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Application of Optimal Control Theory to Inventory Problems That Are Increasing at PT. Canang Indah

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ABSTRACT

This journal discusses optimal control of inventory problems that are increasing. The inventory in the company is needed, to meet every incoming demand. Inventory in minimum quantity can result in shortage of inventory. But inventory quantity maximum can result in losses, due to the minimum demand. The purpose of this research is to determine the level of optimal inventory in PT. Canang Indah. Using the optimal control theory model and analyzing the stability of the dynamic differential equation, to find the optimal inventory level. Obtained optimal inventory levels achieve stability at the time $6.106,0644m^3$. For the planning length of 12 months includes: raw material inventory (logs sengon and rambung), production (finished materials in process) and finished particle board products that are in the warehouse. From this research optimal control theory can be applied in PT. Canang Indah to optimize inventory on the problem of increasing inventory.

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1. INTRODUCTION

Inventory is an important part of the company's production activities, which are consistently obtained, modified and further processed, then sold. With the avaibility of sufficient inventory in the warehouse, aims to facilitate production activities or services to consumers. Supplies of raw logs with sengon logs dan rambung types at PT. Canang Indah is very much needed, because the two woods are the main raw material as well as support for the company's production activities to produce particle board. Inventory in maximum quantity will maintain the company's capability to serve every customer order, but it will have an impact on increasing inventory storage and production costs in the company.

Demand is the amount of commodity desired by the market at a certain price level, certain income level and a certain period of time. Periodically, consumer demand for an item is influenced by the price of the goods needed, income levels, the number of residents in the area, future interests and predictions, and prices of other commodities. Demand for particle board at PT. Canang Indah comes from within and outside the country. Demand is experiencing instability from each consumer with a different quantity every month.

Inventory is experiencing an increase problem, due to the initial inventory in large quantities but for the level of demand for the products offered is minimum and the company continues to carry out production activities in large quantities. The supply of particle board at PT. Canang Indah is experiencing an increasing problem, because the raw materials for logs sengon and rambung are continuously produced resulting in large quantities of particle board products causing inventory accumulation problem due to the minimum quantity of demand for particle board originating from within country and abroad. The company also experiences waste in storage and production costs, due to a lot of capital that must be issued and embedded in the company it is necessary to control within the company, so that the company does not continue to suffer losses due to the accumulation of inventory.

2. RESEARCH METHODE

2.1 Research Sources And Variables

The data collection period is Januari 2021 to Desember 2021.

2.2 Data Analysis Techniques

The research was carried out with a procedure described in nine stages, starting with the initial step namely identifying the problem and ending after finding the following conclusions and suggestions:

1. Collecting inventory stock data, production data, demand data, inventory storage cost data, and particle board production cost data period is Januari 2021 to Desember 2021.
2. Establish a dynamic differential equation model for an increasing product inventory model.

$$\dot{I} = P(t) + v(t)I(t) \qquad t \in [0, t_1] \tag{2.1}$$

With $v(t) = m(t) - \theta(t)$

Description:

3. Establish an objective function model

$$J = \frac{1}{2}\int_0^t \left\{ h\left[I(t) - \hat{I}\right]^2 + k\left[P(t) - \hat{P}\right]^2 \right\} dt \tag{2.2}$$

4. Defined by Hamilton's equation, for an increasing product inventory model

$$H = -\frac{1}{2}\left[h\left(I - \hat{I}\right)^2 + k\left(P - \hat{P}\right)^2\right] + \lambda g \text{ with } g = P + vI \tag{2.3}$$

5. Defined by Lagrange's equation, for an increasing product inventory model

$$L = -\frac{1}{2}\left[h\left(I - \hat{I}\right)^2 + k\left(P - \hat{P}\right)^2\right] + (\lambda + \mu)g \text{ with } g = P + vI \tag{2.4}$$

6. Prove the optimal conditions of the Hamilton dan Lagrange equation
   a) $H_P = 0$ $\quad (2.5)$
   b) $L_I = -\dot{\lambda}$ $\quad (2.6)$
   c) $L_P = 0$ $\quad (2.7)$
   d) $\mu \geq 0; \mu g \geq 0$ $\quad (2.8)$
7. From proving the optimal conditions, it is assumed in two cases in the form of an explicit solution:
   a) The function v is a constant
   b) The function $\frac{h}{k} + \dot{v} + v^2$ is a constant
8. Perform a stability analysis, stability analysis is measured through the completion of an explicit form to find the optimal inventory level.

$$I(t) = F_{11}e^{rt} + F_{12}e^{-rt} + Q_1(t) \qquad t \in [0, T] \tag{2.9}$$

9. Make conclusions and suggestions.

3. RESULT AND ANALYSIS

3.1 Description of Data

The data collection in this study is divided into several data, starting from May 2020 to April 2021, it is known as follows:

1) $\left(I(12)\right)$ = $10.005,02m^3$
2) $\left(I_0\right)$ = $332,23m^3$
3) $\left(M\right)$ = $1.121,50m^3$
4) $\left(m(12)\right)$ = $505,20m^3$
5) $\left(\theta(12)\right)$ = $50,20m^3$
6) $\left(v(12)\right)$ = $175m^3$
7) $\left(h\left(I(12)\right)\right)$ = Rp. 10.001.020
8) $\left(P(12)\right)$ = $332,33m^3$
9) $\left(P_0\right)$ = $118,48m^3$
10) $\left(\hat{P}\right)$ = $128,22m^3$
11) $\left(k\left(P(12)\right)\right)$ = Rp. 200.227.316

3.2 Optimal Control Model on Inventory

This study discusses the optimal control theory on the problem of increasing inventory, carried out with the following steps:

1. Form the following dynamic differential equation model:

$$\dot{I} = P(t) + v(t)I(t) \qquad t \in [0, t_1] \tag{3.1}$$

2. The form of the objective function model is as follows:

$$J = \frac{1}{2}\int_0^t \left\{ h\left[I(t) - \hat{I}\right]^2 + k\left[P(t) - \hat{P}\right]^2 \right\} dt \tag{3.2}$$

Equation (3.1) and (3.2) are non-negative constraints.

3. Defined by Hamilton's equation, based on equation (3.1) and (3.2) it is :

$$H = -\frac{1}{2}\left[h\left(I - \hat{I}\right)^2 + k\left(P - \hat{P}\right)^2\right] + \lambda g \quad \text{with } g = P + vI \tag{3.3}$$

4. Define by the following Lagrange equation:

$$L = -\frac{1}{2}\left[h\left(I - \hat{I}\right)^2 + k\left(P - \hat{P}\right)^2\right] + (\lambda + \mu)g \quad \text{with } g = P + vI \tag{3.4}$$

5. Prove the optimal conditions formed from the Hamilton and Lagrange equations, as follow:

a) First Condition:

$$H_P = 0 \tag{3.5}$$

b) Second Condition:

$$L_I = -\dot{\lambda} \tag{3.6}$$

c) Third Condition:

$$L_P = 0 \tag{3.7}$$

d) Fourth Condition:

$$\mu \geq 0; \mu g \geq 0 \tag{3.8}$$

Equation (3.8) implies $\mu = 0$ based on equation (2.1) and equation (3.5) it is obtained:

$$\dot{I} = \left(\hat{P} + \frac{\lambda}{k}\right) + vI \tag{3.9}$$

With the derivative of the equation (3.9) it is obtained:

$$\ddot{I} = \frac{\dot{\lambda}}{k} + \dot{v}I + v\dot{I} \tag{3.10}$$

By substituting the equation (3.6) and (3.9) into equation (3.10) it is obtained:

$$\ddot{I} = \frac{\left(h\left(I - \hat{I}\right) - (\lambda + \mu)v\right)}{k} + \dot{v}I + v\left(\left(\hat{P} + \frac{\lambda}{k}\right) + vI\right) \tag{3.11}$$

And based on equation (3.7) , then $k\left(P - \hat{P}\right) = (\lambda + \mu)$, so that:

$$\ddot{I} = \frac{\left(h\left(I - \hat{I}\right) - (\lambda + \mu)v\right)}{k} + \dot{v}I + v\left(\hat{P} + \frac{\lambda}{k} + vI\right) \tag{3.12}$$

Based on equation (3.5) then, $P - \hat{P} = \frac{\lambda}{k}$, so that:

$$\ddot{I} - \left(\frac{h}{k} + \dot{v} + v^2\right)I = -\frac{h\hat{I}}{k} + v\hat{P} \tag{3.13}$$

6. From proving the optimal conditions, it is assumed in two cases in the form of an explicit solution.
   a) The function v is a constant.
   If the v function is a constant, it can be seen in the differential equation (3.13) so that the following equation is formed:

$$\ddot{I} - \left(\frac{h}{k} + v^2\right)I = -\frac{h\hat{I}}{k} + v\hat{P} \tag{3.14}$$

   Equation (3.14) is a second order differential equation which is not homogeneous. The first step that must be taken to solve equation (3.14) is to choose a general solution to the homogenous equation, so that the following characteristic equation is formed:

$$r^2 - \left(\frac{h}{k} + v^2\right) = 0 \tag{3.15}$$

   From equation (3.15) , the solution of the differential equation is obtained with different real roots. So that the following equation is formed:

$$r_1 = \sqrt{\left(\frac{h}{k} + v^2\right)} = r \qquad r_2 = -\sqrt{\left(\frac{h}{k} + v^2\right)} = -r$$

For solution (3.15) produces the following equation to analyze the stability of inventory levels:

$$I(t) = F_{11}e^{rt} + F_{12}e^{-rt} + Q_1(t) \qquad t \in [0, T] \qquad (3.16)$$

Where $Q_1(t)$ is an additional solution to the non-homogeneous equation (3.14) then we get:

$$Q_1(t) = \frac{h\hat{I} - vhP}{h + kv^2}$$

Then determine $(P(t))$ from the conditions $I(0) = I_0$ and $I(t_1) = M$ in equation (3.16), so that the following equation is formed:

a. For $t = 0$ obtained: $I_0 = F_{11}(1) + F_{12}(1) + Q_1(0)$
b. For $t = t_1$ obtained: $M = F_{11}e^{rt_1} + F_{12}e^{-rt_1} + Q_1(t_1)$

Value F_{11} and F_{12} can be solved as follows:

$A = Bx + C$

$$\begin{pmatrix} I_0 \\ M \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ e^{rt_1} & e^{-rt_1} \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \end{pmatrix} + \begin{pmatrix} Q_1(0) \\ Q_1(t_1) \end{pmatrix}$$

So, $x = B^{-1}(A - C)$ obtained:

$$F_{11} = \frac{e^{-rt_1}\left(I_0 - Q_1(0)\right) - \left(M - Q_1(t_1)\right)}{e^{-rt_1} - e^{rt_1}} \qquad F_{12} = \frac{-e^{rt_1}\left(I_0 - Q_1(0)\right) + \left(M - Q_1(t_1)\right)}{e^{-rt_1} - e^{rt_1}}$$

Based on equations (3.9) and (3.16) obtained as follows:

$$\lambda = h\left(\dot{I} - \hat{P} - vI\right)$$

$$\lambda = h\left(F_{11}(r - v)e^{rt} - F_{12}(r + v)e^{-rt} + \dot{Q}_1 - \hat{P} - vQ_1\right) \qquad (3.17)$$

From equation (3.9) substituted into equation (3.17) it is obtained as follows:

$$P(t) = \hat{P} + \left(F_{11}(r - v)e^{rt} - F_{12}(r + v)e^{-rt} + \dot{Q}_1(t) - \hat{P} - v\left(Q_1(t)\right)\right) \qquad (3.18)$$

b) The function $\frac{h}{k} + \dot{v} + v^2$ is a constant

Assumed $\frac{h}{k} + \dot{v} + v^2 = k_1^2 \qquad (3.19)$

So, that the equation (3.14) is obtained:

$$\ddot{I} - \left(k_1^2\right)I = -\frac{h\hat{I}}{k} + v\hat{P} \qquad t \in [0, t_1] \qquad (3.20)$$

To obtain a second-order differential equation that is not homogeneous, the solution to equation (3.19) is done by first calculating the value of v to obtain equation (3.20) . in equation (3.19) it is assumed:

$k_1^2 - \frac{h}{k} = z^2$, so that the following equation is formed: $\frac{dv}{z^2 - v^2} = dt$

This form can be solved by integrating both sides, so that the following equation is formed:

$$v(t) = \frac{z\left(e^{2zt}+1\right)}{\left(e^{2zt}-1\right)}$$

Substituting $v(t)$ into equation (3.19) we get:

$$\ddot{I} - \left(k_1^2\right)I = \frac{-h\hat{I}}{k} + \frac{z\left(e^{2zt}+1\right)}{\left(e^{2zt}-1\right)}\hat{P} \tag{3.21}$$

Equation (3.21) is a second-order differential equation that is not homogeneous, the first step to solving equation (3.21) is to determine the general solution to the homogeneous equation. So, that the following characteristic equation will be formed:

$$r^2 - \left(k_1^2\right) = 0 \tag{3.22}$$

So that the roots of the equation are obtained, namely $r_1 = k_1$ and $r_2 = -k_1$ to solve the following equation (3.22) :

$$I(t) = F_{11}e^{k_1 t} + F_{12}e^{-k_1 t} + Q(t) \tag{3.23}$$

Where $Q(t)$ is the solution of the homogeneous equation of equation (3.21), so we get:

$$Q(t) = \frac{k\hat{I}}{kk_1^2} + v_1 e^{k_1 t} + v_2 e^{-k_1 t}$$

With v_1 and v_2 is the antiderivative of v_1 and v_2.

Furthermore, to determine $\left(P(t)\right)$ using the condition $I(0) = I_0$ and $I(t_1) = M$ the following equation will be obtained:

a. For $t = 0$ obtained $I_0 = F_{11} + F_{12} + \frac{h\hat{I}}{kk_1^2}$

b. For $t = t_1$ obtained $M = \left(F_{11} + v_1(t_1)\right)e^{k_1 t_1} + \left(F_{12}v_2(t_1)\right)e^{-k_1 t_1} + \frac{h\hat{I}}{kk_1^2}$

Value F_{11} and F_{12} can be solved as follows:

$$A = Bx + C$$

$$\begin{pmatrix} I_0 \\ M \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ e^{k_1 t_1} & e^{-k_1 t_1} \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \end{pmatrix} + \begin{pmatrix} \frac{h\hat{I}}{kk_1^2} \\ v_1(t_1)e^{k_1 t_1} + v_2(t_1)e^{-k_1 t_1} + \frac{h\hat{I}}{kk_1^2} \end{pmatrix}$$

So $x = B^{-1}(A - C)$ obtained:

$$F_{11} = \frac{e^{-k_1 t_1}\left(I_0 - \frac{h\hat{I}}{kk_1^2}\right) - \left(M\left(v_1(t_1)e^{k_1 t_1} + v_2(t_1)e^{-k_1 t_1} + \frac{h\hat{I}}{kk_1^2}\right)\right)}{e^{-k_1 t_1} - e^{k_1 t_1}} \qquad F_{12} = \frac{-e^{k_1 t_1}\left(I_0 - \frac{h\hat{I}}{kk_1^2}\right) + \left(M\left(v_1(t_1)e^{k_1 t_1} + v_2(t_1)e^{-k_1 t_1} + \frac{h\hat{I}}{kk_1^2}\right)\right)}{e^{-k_1 t_1} - e^{k_1 t_1}}$$

Based on equation (3.9) and equation (3.23) obtained as follows:

$$\lambda = k\left(\dot{I} - \hat{P} - vI\right)$$

$$= k\left(\left((k_1 - v)\left[\left(F_{11} + v_1(t)\right)\right]e^{k_1 t} - (k_1 + v)\left[\left(F_{12} + v_2(t)\right)\right]e^{-k_1 t}\right) - \dot{P} - \frac{h\hat{I}}{kk_1^2}v\right)$$

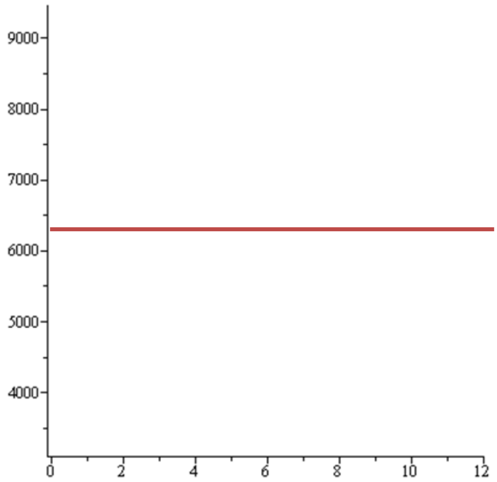
So that from equation (3.23) can be substituted into equation (3.5) and we get:

$$P(t) = \left((k_1 - v(t))\left[\left(F_{11} + v_1(t)\right)\right]e^{k_1 t} - (k_1 + v(t))\left[\left(F_{12} + v_2(t)\right)\right]e^{-k_1 t}\right) - \frac{h\hat{I}}{kk_1^2}v(t) \qquad (3.24)$$

7. Stability Analysis

Stability analysis was carried out to find the optimal inventory level from equation (3.17) for $t \in [0,12]$ using maple software.

Presented in the following graphic image:



Pict.1. Graphic $I(t)$ for $t \to 12$

Based on pict.1, it can be seen that for $t \to 12$ the inventory level value is $(I(t)) \to 6.106,0644m^3$ or $(I(t)) \to M$ it can be concluded that $I(t)$ is stable because $I(t)$ for $t \to 12$ goes to the value with the maximum inventory level (M).

4. CONCLUSION

Based on the result of the research that has been done, the optimal inventory level is obtained at $6.106,0644m^3$ for the planning length of 12 months. The inventories include: Raw materials sengon logs and rambung, particle board products in process, and finished particle board products ready for sale. It is concluded that Optimal Control Theory can be applied at PT. Canang Indah.

Other researchers who are interested in conducting research with optimal control theory models, are expected to develop and modify dynamic differential equation models. So that the resulting mod can be applied to other larger companies in other fields.

REFERENCES

[1] Affandi, P., & Faisal. (n.d.). Peningkatan Atau Penurunan Dengan Menggunakan Kendali Optimal *Abstrak.* 51–55.

[2] Andiraja, N., & Agustina, D. (2020). Aplikasi Kendali Optimal Untuk Model Persediaan yang Mengalami Kerusakan pada Persediaan dan Perubahan Tingkat Permintaan. *Jurnal Sains Matematika Dan Statistika,* 6(2), 12.

[3] P.Sethi, S. (2019). *Optimal Control Theory: Aplications to Management Science and Economics*. Spinger.

[4] Tu, P. N. Van. (1984). *Introductory Optimization Dynamics.* Spinger-Verlag Berlin Hedeiberg.

[5] Usvita, M. L., & Andiraja, N. (2017). *Kendali optimal pada masalah inventori yang mengalami peningkatan.* Seminar Nasional Teknologi Informasi Komunikasi Dan Industri (SNTIK) 9, 2015, 70–76.