r(cos\, t + i\, sin\, t) \text{ with real numbers } r, t \geq 0. \quad (1)$$

The number $\bar{z} = a - ib = re^{it}$is the *conjugate* of $z = a + ib$ and has the property

$$z\bar{z} = a^2 + b^2 = r^2 = |z|^2.$$

For arbitrary sets X and Y, XY is the set of all pairs (x, y) of elements$x \in X, y \in Y. Y^X$is the collection of all functions $f: X \to Y$, which may be thought of as *valuations* of the elements of **X** with values from Y.

The set $\mathbb{C}^X$ of all complex valuations of X is a vector space under the usualaddition and scalar multiplication of complex-valued functions. The *support* of$f \in \mathbb{C}^X$is the set

$$supp\,(f) = \{x \in X | f(x) \neq 0\}.$$

If $X = \{x_1, \dots, x_n\}$is a finite set, $\mathbb{C}^X$ may conveniently be identified with the n-dimensional coordinate space $\mathbb{C}^n$and, similarly, $\mathbb{C}^{X \times X}$with the space of all complex n × n matrices.

a. Polynomials

A *(formal) polynomial* relative to the set X is an element $f \in \mathbb{C}^X$ with finite support. In order to emphasize the polynomial aspect, we write elements $f \in \mathbb{C}^X$as formal sums

$$f = \sum_{x \in X} f_x x \text{ with the coefficients } f_x = f(x)$$

In the case $supp\,(f) \subseteq \{x_0, x_1, \dots, x_n\}$, we write the polynomial f also in the form

$$f \sum_{k=0}^{n} f_k x_k \text{or, uisng superscripts } x^k = x_k, f \sum_{k=0}^{n} f_k x^k.$$

We denote by $\mathbb{C}(X)$ the complex vector space of all polynomials in $\mathbb{C}^X$. The elements $x \in X$are formally just *indeterminates* without a numerical meaningin their own right. However, they can be given particular functional meaning in applications of the polynomial model.

Remark For notational convenience, we will often identify the indeterminate $x \in X$ with the polynomial

b. Polynomial functions

If $X = \{x_0(t), x_1(t), x_2(t), \dots\}$is a family of complex-valued functions $x_k(t)$ in the variable t, one may think of a polynomial p C(X) as a complex-valuedfunction in the variable t,

$$p(t) = \sum_{k \geq 0} c_k x_k(t),$$

which arises as the corresponding linear combination of the functions in X. In thespecial case of the functions $x_k(t) = t^k$, p(t) is a standard polynomial function:

$$p(t) = c_0 + c_1 x + c_2 t^2 + \cdots + c_n t^n.$$

Remark 2 The representation of polynomials as functions allows the applicationof the methods of differentiation and intergration in their analysis. For modelingpurposes, however, it is important to retain the flexibility of formal polynomials.

c. Power series and generating functions

In the case $X = \{x_0, x_1, x_2, \dots,\}$ of indeterminates that are indexed by the nat- ural numbers, we think of $f \in \mathbb{C}^X$as a *(formal power series)* with the notationalrepresentation

$$f = \sum_{k \geq 0} f_x x^k.$$

Assuming $X = \{1, t, t^2, \dots\}$as a set of polynomial functions $x(t) = t^k$, on the other hand, the vector $f \in \mathbb{C}^X$may define a complex function

$$f(t) = \sum_{k \geq 0} f_x x^k.$$

if the sum has a well-defined region of convergence in C. In this case, the function $f(t)$ is interpreted as *generating* the numerical parametes $f_k \in \mathbb{C}$ *via* evaluations of derivatives:

$$f_k = \frac{f^{(k)}(0)}{k!} (k = 0,1,2, \dots)$$

Generating functions have proven useful in probability theory or in the asymp- totic analysis of combinatorial parameter sequences[8]. If X is finite, a generating function is just a polynomial function.

3.2 Algebra of natural numbers

We denote the usual standard sum of natural numbers $i, j \in \mathbb{N}$ by $i + j$ and their product by $i.j$ or simply ij. However, other important algebraic structures on ℕ also offer themselves, depending on the representation of natural numbers.

a. Binary representation and algebra

Every natural number $k < 2^n$ has a unique representation with n binary digits k_i:

$$k = \sum_{i=0}^{n-1} k_i 2^k \quad (k_i \in \{0,1\})$$

So, every (0, 1)-string $\alpha \in \{0,1\}^{\mathbb{N}}$ with finite support corresponds to a uniquenatural number

$$a \sum_{i=0}^{\infty} \alpha_i 2^i$$

Endowing the set {0, 1} with the binary addition rule $1 \oplus 1 = 0$,
one obtains a binary addition rule for all natural numbers:

$$(\sum_{i=0}^{\infty} \alpha_i 2^i) \oplus (\sum_{i=0}^{\infty} \beta_i 2^i) = \sum_{i=0}^{\infty} (\alpha_i \oplus \beta_i) 2^i \; (\alpha_i \beta_i \in \{0,1\})$$

which is commutative and associative. Moreover, 0 is the neutral element (as under the standard addition rule).
Consider now a set $X = \{x^0, x^1, \dots, x^n, \dots\}$ of indeterminates x^n with in- dices $n \in \mathbb{N}$. The binary addition suggests a commutative and associative multi- plication on X:

$$x^i \odot x^j = x^{i \odot j}$$

with the neutral element $\mathit{1} = x^0$. The monoid (X, ⊙) implies the polynomial algebra $(\mathbb{C}(X), \odot)$ where the (binary) product of the polynomials $f, g \in \mathbb{C}(X)$ is the polynomial $f \odot g$ with the coefficients

$$(f \odot g)_n = \sum_{i \odot j = n} f_i g_i$$

IMPARTIAL GAMES. We illustrate binary algebra with the example of the well-known 2-person game *Nim*, which involves two players and a finite set Nthat is partitioned into k blocks N_i with $n_i = |N_i|$ bjects each. A *move* of a player consists in the choice of a non-empty block N_i and the subsequent removalof at least one object from N_i. The players move alternatingly. A player loses if he cannot move on his turn. We associate with the Nim game its *characteristic*
polynomial as the binary monomial

$$p(N) = x^{n_1} \odot x^{n_2} \odot \dots \odot n^{n_k} = x^{n_1 \odot n_2 \odot \dots + \odot n_k}$$

By the Sprague-Grundy theory of combinatorial 2-person games[9] one then finds:

- The *second* player has a winning strategy $if\ p(N) = x^0$).
- If $p(N) \neq 1$, the first player has a winning strategy.

Remark An actual winning strategy for the Nim game N is easily computed: The current player moves, if possible, the game into a Nim situation N' with characteristic polynomial $\mathrm{p}(\mathrm{N}') = 1$. Nim games are prototypical impartial 2-person games as each impartial game isstrategically equivalent to a Nim game.

b. Set representations and Boolean algebra

Each (0, 1)-string $\alpha \in \{0,1\}^{\mathbb{N}}$ with components α_i describes a unique subset A of ℕ *via*

$$A = supp(\alpha) = \{i \in \mathbb{N} | \alpha_i = 1\} \; (9)$$

In fact, (9) establishes a one-to-one correspondence between $\{o, 1\}^{\mathbb{N}}$ and the sub-sets of ℕ. Moreover, strings with finite support correspond to finite subsets and, simultaneously, to natural numbers *via* their binary representation (7).
Consider now an arbitrary n-element set $E = \{e_1, \dots, e_n\}$Each subset $A \subseteq E$ corresponds to a unique (0, 1)-string α of length n,

$$\alpha = \alpha_1 \dots \alpha_n \text{ with } \alpha_i = 1 \leftrightarrow e_i \in A,$$

and also, to a natural number

$$a = \sum_{i=1}^{n} \alpha_i 2^{i-1} < 2^n$$

Consequently, the family ε of all subsets $A \subseteq E$ corresponds to the set

$$\mathbb{N}_n = k \in \mathbb{N} | k < 2^n$$

of natural numbers or to the family $\mathbb{B}_n = \{0,1\}^n$of $(0,1)$-strings of length *n*.
Hence, if $m = 2^n$, each polynomial of the form

$$p = p_0 x_0 + p_1 x_2 + \dots + p_{m-1} x_{m-1}$$

may equally well be indexed by the subsets of the n-element set ε or the binary
n-strings:

$$p = \sum_{k=0}^{m-1} p_k x_k \leftrightarrow \sum_{A \in \varepsilon} p_A x_A \leftrightarrow \sum_{\alpha \in \mathbb{B}_n} p_\alpha x_\alpha$$

The set-theoretic interpretation suggests a polynomial algebra based on Booleanlattice operations:
$x_A \vee x_B = x_{A \cup B}$ and $x_A \wedge x_B = x_{A \cap B}$
(X, V) a monoid with neutral element x_ϕ. (X,∧) is a monoid with neutral element x_E if attention is restricted to polynomials indexed by the subsets of E.

3.3 Interaction Games

Consider a set $N = \{1, \dots, n\}$ of general entities i and specify an interaction game on N as a valuation

$$V: N \times N \to \mathbb{R}$$

where $V_{ij} = V(i,j)$ is the interaction worth of $i, j \in N$. Where X is a set of N indeterminates x_i, the associated characteristic polynomial is

$$X^V = \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} (x_i \otimes x_j),$$

which means that interaction games refer to the tensor product space

$$\mathbb{C}(X \otimes X)$$

(And not to the vector space $\mathbb{C}(X)$) or, more precisely, to the real tensor space $\mathbb{R}(X \otimes X)$. Abstractly, V is given as a real $n \times n$ matrix with the coefficients V_{ij}.

Let $A \in \mathbb{R}^{N\times N}$be an arbitrary real matrix whose coefficients V_{ij} have the interpretation of enhancing the interaction worth relative to V:

If i and j interact at level A_{ij}, their interaction produces the value$V_{ij}A_{ij}$

Hence the interaction instance A produces the game's overall value as

$$X^V(A) = \textstyle\sum_{i,j\in N} V_{ij}A_{ij} = tr(A^T V) \qquad (15)$$

where the *trace* $tr(C)$ of a matrix C is the sum of its diagonal coefficients. In other words, $X^V(A)$ equals the usual euclidean inner product $(V|A)$of the two matrices V, A, considered as n^2 dimensional parameter vectors. This fact yields the dual interpretations:

1. An interaction game V is a linear functional on the space of all interactioninstances A.
2. An interaction instance A is a linear functional on the space of all interac-tion games V.

a. The Hermitian Perspective

Setting $A^+ = (A + A^+)/2$, one finds that a matrix $A \in \mathbb{R}^{n\times n}$ decomposes into a symmetric matrix A^+ and a skew-symmetric matrix A^-:

$$A = A^+ + A^- \text{ where } (A^+)^T = A^+, (A^-)^T = -A^- \quad (16)$$

Moreover, is it straightforward to check that the symmetry decomposition (16) of A is unique. Associate now with $A \in R^{n\times n}$ the well-defined hermitian matrix

$$\hat{A} = A^+ + iA^- \in \mathbb{C}^{n\times n}$$

and let $\mathbb{H}_n \subseteq \mathbb{C}^{n\times n}$ be the family of all hermitian n × n matrices.

Clearly, $\mathbb{H}_n$ is isomorphic to $\mathbb{R}^{n\times n}$with respect to the field $\mathbb{R}$ of real scalars. ($\mathbb{H}_n$ is not a complex vectors space, however.) Recall that the adjoint C^* of the complex matrix $C \subseteq \mathbb{C}^{n\times n}$ is the tranose C^{-T} of the conjugated matrix C and note

Lemma For any matrices $C \subseteq \mathbb{C}^{n\times n}$ and $A, B \in \mathbb{R}^{n\times n}$, one has

1. $C \in \mathbb{H}_n$ if and only if $C^* = C$ (I,e., C is self-adjoint.
2. $(A|B) = tr\,(B^T A) = tr(B\hat{A}) = (\hat{A}|B)$

Hence:

Interaction games can equally well be studied within the context of the her-mitian matrix space $\mathbb{H}_n$.

The adjoint $u^*$of the (column) vector $u \in \mathbb{C}^n$ is the row vector of conjugatedcomponents of n. So $uu^*$is a self-adjoint $n \times n$ matrix. Indeed, an important general characterization of self-adjoint matrices is:

Lemma The matrix $C \in \mathbb{C}^{n\times n}$ is self-adjoint if and only if there are real scalars λ_k and vectors $u_k \in \mathbb{C}^n$ such that

$$C = \sum_k \lambda_k u_k u_k^*.$$

Proof. Because the matrices $u_k u^*k$ are self-adjoint, a matrix C of the form (17) is self-adjoint. To see the converse, recall that $\mathbb{R}^{n\times n}$and $\mathbb{H}_n$ are isomorphic. So, it suffices to consider (0, 1)-matrices $A \in \mathbb{R}^{n\times n}$ with exactly one non-zero entry $A_{ij} = 1$. Moreover, we may assume $n = 2$.

If A is diagonal, one has $\hat{\mathbf{A}} = \mathbf{A}$, for which the claim is obviously true. So, assume for example,

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ and hence } C = \hat{A} = \frac{1}{2}\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$$

With the two real eigenvalues $\lambda_1 = +\sqrt{2}$ and $\lambda_2 = -\sqrt{2}$. Consequently, the 2-dimensional space $\mathbb{C}^2$admits a basis of eigenvectors of C, wjich imply the claim.

Remark The spectral decomposition of self-adjoint matrices shows that thevectors u_k in can be chosen as eigenvectors of C with real eigenvalues λ_k also in the case $n > 2$.

Consider, for example, an activity system A_N as in Section 5.2 relative to the valuation $v: N \rightarrow \mathbb{R}$. Let $u \in \mathbb{C}^N$ be a state vector of norm $\|u\| = 1$ with the associated self-adjoint $m \times m$ matrix $U = uu^*$ of complex coefficients $V = \mathrm{diag}(v)$ is the diagonal interaction matrix with diagonal coefficients $V_{SS} = v(S)$ one now observes:

$$(V|U) = \sum_{\subseteq N} v(S) u_S u_S = \sum_{\subseteq N} v(S) |u_S|^2 = E^v(u)$$

So A_n is recognized as the restriction of an interaction game on with char-acteristic function $V = \mathrm{diag}(v)$to interaction instances of the form uu^*

b. The Hermitian Perspective

A measurement operator on the interaction system N is a functional

$$\mu: \mathbb{R}^{N \times N} \longrightarrow \mathbb{R}$$

which produces the measurement result μ(A) if the system is in a state that cor- responds to the activity instance A. If the functional μ is linear, the measurementacutally represents an interaction game. So there exists a matrix $M \in \mathbb{R}^n \times n$ such that

$$\mu(A) = (M|A) = (M|\hat{A}),$$

which means that μ can be understood as a linear functional on the (real) vectorspace $\mathbb{H}_n$ of all hermitian matrices. Assuming

$$\hat{A} = \sum_k \lambda_k u_k u_k^*$$

for suitable real parameters λ_k and (complex) vectors $u_k \in \mathbb{C}^n$, one has

$$\hat{A} = \sum_k \lambda_k \mu(u_k u_k^*) = \sum_k \lambda_k (M|u_k u_k^*)$$

A Heisenberg measurement operator[22] on the (complex) vector space $\mathbb{C}^n$ is areal-valued functional $\gamma: \mathbb{C}^n \rightarrow \mathbb{R}$ of the form

$$\gamma(u) = u^* G u = (G|uu^*) \quad \text{with } G \in \mathbb{C}^{n \times n}$$

Since γ is real-valued, one may assume that G is self-adjoint. Notice that aHeisenberg measurement operator is not linear. On the other hand, (18) shows:

to the fact that this measurement model is standard in quantum theory.

- A Heisenberg measurement operator on $\mathbb{C}^n$ arises from the restriction ofa linear measurement operator on the interaction system N to interaction instances of the hermitian form uu^*.

Similarly, the linear measurement matrix $\hat{M}$ of the operator μ of the form$M = \sum_j \delta_j w_j w_j^*$ for suitable real numbers δ_j and vectors $w_j \in \mathbb{C}^n$, which implies

$$\mu(A) = \sum_i \sum_j \lambda_i \mu_j (w_j w_j^* | u_i u_j^*)$$

Consequently, the fundamental linear interaction measures are recognized tobe of the form

$$\mu_w(u) = (ww^*|uu^*) = |w^*u|^2 \ with\ w, u \in \mathbb{C}^n.$$

4. CONCLUSION

Linearity plays an important role in mathematical application models. The mathe-matical analysis, however, will reveal more characteristic features of the under- lying systems when the model is not just considered to be a vector space with scalar multiplication, but an algebra, *i.e*, additionally equipped with a multiplica-tion operation for vectors. Suitable multiplication operations are naturally associ-ated with multiplication rules for polynomials, which renders polynomial modelspowerful and flexible.

The present approach shows that mathematical models for cooperation and in-teraction connect with important aspects of classical algebra and combinatorics. For example, the representation of coalitions by natural numbers embeds the rep-resentation of cooperative games into the context of Galois theory, *i.e.*, the theoryof solving algebraic equations. Future work of exploring this area of mathematicalsystem analysis in more detail appears to be promising.

The polynomial model also underlines the aspect of quantum-theoretic modelsas interaction systems and, conversely, embeds cooperation and interaction into the setting of physical quantum systems. The evolution of such systems can be mathematically understood in a far broader context (see, *e.g.* [7]). Moreover, the apparatus of theoretical physics can be brought to bear on general systems ofcooperation and interaction. In particular, Hamiltonians of cooperative games canbe expected to provide considerable insight into fundamental laws according to which such systems behave.

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