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IMPLEMENTATION OF SIR MODEL IN THE SPREAD OF TB INFECTIOUS DISEASES IN PEMATANGSIANTAR CITY

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ABSTRACT

The most commonly-causing disease in developing countries is tuberculosis (TBC or TB). Therefore, it is necessary to conduct an analysis that can be accepted individually or scientifically on the problem of transmission of tb tuberculosis cases in developing countries. One of them can be viewed in the form of a mathematical model. By dividing the human population into 3 groups, namely the group of susceptible individuals (susceptible to disease), hordes of infective individuals (infected with disease), and groups of recovered individuals (cured derived from disease) that's how the SIR model describes the dynamics of the spread of infectious diseases. Therefore, critical point analysis, eigen value and basic reproduction ratio are needed. Then test the parameter analysis by simulating the runge kutta method order 4, from the results of the analysis conducted we will find the rate of transmission and cure rate which is the most influential thing on the spread of tuberculosis (TBC). Thus from the occurrence of epidemics that form or decrease the rate of transmission and increase the cure rate of tuberculosis disease transmission can be controlled.

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1. INTRODUCTION

Mycobacterium tuberculosis germs are the cause of infectious diseases Tuberculosis (TBC) which can be claimed also as acid resistant bacteria (BTA) and have various species of

Mycobacterium, such as M. tuberculosis, M. africanum, M. bovis, M. leprae and others. Most TBC attacks the lungs but TBC also affects other organs of the body. In addition to causing disruption in the channel of stripping other mycobacterium bacterial species can sometimes inhibit tb treatment and enforcement of diagnosis in tuberculosis. Tuberculosis was still ranked 10th in 2016 as the highest cause of death in the world, although the death rate caused by tuberculosis (TBC) decreased by 22%. between 2000 and 2015.

This can make others infected because the size of tuberculosis germs is so small that tuberculosis germs in the inhaled droplet nucleus can enter the alveolus. The origin of transmission is a BTA positive tuberculosis (TBC) patient. Approximately 3000 splashes of sputum (droplet nuclei) in one cough or sneeze that the patient can spread germs into the air in the room where usually transmission can occur. The patient's sputum splash can last for several hours in a dark room condition and feel damp. The number of germs that patients remove from their lungs affects the transmission power that occurs, the degree of positivity of the results of the sputum examination is high, so it can be said that the more contagious the patient. Factors that allow a person to contract or be exposed to tuberculosis (TBC) germs are influenced by the concentration of the patient's sparks in the air and the length of breathing of the air.

Tuberculosis prevention efforts in Indonesia have been carried out since the Dutch colonial period until now. The effort has also shown success, although not yet maximally. Some of the successes that have been seen are the enforcement of TBC disease diagnosis that has increased in line with technological developments as well, but there are still weak points in tb prevention in Indonesia and other developing countries such as diagnosis enforcement, as well as low cure rates. Things that can be done when a person is stricken with TBC disease so as not to share the virus with other oaring is to close their mouth when coughing and sneezing, do not spit or throw phlegm carelessly, reduce social interaction, let sunlight into the room, limit direct contact with people.

According to the Ministry of Health, which can cause the increasing problem of TBC disease, among others, is the cause of poverty in various communities, for example in developing countries today, about the failure of the existing TBC program so far. This is due to the State being unable to adequate political commitment and funding or costs, the State cannot adequate TBC service organization (less accessible by citizens let alone those in the village, the discovery of problems/diagnoses that are not standard or not guaranteed, drugs are not guaranteed provision, not done thorough monitoring, recording and reporting standards, and so on). The state cannot adequately conduct cases (diagnosis and mixture of drugs that are not standard, fail to cure cases that have been diagnosed from previous cases), all of these are the causes of the increase in tb or TBC disease that exists today.

Pematangsiantar is the 2nd city that has the most TBC disease problems in 2017 in north Sumatra province with 625 cases of tuberculosis based on the central statistics agency of north Sumatra province. The SIR model in this case cannot accurately describe all aspects of the epidemic, but with the mathematical modeling needed to convey good results to compare strategies that can be done to minimize the rate of epidemic disease infection. Although, mathematical models are not enough to predict and control epidemic diseases in the future. Research in this mathematical model can be developed from the SIR model studied by a researcher named Side, therefore it is expected that our awareness, especially the Pematangsiantar community, using sir mathematical model is expected to convey solutions about the spread of tuberculosis (TBC) disease in the Pematangsiantar.

The purpose of this study is to analyze the SIR model on the spread of TBC infectious diseases in the Pematangsiantar city area in 2017-2018 and test the SIR model can be used as a

solution to reduce the spread of TBC disease in Pematangsiantar.

Based on the research variables conducted, the hypothesis in this study is that there is an impact of the implementation factor of the SIR model in the spread of TBC infectious diseases in the Pematangsiantar of 2017 and 2018.

The benefit of this study is to gain knowledge about the behavior of the spread of tuberculosis (TBC) disease in the population in Pematangsiantar city area in 2017-2018 in using SIR mathematical model.

2. RESEARCH METHODE

This research is presented using a type of quantitative research because the data is in the form of numbers that can be calculated to show the results of the data obtained in this study. This research is descriptive because this study uses research methods with the data process able to identify why, what and how a thing happens. By identifying the formation of the SIR model on the spread of tuberculosis (TBC), a way to find out the formation of the SIR model on the spread of TBC disease, how efforts to prevent TBC disease so that not more people are affected by this infectious disease, what causes TBC disease, and in this study can describe the spread of TBC infectious diseases in the city of Pematangsiantar which can be shown from the calculation of SIR model conducted.

The data used in this study is secondary data, because data and explanations about TBC disease are obtained from books, journals, and data on the spread of TBC disease in Pematangsiantar City obtained from the Census of the Central Bureau of Statistics of North Sumatra in 2017 to 2018. In this study, all TBC disease patients who sought treatment at

Pematangsiantar City Hospital were population, recorded in the Census Data of the Central Bureau of Statistics of North Sumatra in 2017 amounted to 625 people and in 2018 amounted to 451 people. A sample is part of the number and charactericity of a population capable of resembling the population in the study. In the SIR model population of TBC disease spread in

Pematangsiantar there is a population N which is divided into 3 subpopulations, namely: Susceptible $(S(t))$, Infectious $(I(t))$, and Recovered $(R(t))$.

The procedures in this study to achieve the research objectives in the formation of mathematical models on the spread of TBC disease are: (a) identifying the problem by reading and understanding about TBC disease and the mathematical model used in this study; (b) determine and form the SIR model to obtain the equilibrium point of the models and parameters used in the spread of TBC disease; (c) analyze the SIR model to obtain a fixed point model and stability of a fixed point based on the eigenvalues in the spread of TBC disease, and (d); perform calculations and analyze the results of simulations from TBC patient data obtained from the Census Data of the Central Statistics Agency of Sumatra.

3. RESULT AND ANALYSIS

SIR Model for The Spread of Disease

The SIR (Susceptible, Infected, Recovered) model is one of the most widely used epidemiological models developed in the modeling of infectious diseases or the like for use as a simulation and prediction of the number of cases of a particular disease. This SIR model is widely applied to dengue and tuberculosis and generally, sir models made or modified do not involve diffusion factors or spatial effects of the intended population. The SIR model is used to model the spread caused by a specific disease in a population in a given area and can be applied to estimate the likelihood if a disease becomes an outbreak so that an analysis of preventive measures to regulate the outbreak can be performed.

$S(t)$,$I(t)$, and $R(t)$ are functions of the SIR model which are the number of susceptible, infected, and removed populations for each function. The total population of $N(t)$ can be constant or dynamic over time and meet $N(t) = S(t) + I(t) + R(t)$. Between population types in the model there is an interaction, namely the interaction of S with I which results in a positive rate for I and a negative for S. The development of the number of R occurs simultaneously with a decrease in the rate of the number of I modeled occurs exponentially negative.

SIR Model equations can generally be written as follows:

$$\frac{dS}{dt} = A - \mu S - \beta SI$$

$$\frac{dI}{dt} = \beta SI - \mu I - \gamma I$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

The SIR model has a disease-free equilibrium point $(TE_E) = \left(\frac{A}{\mu}, 0, 0\right)$ and an endemic balance point of the disease $(TE_E) = \left(\frac{\mu+\gamma}{\beta}, \frac{A}{\mu+\gamma} - \frac{\mu}{\beta} . \gamma\left(\frac{A}{\mu(\mu+\gamma)} - \frac{1}{\beta}\right)\right)$

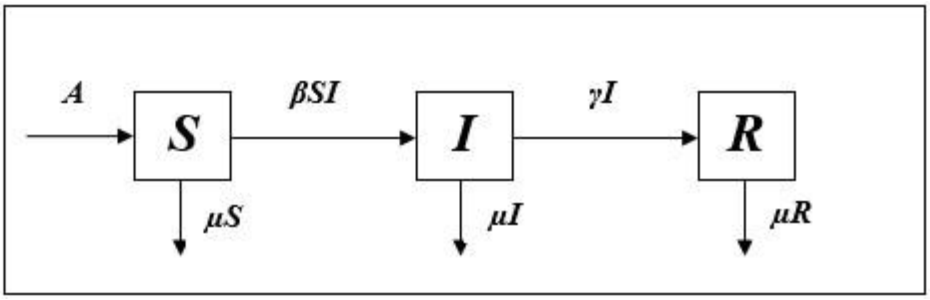

Figure 3.1 Flow of Disease Spread sir model

The flow of the spread of SIR model disease, as follows:

a. Susceptible individuals

The number of increases in the number of vulnerable individuals caused by the birth of vulnerable individuals with the number of births amounting to A and the number decreased because of the presence of vulnerable individuals infected with a large number of transmissions as much as β. In addition, the decrease in the number of vulnerable individuals is also influenced by the presence of deaths of vulnerable individuals with a large number of natural deaths of μ. b. Infected individuals

The number of infected individuals has increased due to the presence of vulnerable individuals infected with a large number of transmissions of β. Treatment is done in infected individuals to reduce the number of deaths or increase recovery with a cure rate of γ and the number of infected individuals will decrease due to natural deaths with many death rates of μ. c. Recovered Individual

The number of increases in the number of individuals recovered due to the cure rate of infected individuals with a large number of cures amounted to γ, but the number of reductions in the number of individuals who recovered due to the death of individuals who recovered with a large number of natural deaths amounted to μ.

Establishment of SIR Tuberculosis Model

The number of S populations will increase due to the onset of π , with π is constant. Then S will be reduced if death at a rate of μ. If meeting directly with someone who is infected with TBC will cause the person in the vulnerable population will be infected and will enter into population I. This leads to a decrease in population S. The rate of transmission of TBC disease is
b.

Population I is said to be where a person is affected by TBC disease and will transmit it to others. Death due to other factors with the rate of μ and death due to TBC disease with a rate of μ_t cause the population to decrease with a rate of c and entering the R population can make a
person affected by TBC can recover spontaneously. It also leads to a decrease in population I . A person can be expected not to relapse and will enter the population R caused by death at a rate of
μ.

Thus, a flowchart on the mathematical model of Tuberculosis (TBC) can be inferred from the discussion above. Can be seen in figure 3.2

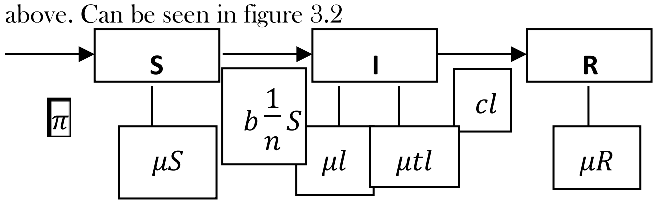

Figure 3.2 Flow Diagram of Tuberculosis Mathematical Model

Mathematical models of the spread of tuberculosis (TBC) based on figure 3.2 above are:

$$\frac{dS}{dt} = -b\frac{I}{N}S - \mu S + \pi$$

$$\frac{dI}{dt} = b\frac{I}{N}S - (\mu + \mu_t + c)I$$

$$\frac{dR}{dt} = cI - \pi R$$

With $N = S + I + R$

Analysis of SIR Model for The Spread of TBC Disease

a. Tipping Point

Create the critical point itself, the system is carried out in a constant position against time i.e. a condition where $\frac{dS}{dt} = 0$, $\frac{dI}{dt} = 0$, and $\frac{dR}{dt}$

Then the following two critical points are obtained:

i. $S\ (= \frac{\pi}{\mu}, I = 0, R = 0)$, Which conveys disease-free equilibrium,

ii. $(S = \frac{J\pi}{\mu K}, I = \frac{\pi L}{MK}, R = \frac{c\pi L}{\mu MK})$, with :

$J = \mu + c$,

$K = b - \mu_t$, $L = b - \mu - \mu_t - c$, $M = \mu + \mu_t + c$.

This tipping point must be conditioned to be positive because basically the human population cannot be negative in real life. The values J and M have been positive, therefore what needs to be known is the original value of L in order to have a positive value. So, K will definitely have a positive value as well. Suppose there is a differential equation as the following equation:

$$\frac{dx}{dt} = f(x), x \in R^n$$

A point is called a fixed point if it meets $f(x) = 0$. Fixed points are solutions that depend on t (constant to time). The fixed point, the tipping point and the equilibrium point are the same point. For the next used the term fixed point.

b. Eigen value

Eigen values can be reviewed from a data or system by searching for the Jacobian matrix (MJ). The Jacobian matrix of the following systems is:

$$MJ = \begin{matrix} \frac{\partial S}{\partial S} & \frac{\partial S}{\partial I} & \frac{\partial S}{\partial R} \\ \frac{\partial I}{\partial S} & \frac{\partial I}{\partial I} & \frac{\partial I}{\partial R} \\ \frac{\partial R}{\partial S} & \frac{\partial R}{\partial I} & \frac{\partial R}{\partial R} \end{matrix} \rightarrow$$

$$MJ = \begin{bmatrix} -\frac{bI}{N} + \frac{bSI}{N^2} - \mu & -\frac{bS}{N} + \frac{bSI}{N^2} - \mu & \frac{bSI}{N^2} \\ \frac{bI}{N} - \frac{bSI}{N^2} & \frac{bS}{N} - \frac{bSI}{N^2} - \mu - c & -\frac{bSI}{N^2} \\ 0 & c & -\mu \end{bmatrix} \quad (1)$$

The critical point that has been obtained and then subtitled in the MJ, then will be obtained two Jacobian matrices. Because you already know the Jacobian matrix, it can be found the value of eigen as follows:

$$\det\ (MJ - \lambda I) = 0$$

Found λ and eigen value to find the critical point (i) are:

$$\lambda_1 = -\mu, \quad \lambda_2 = -\mu, \quad \lambda_3 = L \quad (2)$$

Meanwhile, the value of eigen to find the critical point (ii) is:

$$\lambda_1 = -\mu, \quad \lambda_2 = \frac{-B+\sqrt{B^2-4AC}}{2A}, \lambda_3 = \frac{-B-\sqrt{B^2-4AC}}{2A} \quad (3)$$

with $A = B\ J, B = b(\mu_t\ \mu - b)$, and $C = \mu\ M\ K\ L$

All eigenvalues of a system that are negative or $\lambda_i < 0$ with i = 1, 2, and 3 are systems that can be said to be stable. The onset of opposition with the condition for the critical point (ii) caused by the value of eigen (2) will stabilize and go to the critical point $(S = \frac{\pi}{\mu}, I = 0, R = 0)$ if $L < 0$ or $b < \mu + \mu_t + c$. This can cause populations I and R will be negative, if this becomes a condition then the tipping point (ii) can be ignored. So it is concluded that the tipping point (ii) will exist and be stable if the value of eigen (2) is unstable.

The three eigen values must be negative, the value of λ_1 is already negative, so the values of λ_2 and λ_3 need to be conditioned in order to produce negatives as well. The value $A = bJ$ will definitely produce a positive, so on the denominator of the eigen value will definitely produce a positive. Therefore the numerator must produce a negative value. This is a way to make the eigen value (3) stable.

$$-B + \sqrt{B^2 - 4AC} < 0$$
$$\sqrt{B^2 - 4AC} < B$$
$$B^2 - 4AC < B^2$$
$$-4AC < 0$$
$$AC > 0$$

When A is definitely positive then C should also be positive value so that $AC > 0$. In addition, B must also be positive, because otherwise the eigenvalues will be positive and become unstable. If the requirement for λ_2 is met, the value for λ_3 is also met. This is because if $-B + \sqrt{B^2 - 4AC} < 0$ then the value of $-B - \sqrt{B^2 - 4AC} < 0$ is also less than zero. Then, the stable value of eigen (3), the value of eigen (2) will become unstable. Can be seen (3) will be stable if it qualifies $L > 0$ or $b > \mu + \mu_t + c$ which causes the value of eigen (2) to be positive and unstable. stable can be aimed at critical point (i) then the critical point (ii) will become unstable, and so on.

c. Basic Reproduction Ratio (A.)

The rate of spread of an infectious disease is usually measured by a value called the basic reproduction ratio (R_0) according to epidemiology. In order to be free from TBC infection, it must be made $R_0 < 1$ in order to be spared or free from TBC disease. In this case each patient can only spread the disease to an average of less than one new sufferer, so that eventually the disease will slowly disappear. But if every tb sufferer can spread TBC disease to an average of more than one new sufferer it all happens if $R_0 < 1$, so that eventually there will be an epidemic.

If looking for R_0, then the controlled population that spreads TBC infection then only model I is needed in the equation (1). Suppose A is a derivative of $\frac{dI}{dt}$ to I, where $A = M - D$.

Thus it can be known $R_0 = MD^{-1}$ Until it is obtained:

$$A = \frac{bS}{N} - \frac{bSI}{N^2} - (\mu + \mu_t + c)$$

Substitute a critical point $(S = \frac{\pi}{\mu}, I = 0. R = 0)$ in the equation (4), so:

$$A = b - (\mu + \mu_t + c)$$

From the equation above can be known $M = D$ and $D = \mu + \mu_t + c$. From here it can be obtained:

$$R_0 = MD^{-1}$$

$$R_0 = \frac{b}{(\mu + \mu_t + c)}$$

$R_0 < 1$ occurs when $b < \mu + \mu_t + c$ while $R_0 > 1$ occurs when $b > \mu + \mu_t + c$. As per the results R_0 obtained, to produce $R_0 < 1$, the denominator is greater than the numerator. Deaths due to other factors (μ) and deaths caused by tuberculosis (μ_t) cannot be increased. As a result, the cure rate (c) will be higher the need for healing or treatment for TBC sufferers. Also the rate of transmission of TBC disease (b) needs to be lowered, will result in a reduced or decreased rate of TBC spread so that the disease can be more controlled from epidemic conditions. So it can be said, parameters b and c are the most influential parameters of all parameters contained in the example of tuberculosis spread.

Simulation of Numerical Analysis

Based on data from people exposed to TBC disease in Pematangsiantar city in 2017 and 2018, there were 1078 people in two years. With the life expectancy in 2017 was 72,63 and in 2018 it was 72,93. TBC patient data of Pematangsiantar city in 2017-2018 as follows:

Variable	Number
S	482
I	219
R	375
N	1076

In determining the value of the parameters of the data can use the formula, as follows:

$$\beta(t) = \frac{I}{N} = \frac{219}{1076} = 0{,}203531599$$

$$\gamma(t) = \frac{R}{N} = \frac{375}{1076} = 0{,}348513011$$

$$K(t) = \frac{S}{N} = \frac{482}{1076} = 0{,}44795539$$

$$\mu(t) = \frac{1}{angka\ harapan\ hidup} = \frac{1}{145{,}56} = 0{,}00687001924$$

Then, it can be obtained:

$$R_0 = \frac{\beta}{(\mu + \gamma)}$$

$$R_0 = \frac{0{,}203531599}{(0{,}00687001924 + 0{,}348513011)}$$

$$R_0 = \frac{0{,}203531599}{0{,}35538303}$$

$$R_0 = 0{,}572710517$$

Based on the terms of $R_0 < 1$, from the results obtained $R_0 = 0{,}572710517$, meaning

$R_0 < 1$, so that the critical point is stable. To show the stability of the equilibrium point TE_1can use the formula, as follows:

$\lambda_1 = -\mu = -0{,}00687001924$

$\lambda_2 = (R_0 - 1)(\gamma + \mu) = (0{,}572710517 - 1)(\ 0{,}348513011 + 0{,}00687001924) = \ -0{,}151851431$

$\lambda_3 = \ -\mu = -0{,}00687001924$

Since $\lambda_1, \lambda_2, \lambda_3$ is obtained with a negative result, the equilibrium point is asymptotic stable. It can be interpreted from the results obtained that people infected with TBC can not transmit to healthy people (not infected with TBC), so there can be no epidemic TBC and TBC disease can be controlled or handled properly in the city of Pematangsiantar.

4. CONCLUSION

Based on the discussion of this study it can be concluded that TBC disease in Pematangsiantar City has a population of $R_0 = 0{,}572710517$which indicates $R_0 < 1$means that everyone infected with TBC is not enough to transmit the disease to others. In this study also obtained a critical point (balance point) $TE_1 = \ \left(\frac{K}{\mu}, 0{,}0\right)$ by obtaining negative results that mean free from TBC disease where the absence of a new person or patient infected with TBC in a population in Pematangsiantar City. The balance point obtained in this study based on existing data is stable asymptotic. Thus, it can be said that TBC disease in Pematangsiantar City can be controlled properly and will slowly decrease which will not be able to cause a pandemic of TBC disease in Pematangsiantar City.

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