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How the Pigeonhole Principle Can be Applied to Verify the Number of Classrooms Needed

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ABSTRACT

Classrooms are one of the main needs for educational institutions to carry out learning activities and are an important element in creating an optimal learning environment for educators and students. Determining the number of classrooms should be done through the thorough calculation. The aim of this research is to perform how pigeonhole principle can be applied to verify the numbers of classroom needed. In working with pigeonhole principle, it should be clear what will be "pigeon", and what will be "pigeonhole". According to the results of this research, as we took the case study for Faculty of Science and Technology Universitas Jambi which currently build a new building, if FST wants to allocate the rooms for each department, it will need 52 classrooms based on scheme 1 and 58 classrooms based on scheme 2. While if FST does not consider the allocation for each department, then it will need 45 rooms based on scheme 1 and 46 classrooms based on sheme 2.

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1. INTRODUCTION

The existence of educational facilities is absolutely necessary in the educational process and learning activity, so it is included in the components which must be fulfilled in the educational process and learning activity. Without educational facilities, the educational process and learning activity will experience difficulties and the goals of education will not be achieved [1]. One of the facilities should be provided by educational institutions is classroom. Classroom play a crucial role in educational institutions and significantly contribute to the overall learning activity and experience, especially for the educational institutions which still promote and prioritize the interaction between educators and students. The interactions, resources, and environment within classrooms collectively contribute to a holistic educational experience for students in educational institutions, likewise in a university.

Determining the number of classrooms needed for an educational institution, just like in a university, involves a comprehensive analysis of various factors. Some consideration to determine the appropriate number of classrooms needed are the numbers of students, the numbers of course available, the time period of each courses and total hours of academic activity per day. Likewise in Faculty of Science and Technology (FST) Universitas Jambi which houses 14 departments. Currently, academic activities for these 14 departments are organized and held in two separated locations which are quite far apart. Therefore, FST continues to develop adequate academic activity facilities. One important aspect in this development stage is ensuring that existing physical resources, such as classrooms, need to be available to accommodate all departments under the auspices of FST and it will be in one location instead of in two different locations. Therefore, FST currently is building a new building for learning activities which accommodate the learning activities by all departments and for all theoretical courses. In order for the construction of lecture buildings and classrooms to suit the needs and situation of each department, courses and the weight of credits for each course, it is necessary to carry out a scientific analysis of the number of classrooms required by FST and FST needs recommendation about this.

One scientific approach that can be used to verify and confirm the appropriate number of classrooms is to apply the pigeonhole principle or Dirichlet's principle. The pigeonhole principle is one of the basic techniques in the topic of combinatorics in mathematics. This principle looks simple, but it is an important "tool", very powerful and very useful in the development of mathematical theory studies and proofs as well as in counting techniques [2], [3]. The pigeonhole principle is also known as the Dirichlet drawer principle, as German mathematician G. Lejeune Dirichlet implement this principle in his work frequently [4], [5], as well as in his famous book on the theory of numbers related to rational approximations of irrational numbers [6]. There are many problems which can be solved and proven effectively using the pigeonhole principle as described in [7] and [8]. Among them Rebman showed how pigeonhole principle works and applies for coloring the maps [7]. According to Rebman, as many as 6 colors is enough for coloring any map. The pigeonhole principle can also be applied in setting athlete training and game combinations in a tournament [9]. The pigeonhole principle is even used to estimate accurately the resonance frequency in surface acoustic wave resonator (SAWR) sensors [10] and is a basic principle in hyperspectral imaging [11]. We have not found a scientific article which discussed about how the pigeonhole principle works and applies to verify the number of classrooms required by educational institutions. However, we believe that the pigeonhole principle can be applied to verify this. Therefore, the aim of this research is to carry out a scientific analysis about how pigeonhole principle works and applies to verify the number of classrooms required. The case study to describe this in our research is verifying the number of classroom needed by FST Universitas Jambi as this faculty needs recommendation on this.

2. RESEARCH METHOD

The technique of problem solving used in this research is a counting technique in combinatorial, namely the pigeonhole principle or Dirichlet drawer principle. The pigeonhole principle is one of the basic principles in combinatorials and is an important combinatorial tool. The basic statement of the pigeonhole principle is: if there are at least $n + 1$ objects (or pigeon) to be placed into n boxes (or pigeonhole), then one of the boxes must contain at least two objects [12], [4]. More generally, the pigeonhole principle states that if there are N objects to be placed into r boxes, with $N > r$, then there are at least 2 boxes placed into the same box [7]. In other words, if objects are placed in boxes, and there are more number of objects than number of boxes, then there will be a box containing at least two objects.

In this research, we will refer to the generalized pigeonhole principle or Dirichlet drawer principle to be applied to verify the number of classrooms needed.

The generalized pigeonhole principle [4], [13]: If N objects are to be placed in boxes with the number of boxes being r, then there is at least one box containing at least $\lceil N/r \rceil$ objects.

In this case $\lceil N/r \rceil$, read "ceiling of N/r", is the smallest integer that is greater than or equal to N/r [14]. The generalized pigeonhole principle theorem can be proven by using the contradiction method, as follows. Proof of the generalized pigeonhole principle: it is known that there are N objects that will be placed into boxes with the number of boxes being r. Assume that no box contains $\lceil N/r \rceil$ or more objects. Or in other words, no box contains more than $\lceil N/r \rceil - 1$ objects. Because of this, the total number of objects in r boxes is at most M objects with $M = r(\lceil N/r \rceil - 1)$. We will show that in fact $M \neq N$ so that our assumption is wrong. Since $\lceil N/r \rceil < N/r + 1$ [14], we obtain

$$M = r\left(\left\lceil \frac{N}{r} \right\rceil - 1\right) < r\left(\left(\frac{N}{r} + 1\right) - 1\right) = r\left(\frac{N}{r}\right) = N,$$

so that $M < N$. This result contradicts the number of objects we have, which should be N (not less than N objects). By that way, we can not say that there are at most M objects in r boxes. Thus there must be at least 1 box containing $\lceil N/r \rceil$ objects or more.

3. RESULT AND ANALYSIS

The data used in this research is presented in Table 1. This data was taken based on academic activities in the odd semester 2023/2024, especially lecture activities that require classrooms for theoretical (not practical) courses in each department at FST Universitas Jambi. The number of courses presented in Table 1 are combination of the number of different courses and the number of classes that enrol and attend these courses. For example, if course A is held for 2 different classes, then we assume to count it as 2 different courses. The reason of this assumption is that we need to split the calssroom for 2 different classes eventhough both classes are taking the same course at the same schedule. Lecture activities held at FST are expected to be held on 5 working days every week, i.e. Monday to Friday, from 7.30AM to 05.00PM (equal to 570 minutes per day including break time). Each course of 3 credits and course of 2 credits is held once a weak, while course of 4 credits is organized twice or in 2 different schedules in a week.

Table 1. Department and the number of courses in each department at FST

No.	Department	number of courses	number of courses of 2 credits	number of courses of 3 credits	number of courses of 4 credits
1	Mathematics	56	17	39	-
2	Chemistry	52	42	10	-
3	Physics	45	19	26	-
4	Biology	69	37	24	8
5	Chemical Analyst	27	23	5	-
6	Chemical Industry	31	26	5	-
7	Information Systems	149	69	80	-
8	Electrical Engineering	52	27	24	-
9	Mining Engineering	82	40	42	-
10	Geological Engineering	63	32	31	-
11	Geophysical Engineering	68	37	27	4
12	Chemical Engineering	68	50	18	-
13	Civil Engineering	59	47	12	-
14	Environmental Engineering	64	35	29	-

Note: 1 credit is equal to 50 minutes.

When thinking about how to implement the pigeonhole principle, we will need to determine which ones will act as objects or pigeon, and which ones will act as boxes or pigeonholes and this should be thought carefully and thorough [15]. In working with pigeonhole principle to verify the number of classrooms needed, we claim the different courses held within a week acts as “pigeon” or “object” and let denote its number by N, while the schedule options within a week acts as “pigeonholes” or “boxes” and let denote its number by r. Then pigeonhole principle works to verify that if N different courses will be organized in r schedule options, then there is at least one schedule option that carry out at least $\lceil N/r \rceil$ different courses. To organize $\lceil N/r \rceil$ different courses at the same schedule, of course as many as $\lceil N/r \rceil$ classrooms must be provided. That’s how the pigeonhole principle is applied to verify the number of classrooms needed.

In this research, we present two options proposed. They are as follows.

1) Option 1: verify the number of classrooms needed by each department at FST

   To verify the number of classrooms needed by each department, we propose two schemes as below:

   a) Scheme 1: assume 3 courses of 3 credits and 1 course of 2 credits are organized everyday during working hours so that there are 4 schedule options for a room everyday, or equivalent to 20 schedule options for a room in a week. Therefore based on pigeonhole principle, the number of classrooms needed by each department is :

   $$\left\lceil \frac{N_i}{20} \right\rceil$$

   where N_i is the number of courses in department i.

   b) Scheme 2: verification of the number of classrooms is doing based on the credit of the course.

      i) Within 1 day, there are 5 courses of 2 credits that can be organized during working hours everyday, so that there are 25 schedule options within a week for organizing the courses of 2 credits. Based on the pigeonhole principle, the number of classrooms required for each department is at least

$$\left\lceil \frac{M_j}{25} \right\rceil$$

where M_i is the number of courses of 2 credits in department i.

ii) Within 1 day, there are 3 courses of 3 credits that can be organized during working hours everyday, so that there are 15 schedule options within a week for organizing the courses of 3 credits. Based on the pigeonhole principle, the number of classrooms required for each department is at least

$$\left\lceil \frac{P_i}{15} \right\rceil$$

where P_i is the number of courses of 3 credits in department i.

Based on these 2 schemes in option 1, the the number of classrooms needed for each department at FST are shown in Table 2. As for the course of 4 credits, as it is organized in 2 different schedules in a week, then it is considered that 1 course of 4 credits is equivalent to 2 courses of 2 credits. Therefore for department of Biology, it is assumed to have 52 courses of 2 credits (after combining courses of 2 credits and courses of 4 credits). While for department of Geophysical Engineering, it is assumed to have 45 courses of 2 credits (after combining courses of 2 credits and courses of 4 credits)

Table 2. The number of classrooms needed by each department based on option 1

No.	Department	Number of classrooms based on scheme 1 $\lceil N_i/20 \rceil$	Scheme 2		
			course of 2 credits $\lceil M_i/25 \rceil$	course of 3 credits $\lceil P_i/15 \rceil$	Number of classrooms based on scheme 2
1.	Mathematics	$\lceil \frac{56}{20} \rceil$ = 3 rooms	$\lceil \frac{17}{25} \rceil$ = 1 room	$\lceil \frac{39}{15} \rceil$ = 3 rooms	4
2.	Chemistry	$\lceil \frac{52}{20} \rceil$ = 3 rooms	$\lceil \frac{42}{25} \rceil$ = 2 rooms	$\lceil \frac{10}{15} \rceil$ = 1 room	3
3.	Physics	$\lceil \frac{56}{20} \rceil$ = 3 rooms	$\lceil \frac{19}{25} \rceil$ = 1 room	$\lceil \frac{26}{15} \rceil$ = 2 rooms	3
4.	Biology	$\lceil \frac{77}{20} \rceil$ = 4 rooms	$\lceil \frac{52}{25} \rceil$ = 3 rooms	$\lceil \frac{24}{15} \rceil$ = 2 rooms	5
5.	Chemical Analyst	$\lceil \frac{27}{20} \rceil$ = 2 rooms	$\lceil \frac{23}{25} \rceil$ = 1 room	$\lceil \frac{5}{15} \rceil$ = 1 room	2
6.	Chemical Industry	$\lceil \frac{31}{20} \rceil$ = 2 rooms	$\lceil \frac{26}{25} \rceil$ = 2 rooms	$\lceil \frac{5}{15} \rceil$ = 1 room	3
7.	Information Systems	$\lceil \frac{149}{20} \rceil$ = 8 rooms	$\lceil \frac{69}{25} \rceil$ = 3 rooms	$\lceil \frac{80}{15} \rceil$ = 6 rooms	9
8.	Electrical Engineering	$\lceil \frac{51}{20} \rceil$ = 3 rooms	$\lceil \frac{27}{25} \rceil$ = 2 rooms	$\lceil \frac{24}{15} \rceil$ = 2 rooms	4
9.	Mining Engineering	$\lceil \frac{82}{20} \rceil$ = 5 rooms	$\lceil \frac{40}{25} \rceil$ = 2 rooms	$\lceil \frac{42}{15} \rceil$ = 3 rooms	5
10.	Geological Engineering	$\lceil \frac{63}{20} \rceil$ = 4 rooms	$\lceil \frac{32}{25} \rceil$ = 2 rooms	$\lceil \frac{31}{15} \rceil$ = 3 rooms	5
11.	Geophysical Engineering	$\lceil \frac{72}{20} \rceil$ = 4 rooms	$\lceil \frac{45}{25} \rceil$ = 2 rooms	$\lceil \frac{27}{15} \rceil$ = 2 rooms	4
12.	Chemical Engineering	$\lceil \frac{68}{20} \rceil$ = 4 rooms	$\lceil \frac{50}{25} \rceil$ = 2 rooms	$\lceil \frac{18}{15} \rceil$ = 2 rooms	4
13.	Civil Engineering	$\lceil \frac{59}{20} \rceil$ = 3 rooms	$\lceil \frac{47}{25} \rceil$ = 2 rooms	$\lceil \frac{12}{15} \rceil$ = 1 room	3
14.	Environmental Engineering	$\lceil \frac{64}{20} \rceil$ = 4 rooms	$\lceil \frac{35}{25} \rceil$ = 2 rooms	$\lceil \frac{29}{15} \rceil$ = 2 rooms	4
	TOTAL	52 rooms			58 rooms

Based on scheme 1, the total classrooms required by FST as a whole are at least 52 classrooms, while based on scheme 2, at least 58 classrooms are needed. The results in Table 2 are presented in the form of a bar chart in Figure 1.

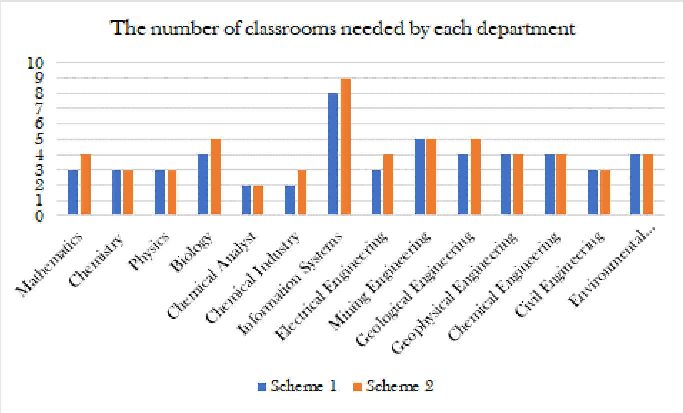

Figure 1. The number of classrooms needed by each department

2) Option 2: verify the number of classrooms needed by FST without considering the allocation for each department.
Verification the number of classrooms needed without considering the allocation for each department also considers the 2 schemes just like in option 1, and the calculation is presented in Table 3. The number of courses in all departments under the auspices of FST whose schedule needs to be plotted or provided is 897 courses.

Table 3. The number of classrooms needed by FST based on option 2

Number of classrooms based on scheme 1 $\lceil N/20 \rceil$	Scheme 2		
	course of 2 credits $\lceil M/25 \rceil$	course of 3 credits $\lceil P/15 \rceil$	Number of classrooms based on scheme 2
$\left\lceil \frac{897}{20} \right\rceil$ = 45 rooms	$\left\lceil \frac{525}{25} \right\rceil$ = 21 rooms	$\left\lceil \frac{372}{15} \right\rceil$ = 25 rooms	46 rooms

4. CONCLUSION

When thinking about how to implement the pigeonhole principle, we will need to determine which ones will act as objects or pigeon, and which ones will act as boxes or pigeonholes. In working with pigeonhole principle to verify the number of classrooms needed, we claim the different courses held within a week acts as "pigeon" or "object" and lets denote its number as N, while the schedule options within a week acts as "pigeonholes" or "boxes" and lets denote its number as r. Then pigeonhole principle works to verify that there is at least one schedule option that perform at least $\lceil N/r \rceil$ different courses. To organize $\lceil N/r \rceil$ different courses at the same schedule, of course as many as $\lceil N/r \rceil$ classrooms must be provided. That's how the pigeonhole principle is applied to verify the number of classrooms needed.

Based on the result of this research, as for FST Universitas Jambi, we provide 2 options with 2 schemes for each options regarding the number of classroom need to be provided in new building. By implementing the pigeonhole principle, we come with the result that if FST wants to allocate the classroom for each department, then it will need 52 classrooms based on scheme 1 and 58 classrooms based on scheme 2. While if FST does not consider the allocation for each department, then it will need 45 rooms based on scheme 1 and 46 classrooms based on sheme 2. These results can be a reference for the FST or Universitas Jambi to make decisions about the number of classrooms built in new buildings for the sake of running academic activities smoothly.

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