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Determination Of Dominant Side Contraction Numbers In Banana Tree And Firecracker Grapes

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ABSTRACT

This research aims to determine the pattern of dominance side contraction numbers in banana tree graphs and fireworks graphs. Banana tree graph $B_{n,k}$ is a graph obtained by connecting one leaf from each n-copies of a k star graph with one vertex that is different from all the star graphs. The fireworks graph is a tree that is similar to the caterpillar graph, the difference lies in the backbone node n which is connected to the earring node k from the caterpillar graph. The dominating number is denoted by $\gamma(G)$ is the smallest cardinality of a dominating set. Domination set the minimum is a dominating set from which no points can be removed without changing its dominance. Domination contraction number of a graph $ct\gamma(G)$ is defined as the minimum number of edges that must be contracted to reduce the dominance number.

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1. INTRODUCTION

One branch of mathematics that is widely used to solve problems in other sciences is graph theory. Graph theory was born in 1736 through the writings of L. Euler, a Swiss mathematician, who contains solutions to the problems of the Konigsberg bridge which is very famous in Europe at that time. Euler drew a path problem through a bridge and an island in the center of the city of Konisberg. This problem is described by lines that are used to connect points and eventually develops and is known as a graph to this day.

One of the topics studied in graph theory is domination sets and domination numbers. The dominance set (S) in graph G is a subset of $V(G)$ such that every vertex of G that is not an element of S is connected and is one distance from S. The minimum cardinality between the dominance sets in a graph G is called the dominance number of the graph G and is denoted $\gamma(G)$. Therefore, dominance numbers are closely related to dominance sets[1].

One of the interesting topics to study is side dominance numbers. Edge dominance was first introduced in 1977 by Mitchell and Hedetniemi as an extension of the topic of point dominance in graphs. Let $G = (V, E)$ be an undirected connected graph and F is a subset of the set EF , said to be an edge-dominated set if every edge in E is 3 in F or adjacent to an edge in F , and F is said to be a minimum edge dominance set if there is no subset F' of F which is an edge dominance set, the edge dominance number $\gamma'(G)$ is the minimum cardinality of all minimum edge dominance sets[2].

Mathematical domination was introduced in the early 1960s. Since then Both dominance sets and dominance numbers have been widely used in various applications, including determining the number of camera placements supervisor in the corners of the hallway in a building and determining the location as well there are many traffic police posts at the corners of the city so that every road can be accessed monitored well. Another application example of dominance numbers is to determine the number of fire trucks needed in a housing circle to deal with if a fire occurs. On route bus stop to pick up school students. Bus stop locations are considered as a domination set where the bus stopping point is determined so that each student does not walk far to the bus [3].

Based on this background and considering previous research, in this research the author is interested in studying further the dominance side contraction numbers in a special graph entitled "Determining the Domination Side Contraction Numbers in Banana Tree Graphs and Firecracker Graphs)".

2. RESEARCH METHODE

Tree Graph (Banana Tree)

The banana tree graph $B_{n,k}$ is a graph obtained by connect one leaf from each n-copies in a k star graph with one vertex that is different from all star graphs [4]. different vertices This is called the root vertex. Because the star graph that makes up the graph B is n,k has the same order, the graph $B_{n,k}$ is called a homogeneous (ordered) banana tree graph [5].

Fireworks Graphic (Firecracker)

The fireworks graph is a tree that is similar to the caterpillar graph, the difference lies in the backbone node n which is connected to the earring node k of the caterpillar graph. Before the backbone node n which is connected to the earring node k there is one node which connects the backbone node to the earring node of the caterpillar graph. A fireworks graph can be obtained by adding an edge and a vertex to each backbone vertex which will connect the backbone vertices with the leaf vertices of a caterpillar graph. It is denoted by $F_{n,k}$ where n is the number of backbone nodes and k is the number of earrings [4].

Domination

It is known that the graph $G = V.E$. Let D be a subset of V. If every vertex of V - D is adjacent to at least one vertex of D, then D is said to be the dominating set in the graph G. The dominating number, denoted by γ(G), is the smallest cardinality of a dominating set. A minimal dominance set is a dominance set where no points can be removed without changing its dominance [6].

Side Contraction

The dominance contraction number of a graph ctγ (G) is defined as the minimum number of edges that must be contracted to reduce the dominance number [7].

Method

The research method used in this writing is library research to obtain data and information used in discussing the problem. The research was conducted at the Medan State University Digital Library for approximately two months. The procedure for this research is as follows:

1. Draw a Banana Tree Graph from n = 2 to n and k = 4
2. Determine the dominating points on the graph
3. Determining the dominance set based on the dominating points on the graph
4. Determine the dominance number based on the smallest cardinality of the dominance set
5. Contracting the edges based on the set of domination numbers that have been chosen
6. Determine the dominance set based on the dominating points on a graph that has been contracted
7. Create a pattern of contraction numbers for the dominant side of the graph under study
8. Prove the correctness of the pattern mathematically
9. Do the same steps on the Fireworks Graphic
10. Making Conclusions

3. RESULT AND ANALYSIS

The discussion will start from graphs with n = 2 to 5 and k = 4, then the pattern obtained will be generalized for graphs with n vertices.

Domination Number and Domination Side Contraction Number

Banana Tree Graph n Points ($B_{n,k}$)

Based on the definition which states that the dominance contraction number is the minimum side that must be contracted to reduce the amount of dominance. Because the problem in this research is limited to the n = 2 banana tree graph, the discussion will start from n = 2 to n = 5 with k = 4 which will then be generalized to n and k = 4 banana tree graphs.

Banana Tree Graph $B_{2,4}$

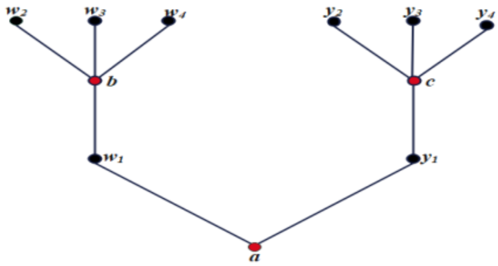

Figure 1. Banana Tree Graph $B_{2,4}$

From Figure 1, it can be seen that *a* dominates w_1 and y_1, *b* dominates w_1, w_2, w_3, and w_4, *c* dominates y_1, y_2, y_3, and y_4, w_1 dominates *a* and *b*, y_1 dominates *a* and *c*. Then we can see the domination set D_1 = {*a, b, c*}.

According to the definition which states that the dominance number is the minimum cardinality of the dominance set, then $\gamma(B_{2,4})$ = 3.

Next, to find the dominance contraction number, $B_{2,4}$ with the dominance set D_1 = {a, b, c} and side (b, w_1) and side (c, y_1) as the sides that will be contracted so that it becomes:

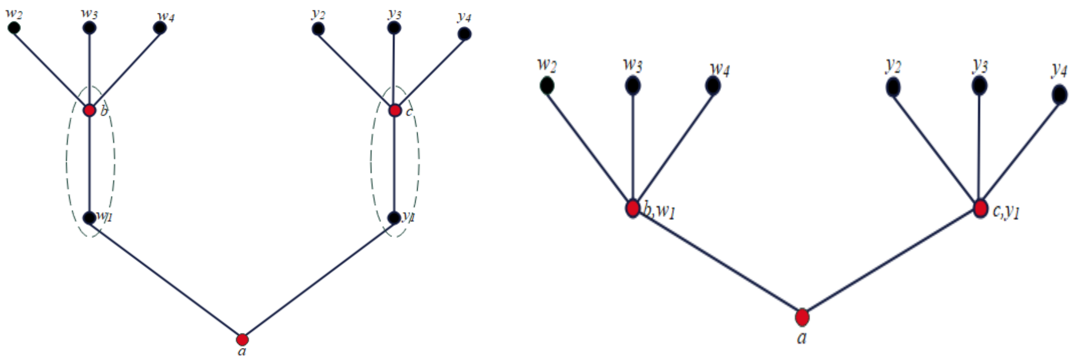

Figure 2. Domination Side Contraction $B_{2,4}$

Next, the sides (b, w_1) and *a* as well as (v, y_1) and *a* will be contracted to become:

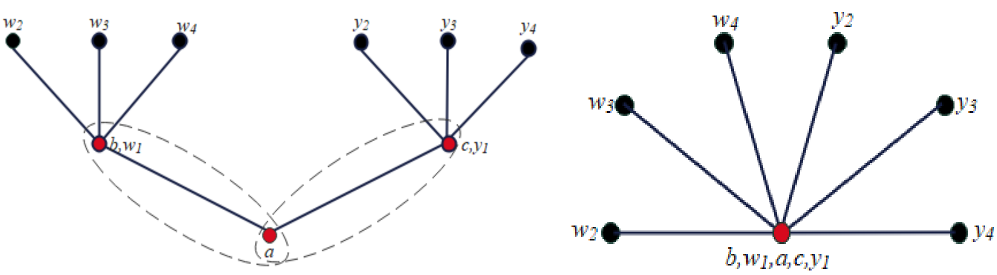

Figure 3. Contraction of the Dominant Side of Graph $B_{2,4}$ to become Graph S_6

Based on the definition, the dominance contraction number is the minimum side that must be contracted to reduce the dominance number, so it is true that the dominance number has been reduced. So ct γ $(B_{2,4})$= 4 because with γ $(B_{2,4})$ = 3 it requires four contractions to produce S_6 with a smaller dominance number, namely $\gamma(S_6)$ = 1. From the results of the contraction of the dominance side in the banana tree graph $B_{2,4}$ produces a star graph S_6.

If we continue in the same way for the n- point banana tree graph ($B_{n,4}$), we get the following results:

Table 1. Pattern of Domination Numbers and Contraction Numbers of the Dominant Side of Graph $B_{n,k}$

Graph Name	Dominance Number Before Contraction ($\gamma(B_{n,k})$)	Dominance Side Contraction Number (ct$\gamma(B_{n,k})$)	Dominance Number After Contraction ($\gamma(B_{n,k})$)
$B_{2,4}$	3	4	1
$B_{3,4}$	4	6	1
$B_{4,4}$	5	8	1
$B_{5,4}$	6	10	1
$B_{6,4}$	7	12	1
$B_{7,4}$	8	15	1
$B_{8,4}$	9	16	1
⋮	⋮	⋮	⋮
$B_{n,4}$	n+1	2n	1

a. $\gamma(B_{n,k}) = n + 1$

To prove the pattern $\gamma(B_{n,k}) = n+1$ proof is used by mathematical induction. For $n = 1$ then $\gamma(B_{1,k}) = 1 + 1 = 2$ and for $n = 2$ then $\gamma(B_{2,k}) = 2 + 1 = 3$ is correct. Assuming it is true for $n = k$, then $\gamma(B_{k,k}) = k + 1$. Next it will be shown that it is true for $n = k + 1$ then $\gamma(B_{k+1,k}) = k + 1 + 1 = k + 2$. This It can be shown from the assumption that for $n = k$ then $\gamma(B_{k,k}) = k + 1$ because every time n increases by 1, the number of $\gamma(B_{n,k})$ increases by 1. This means that every time the number of backbone nodes increases by 1, the dominance number increases by 1. So proven $\gamma(B_{n,k}) = n + 1$

b. $ct\gamma(B_{n,k}) = 2n$

To prove the pattern $ct\gamma(B_{n,k}) = 2n$ proof is used by mathematical induction. For $n = 1$ then $ct\gamma(B_{1,k}) = 2(1) = 2$ and for $n = 2$ then $ct\gamma(B_{2,k}) = 2(2) = 4$ is correct. Assuming it is true for $n = k$, then $ct\gamma(B_{k,k}) = 2(k) = 2k$. Next it will be shown that it is true that for $n = k + 1$ then $ct\gamma(B_{k+1,k}) = 2(k + 1) = 2k + 2$. This can be shown from the assumption that for $n = k$ then $ct\gamma(B_{k,k}) = 2k$ because every time n increases by 1, the number of $ct\gamma(B_{n,k})$ is multiplied by 2. This means that every time the number of backbone nodes increases by 1, the dominance number is multiplied by 2 because there will be 2 contractions for every additional 1 backbone node. So it is proven that $\gamma(B_{n,k}) = 2n$.

c. $\gamma'(B_{n,k}) = 1$

To prove $\gamma'(B_{n,k}) = 1$ proof is used by mathematical induction. For $n = 1$ then $\gamma'(B_{1,k}) = 1$ with n natural number elements is true. For $n = k$ then $\gamma'(B_{k,k}) = 1$ with n natural number elements. For $n = k + 1$ then $\gamma'(B_{k+1,k}) = 1$ with n natural number elements. This can be shown from the assumption that for $n = k$ then $\gamma'(B_{k,k}) = 1$ because every time n increases by 1 it does not affect the number of domination points. Contraction results of the banana tree graph $B_{n,k}$ is a star graph (S_n) which has 1 central point connected to all points in (S_n) . So it is proven that $\gamma'(B_{n,k}) = 1$.

Fireworks Graph n Points ($F_{n,k}$)

Based on the definition which states that the dominance contraction number is the minimum side that must be contracted to reduce the amount of dominance. Because the problem in this research is limited to $n = 2$ fireworks graphs, the discussion will start from $n = 2$ to $n = 5$ with $k = 4$ *which will then be generalized to* n and $k = 4$ fireworks graphs.

Fireworks Graph $F_{2,4}$

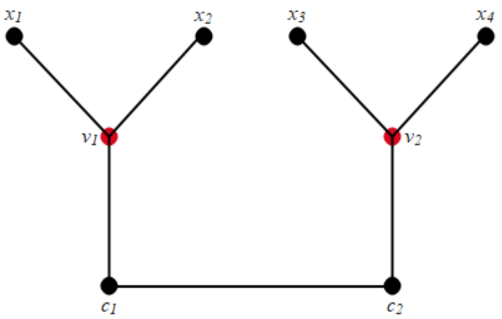

Figure 4. *Fireworks graph $F_{2,4}$*

From the picture above, it can be seen that c_1 dominates v_1 and c_2, c_2 dominates c_1 and v_2, v_1 dominates c_1, x_1, and x_2, v_2 dominates c_2, x_3 and x_4. Then we can see the domination set $D_1=\{v_1, v_2\}$.

According to the definition which states that the dominance number is the minimum cardinality of the dominance set then $\gamma(F_{2,4}) = 2$.

Next, to find the dominance contraction number, then $F_{2,4}$ with the domination set $D_1=\{v_1, v_2\}$ and side (v_1 , c_1) and side (v_2, c_2) as the sides that will be contracted so that it becomes:

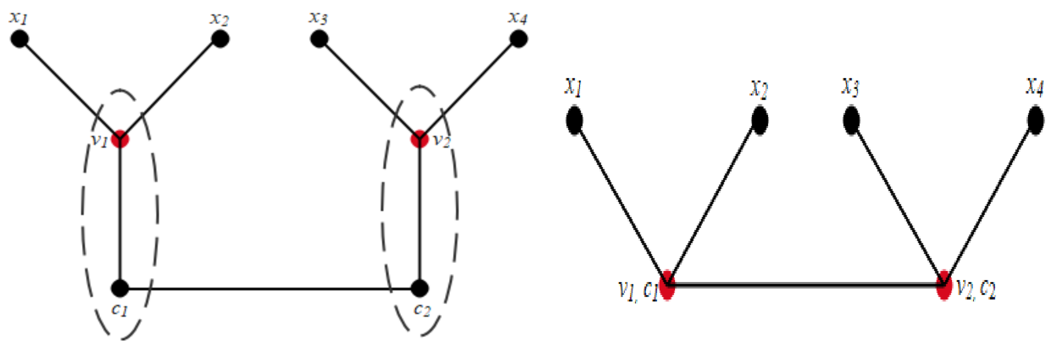

Figure 5. Contraction of the Dominant Side of Graph $F_{2,4}$

Next, the sides (v_1,c_1) and (v_2, c_2) will be contracted to become:

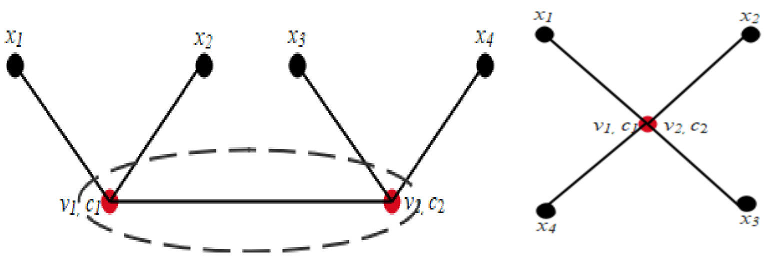

Figure 6. Contraction of the Dominant Side of Graph $F_{2,4}$ to become Graph S_4

Based on the definition of the dominance contraction number, which is the minimum side that must be contracted to reduce the dominance number, it is true that the dominance number has been reduced.

So $ct\gamma$ $(F_{2,4})$ = 3 because with $\gamma(F_{2,4})$ = 2 it requires three contractions to produce S_4 with a smaller dominance number, namely $\gamma(S_4)$ = 1.

From the results of the dominance side contraction of the banana tree graph $F_{2,4}$ it produces a star graph S_4.

If we continue in the same way for the n-point fireworks graph ($F_{n,4}$), we get the following results:

Table 2. Pattern of Domination Numbers and Contraction Numbers of the Domination Side of the $F_{n,k}$ Graph

Grapg Name	Dominance Number Before Contraction ($\gamma(F_{n,k})$)	Dominance Side Contraction Number (ct$\gamma(F_{n,k})$)	Dominance Number After Contraction ($\gamma(F_{n,k})$)
$F_{2,4}$	3	4	1
$F_{3,4}$	4	6	1
$F_{4,4}$	5	8	1
$F_{5,4}$	6	10	1
$F_{6,4}$	7	12	1
$F_{7,4}$	8	15	1
$F_{8,4}$	9	16	1
⋮	⋮	⋮	⋮
$F_{n,4}$	n+1	$2n$	1

a. **γ ($F_{n,k}$) = n**

To prove the pattern $\gamma(F_{n,k})$ = n , proof is used by mathematical induction. For n=1 then $\gamma(F_{1,k})$ = 1 and for n = 2 then $\gamma(F_{2,k})$ = 2 is correct. Assuming it is true for $n = k$ then $\gamma(F_{k,k}) = k$. Next it will be shown that it is true that for $n = k + 1$ then $\gamma(F_{k+1,k}) = k + 1$. This can be shown from the assumption that for $n = k$ then $\gamma(F_{k,k}) = k$ because every n increases 1 then the number of γ ($B_{n,k}$) increases by 1. This means that the number of backbone nodes is the same as the dominance number. So it is proven that γ ($B_{n,k}$)=n.

b. **$ct\gamma(F_{n,k}) = 2n - 1$**

To prove the pattern $ct\gamma$ ($F_{n,k}$) = 2 n - 1 proof is used by mathematical induction. For n = 1 then $ct\gamma(F_{1,k})$ = 2(1) - 1 = 1 and for n = 2 then $ct\gamma$ ($F_{2,k}$) = 2(2) - 1 = 3 is correct. Assuming it is true for $n = k$ then $ct\gamma$ ($F_{k,k}$) = 2(k) - 1 = 2 k- 1. Next it will be shown that it is true for $n = k + 1$ then $ct\gamma$ ($F_{k+1,k}$) = 2(k + 1) - 1 = 2 k + 2 - 1 = 2 k + 1. This can be shown from the assumption that for $n = k$ then $ct\gamma(F_{k,k})$ = 2 k - 1 because every time n increases by 1 the number $ct\gamma(F_{n,k})$ = 2 n- 1 . This means that every time the number of backbone nodes increases by 1, the dominance number becomes 2 n - 1 because there will be 2 contractions for each addition of 1 backbone node but 1 side contraction will decrease in the second contraction. So it is proven *that* $\gamma(B_{n,k})$ = 2n - 1.

c. **γ '($F_{n,k}$) = 1**

To prove γ '($F_{n,k}$) = 1 proof is used by mathematical induction. For n = 1 then γ ' ($F_{1,k}$) = 1 with n natural number elements is true. For $n = k$ then γ '($F_{k,k}$) = 1 with n natural number elements. For $n = k + 1$ then γ '($F_{k+1,k}$) = 1 with n natural number elements. This can be shown from the assumption that for $n = k$ then γ '($F_{k,k}$) = 1 because every time n increases by 1 it does not affect the number of domination points. The contraction result of the fireworks graph $F_{n,k}$ is a star graph (S_n) which has 1 central point connected to all points in (S_n). So it is proven that γ '($F_{n,k}$) = 1.

4. CONCLUSION

Based on the discussion in this thesis, conclusions can be drawn regarding the pattern of contraction numbers on the dominant side of several special graphs studied, namely as follows:

1. The dominance number pattern in the banana tree graph ($B_{n,k}$) with n ≥ 2 and k = 4 is $\gamma(B_{n,k}) = n + 1$.

The dominance number pattern after contraction in the banana tree graph ($B_{n,k}$) with n ≥ 2 and k = 4 is $\gamma'(B_{n,k}) = n + 1$.
The pattern of dominance side contraction numbers in a banana tree graph ($B_{n,k}$) with n ≥ 2 and k = 4 is ct γ ($B_{n,k}$) = 2 n.

2. The pattern of dominance numbers in the fireworks graph ($F_{n,k}$) with n ≥ 2 and k = 4 is γ ($F_{n,k}$) = n
   The pattern of dominance numbers after being contracted in the fireworks graph ($F_{n,k}$) with n ≥ 2 and k = 4 is $\gamma'(F_{n,k})$ = 1
   The pattern of dominance side contraction numbers in the fireworks graph ($F_{n,k}$) with n = 2 and k = 4 is ct $\gamma(F_{n,k})$ = 2 n – 1

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