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Integrated Multi-Period Linear Programming for Production Mix and Raw-Material Procurement in an NPK Fertilizer Plant

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ABSTRACT

Raw-material prices for NPK fertilizer are volatile, yet plant-level planning studies seldom link procurement timing (when raw materials are bought and stocked) to the production mix (how much of each grade is made). This study measured the value of integrating both decisions, its split between timing and mix, and the share attainable with forecasts. A multi-period linear programming model, solved with HiGHS, optimized the monthly mix of six products and the purchases and stocks of seven raw materials at an Indonesian NPK plant in 2021–2024. The objective is product value at international prices minus raw-material purchase and holding costs. With perfect foresight, integration raised the 2024 margin by USD 9.58 million (3.73% of the USD 256.98 million baseline), 63% from timing, and the two effects were additive. With forecasts on a rolling horizon, timing recovered at most 13% of its bound, whereas monthly mix re-optimization remained the robust gain

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1. INTRODUCTION

NPK compound fertilizer is a main input of Indonesian food agriculture, and since 2022 the subsidized-fertilizer program has concentrated on urea and NPK [1] ; fertilizer prices and availability affect rice output and food security [2]. One granulation line can produce several grades, such as NPK 15-10-12, NPK 15-15-15, NPS 20-20-0+13S, and DAP, from the same raw materials (ammonia, phosphoric acid, sulfuric acid, KCl, ZA, and urea) at different ratios, and raw materials are the largest part of the variable cost [3]. Their prices spiked in 2020–2022 and stayed high through 2024 [4], driven largely by energy prices [5], [6], and they are hard to forecast [7].

Joint optimization of procurement, processing, and inventory under time-varying prices is an established topic. Multi-period models with inventory are standard in the process industries [8] and have been applied to fertilizer manufacturing under stochastic demand [9]. Spot and contract buying trade price against supply risk [10], integration pays most under tight capacity and high volatility [11], purchase quantities can be learned from price features [12], and related models cover pricing, replenishment, and production planning [13], [14], [15], [16], usually on a rolling horizon [17], [18], [19]. Linear programming (LP) gives a global optimum and the shadow prices of resources and has been applied to the product mix of a small manufacturer [20], fertilizer blending [21] and purchasing [22], phosphate supply chains [23], [24], and fertilizer logistics [25]; in Indonesia, NPK production planning has used forecasting and lot sizing [26]. These fertilizer studies treat input prices as fixed, whereas the procurement studies do not model a multi-product plant whose mix is itself a decision.

The contribution therefore lies in what a standard multi-period LP is used to measure. RQ1: what is the value of integrating procurement timing with the production mix when future prices are known (the perfect-

foresight bound)? RQ2: how does it split between timing and mix, and are the effects additive? RQ3: what share of the bound is attainable with the forecasts available at decision time? The gap between the bound and a rolling-horizon policy shows how far a deterministic multi-period LP can overstate what is achievable. A monthly history of the shadow price of granulation capacity also indicates when capacity is worth most, under the assumptions of the model.

2. RESEARCH METHOD

This retrospective study used records of PT. X, an Indonesian producer of NPK and phosphate fertilizers, and international reference prices; the company is not named, and its permission and review are described in the Conflict of Interest statement. The research flowchart is Figure S1 of the supplementary appendix.

2.1 Data and Preprocessing

The data are (a) the company's consumption ratios and capacity factors for the six products of its NPK and phosphate plant (Table 1); (b) company-wide monthly procurement records for 2014–September 2025 (quantity, value, and price); (c) annual production by line for 2021–2024, so the monthly profile of the baseline is flat; and (d) monthly CRU Fertilizer Week prices for 2013–2025 [27]. In 2024, phosphoric acid and KCl were 68% of the plant's raw-material spending.

Table 1. Raw-material consumption ratios (ton/ton of product), capacity factors, and production in 2024

Raw material	NPK 15-10-12	NPK 15-15-15	NPS	DAP	SP-36	SP-20
Urea	0.001	0.03	0.04	–	–	–
ZA	0.27	0.13	0.25	0.08	–	–
Sulfuric acid	0.25	0.20	0.24	0.06	0.15	0.03
KCl	0.21	0.255	–	–	–	–
Phosphoric acid	0.23	0.285	0.38	0.84	0.40	0.08
Ammonia	0.12	0.137	0.17	0.21	–	–
Phosphate rock	–	–	–	–	0.53	0.556
Capacity factor κ_p (T15)	0.95	1.00	0.70	2/3	n/a	n/a
2024 production (ton)	2,138,918	59,560	32,012	85,960	6,899	0

Note: one ton of product p uses $1/\kappa_p$ t of NPK 15-15-15 (T15) equivalent capacity; SP-36 and SP-20 are made on a separate phosphate line (n/a).

The ratios close the nutrient balance, except that NPK 15-10-12 receives about 12% P_2O_5 instead of 10% (a balanced ratio of 0.19 is tested in Table S3). Realized prices are landed costs at the plant, including freight, port charges, duties, and taxes. They were preprocessed in four steps. (i) Phosphoric acid prices of 2015–2016, recorded per ton of P_2O_5, were converted to product acid with the factor 0.546, the median ratio of 2017–2025. (ii) Zero prices and prices more than five times above or below the 13-month rolling median were treated as missing. Six such records were found: ammonia in August 2021, sulfuric acid in April–June 2020, phosphate rock in January 2025, and ZA in September 2025. Rupiah records of 2025 were converted at the monthly average exchange rate [28]. (iii) The CRU series were calibrated to the realized prices by a ratio (1) or, for sulfuric acid, whose FOB price can approach zero, additively (2):

$$\hat{c}_{m,t} = \rho_m \, q_{m,t}, \qquad \rho_m = median_t\left(\frac{c_{m,t}^{real}}{q_{m,t}}\right) \tag{1}$$

$$\hat{c}_{m,t} = q_{m,t} + \beta, \qquad \beta = median_t(c_{m,t}^{real} - q_{m,t}) = 17.34\ USD/ton \tag{2}$$

where $q_{m,t}$ is the CRU price and $c_{m,t}^{real}$ the realized price of raw material m in month t (for phosphoric acid, the CRU price per ton of P_2O_5 times 0.52, so that $\rho_m = 1.050$ gives the factor 0.546). (iv) Realized prices follow term contracts, so gaps of up to two months were interpolated and only longer gaps used the calibrated proxy. Urea, produced within the company, is priced at FOB Indonesia as an opportunity cost. Sources and filled months are given in Table S1. Product values $v_{p,t}$ are international reference prices. The NPK grades are valued from NPK 16-16-16 (B) with the nutrient value ratio (3):

$$v_{A,t} = v_{B,t}\,\frac{n_A^N u_t^N + n_A^P u_t^P + n_A^K u_t^K}{n_B^N u_t^N + n_B^P u_t^P + n_B^K u_t^K} \tag{3}$$

where n_A^j is the content (%) of nutrient j in product A and u_t^j the value of one percentage point of j per ton, from urea (46% N), TSP (46% P_2O_5), and KCl (60% K_2O). NPS and DAP have their own quotations; SP-36 and SP-20 are valued from the TSP and SSP quotations scaled to their P_2O_5 content, by 36/46 and 20/19, where 19% is the midpoint of the 18–20% P_2O_5 basis of the CRU SSP quotation (Brazil, carriage paid).

2.2 Linear Programming Model

The model (Figure 1, Table 2) covers six products $p \in P$, of which those in P^{NPK} use the NPK granulation lines, seven raw materials $m \in M$, and the months $t \in T = \{1, \ldots, 12\}$ of planning year τ. It maximizes the raw-material contribution margin (4) subject to inventory balance (5), storage and end conditions (6), line capacity (7), the mix band (8), and non-negativity (9):

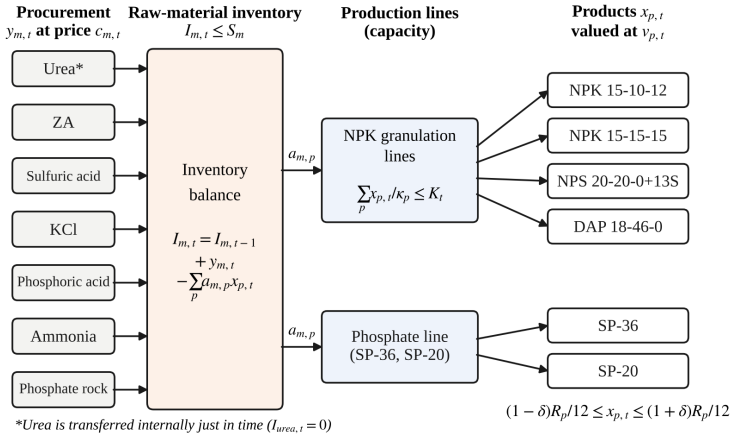


Figure 1. Structure of the production mix, procurement, and raw-material inventory model

Table 2. Nomenclature of the model and overview of the scenarios

Symbol	Description	Symbol	Description
P, P^{NPK}	Products; products made on the NPK lines	M, T	Raw materials; months $t = 1, \ldots, 12$ of year τ
$a_{m,p}$	Consumption of m per ton of p (t/t, Table 1)	$c_{m,t}$	Price of m in month t (USD/t)
$v_{p,t}$	Reference value of p in month t (USD/t)	h_m	Holding cost $\eta\, \underline{c}_m$, $\underline{c}_m$ mean price of year $\tau - 1$
η	Monthly holding-cost rate (2%)	κ_p	Capacity factor to T15-equivalent (Table 1)
K_t	Capacity of the NPK lines (t T15-eq./month)	R_p	Actual production of p in year τ (t)
δ	Mix band around $R_p/12$ (20%)	S_m	Storage, one month of requirement (t)
$x_{p,t}$, $y_{m,t}$	Production of p; procurement of m (t)	$I_{m,t}$	End-of-month inventory of m (t)
Z, λ_t	Margin (4); shadow price of capacity (11)	$\tilde{c}_{p,t}$, $\mu_{p,t}$	Raw-material cost and margin per ton of p (10)
Scenarios (Section 2.3): what is optimized and what is known			
S0	Mix $R_p/12$, buying just in time	S0-actual	S0 with the actual purchase profile
S1	Buying optimized at S0 mix; all prices known	S2	Mix optimized; current prices and R_p known
S3	S1 and S2 together; all prices known	Rolling	S3 re-solved monthly with forecasts
Timing only	Rolling horizon at the S0 mix	Rule (13)	Monthly mix; buy ahead if (13) holds

$$maxZ = \sum_{t \in T} \sum_{p \in P} v_{p,t} x_{p,t} - \sum_{t \in T} \sum_{m \in M} c_{m,t} y_{m,t} - \sum_{t \in T} \sum_{m \in M} h_m I_{m,t} \tag{4}$$

$$I_{m,t} = I_{m,t-1} + y_{m,t} - \sum_{p \in P} a_{m,p} x_{p,t}, \qquad \forall m \in M,\ t \in T \tag{5}$$

$$0 \le I_{m,t} \le S_m, \quad I_{m,0} = I_{m,12} = 0, \quad I_{urea,t} = 0, \qquad \forall m \in M,\ t \in T \tag{6}$$

$$\sum_{p \in P^{NPK}} \frac{x_{p,t}}{\kappa_p} \le K_t, \qquad \forall t \in T \tag{7}$$

$$(1-\delta)\frac{R_p}{12} \le x_{p,t} \le (1+\delta)\frac{R_p}{12}, \qquad \forall p \in P,\ t \in T \tag{8}$$

$$x_{p,t} \ge 0, \quad y_{m,t} \ge 0, \qquad \forall p \in P,\ m \in M,\ t \in T \tag{9}$$

Z is not profit: energy, labor, packaging, and distribution costs were not available by product and month and scale with throughput, which is almost equal in all scenarios. Internal supplies of ammonia, phosphoric acid, ZA, and urea are valued at purchase prices, that is, at opportunity cost; in 2024, purchases covered 77%, 67%, and 54% of the requirements of the first three. In the main model the internal share can also be bought ahead (a variant fixes it, Section 3.5), whereas urea is transferred internally and cannot be stocked. Opening and closing stocks are zero, and the holding rate $\eta = 2\%$ per month covers capital (about 1%) and storage, insurance, and losses (1–5% tested). K_t equals the realized 2024 throughput of 207,144 t T15-equivalent per month (93.1% of the installed capacity; for 2021–2023 that of each year), ignoring shutdowns and seasonality. The band δ represents supply commitments, including subsidized allocations; an annual commitment $(1-\delta_a)R_p \le \sum_t \quad x_{p,t} \le (1+\delta_a)R_p$ with monthly bounds of ±50% is also tested. Equation (10) defines the cost and margin per ton:

$$\tilde{c}_{p,t} = \sum_{m \in M} \quad a_{m,p} c_{m,t}, \qquad \mu_{p,t} = v_{p,t} - \tilde{c}_{p,t} \tag{10}$$

Without inventory, the LP favors the product with the highest margin per unit of capacity, $\kappa_p \mu_{p,t}$.

2.3 Scenarios and Analyses

The scenarios are summarized in Table 2; S0 is a reference plan, not actual operation. S1 and S3 use the prices of the whole year and are therefore perfect-foresight upper bounds, whereas S2 needs only current prices, because it separates by month. All scenarios were solved for 2021–2024 with year-specific inputs by the dual simplex method of HiGHS 1.12.0 [29] through SciPy 1.18.1 [30] (240 variables, 96 constraints; tolerance 10^{-7}); minimum-lot variants are mixed-integer programs. The shadow price of capacity is the dual of (7):

$$\lambda_t = \frac{\partial Z^*}{\partial K_t} \tag{11}$$

Its uniqueness was checked with left and right finite differences of Z^* (±100 t in K_t), and its range of validity was found month by month by bisection. Price-level sensitivity re-solves S3 with the prices $c_m = (c_{m,1}, \ldots, c_{m,12})$ of raw material m raised by 10%:

$$\Delta_m = \frac{Z^*(1.1\, c_m) - Z^*(c_m)}{Z^*(c_m)} \times 100\% \tag{12}$$

Single-month 10% shocks and a rescaling $c_{m,t}^{(k)} = \underline{c}_m + k(c_{m,t} - \underline{c}_m)$, $0 \le k \le 2$, test the timing incentives. Robustness was tested for 2024 with seven operational variants (Table 6, Table S3) and with NPK 15-10-12 valued at 80% and 120% of its reference value. A dynamic analysis solved a single-period LP for each month of 2015–September 2025 with the 2024 plant, a counterfactual at historical prices.

RQ3 was tested on a rolling horizon for 2021–2024 [17], [18]. Each month, the price series is rebuilt from the data available up to that month and the remaining months are forecast. The LP (4)–(9) is then solved from the current inventory, and only the current decisions are implemented and evaluated at actual prices. The forecasts were naive, three-month moving average (MA), three-month drift, and ARIMA selected by the Akaike information criterion (statsmodels 0.15.0). The timing part was measured by separate runs with the mix fixed at S0. A threshold rule re-optimizes the mix monthly and buys next month's requirement of m one month ahead only if

$$\frac{\hat{c}_{m,t+1}}{c_{m,t}} - 1 > \eta + \theta \tag{13}$$

where $\hat{c}_{m,t+1}$ is the one-step forecast and $\theta \in \{0{,}2\%, 5\%\}$ a safety margin; its timing part uses the actual mix. Finally, 300 price paths for a 2024-like year were simulated by a stationary block bootstrap of the monthly log returns of 2015–2024.

3. RESULT AND ANALYSIS

3.1 Prices, Margins, and the 2024 Scenarios

Prices rose sharply in 2021–2022 and corrected in 2023 (Figure S2). In 2024, phosphoric acid was flat within quarters (USD 518–578/ton), ammonia ranged from USD 362 to 438 and ZA from USD 130 to 172/ton, and KCl hardly moved. The 2024 margins per ton were USD 167.9 for NPK 15-15-15, 112.8 for NPK 15-10-12, 46.5 for NPS, and 42.3 for DAP.

Table 3. Results of the optimization scenarios in 2024

Scenario	Product value (million USD)	Procurement cost (million USD)	Procurement cost change (%)	Holding cost (million USD)	Margin Z (million USD)	ΔZ vs. S0 (%)
S0 Baseline	936.80	679.82	–	0.00	256.98	–
S1 Procurement†	936.80	672.34	−1.10	1.44	263.02	+2.35
S2 Mix	939.21	678.69	−0.17	0.00	260.52	+1.38
S3 Integrated†	939.21	671.21	−1.27	1.45	266.56	+3.73

†Perfect-foresight upper bound. Components are rounded independently, so differences may deviate by 0.01.

S1 cut the procurement cost by USD 7.49 million (1.10%) at an extra holding cost of USD 1.44 million and raised the margin by USD 6.04 million (2.35%); S2 raised it by USD 3.54 million (1.38%), and S3 by USD 9.58 million (3.73%), the sum of the two within USD 0.01 million (Table 3). Timing thus gives 63% of the integrated gain, and S1 and S3 are upper bounds (RQ1, RQ2). Additivity is structural: S2 and S3 choose the same mix (product value USD 939.21 million), because the mix follows the ranking of $\kappa_p \mu_{p,t}$, which the small cost changes from timing do not alter. The same holds in 2022 and 2023 and for bands up to ±50% (interaction below USD 0.08 million); only in 2021, when prices rose steadily, did buying ahead change the mix (interaction USD −0.45 million). In S3, purchases concentrate in low-price months (Figure 2); the largest, about 99 thousand tons of phosphoric acid in September, is feasible only if the contract allows fourth-quarter volume at the third-quarter price (Section 3.5).

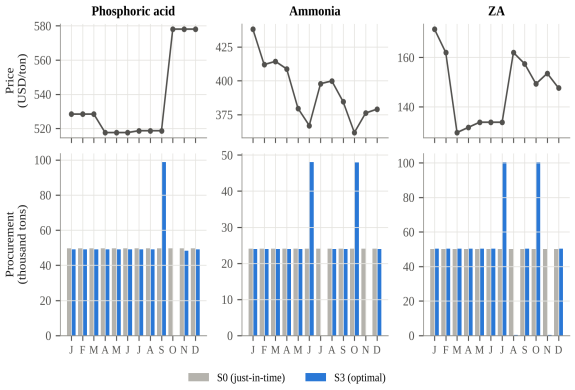


Figure 2. Monthly prices (top, USD/ton) and procurement in scenarios S0 and S3 (bottom, thousand tons) of phosphoric acid, ammonia, and ZA in 2024

3.2 Optimal Production Mix and Shadow Prices

Table 4. Actual and optimal (S3) production mix in 2024

Product	Actual (ton)	Optimal S3 (ton)	Change (%)	Margin (USD/ton)	Margin per ton T15-eq. (USD)	Solution status
NPK 15-10-12	2,138,918	2,160,789	+1.0	112.8	107.2	Balancing
NPK 15-15-15	59,560	71,472	+20.0	167.9	167.9	Upper bound
NPS	32,012	25,610	−20.0	46.5	32.6	Lower bound
DAP	85,960	68,768	−20.0	42.3	28.2	Lower bound
SP-36	6,899	8,279	+20.0	69.3	n/a	Upper bound
SP-20	0	0	–	174.1	n/a	Zero ($R_p = 0$)
Total	2,323,349	2,334,918	+0.5			

NPK 15-15-15 and SP-36 rose to the upper band, NPS and DAP fell to the lower band, and NPK 15-10-12 filled the remaining capacity (Table 4). SP-20 has the highest margin per ton (USD 174.1), because it consists mainly of phosphate rock and sulfuric acid while its reference is a delivered price in Brazil, but it was not produced in 2024 ($R_p = 0$). The shadow price of capacity ranged from USD 56 to 135 per ton T15-equivalent (mean 107.5), close to the margin per unit of capacity of the balancing product (USD 107.2); it is unique and constant for capacity increases of up to 17% in nine months (Table S6).

3.3 Multi-Year Results amd Dynamic Analysis

The The bound is much larger in volatile years (Table 5): S3 raised the margin by USD 67.01 million on a small base of USD 112.41 million in 2021 (+59.6%), and by USD 64.28 million (+22.3%) and USD 14.23 million (+10.4%) in 2022 and 2023. The S2 gain was USD 12.68 million (+11.28%) in 2021, when NPK 15-15-15 and SP-36 were 28% of output, but USD 3.54 million (+1.38%) in 2024. With the CRU series in every month, the four-year S1 bound falls from USD 120.05 to 91.33 million (Table S2).

Table 5. Margin increase relative to S0 (%) with perfect foresight (PF) and with rolling-horizon forecasts, 2021–2024

Year	Margin S0 (million USD)	S3 PF ΔZ (million USD)	S1 PF	S2	S3 PF	S3 naive	S3 MA	S3 drift	S3 ARIMA	Rule (13)
2021	112.41	67.01	+48.73	+11.28	+59.61	+11.28	+14.58	+29.93	+29.11	+22.25
2022	288.35	64.28	+18.31	+3.99	+22.29	+3.98	+10.20	−5.61	+3.87	+5.94
2023	136.50	14.23	+4.71	+5.71	+10.42	+5.69	−9.90	−3.47	+5.48	+5.50
2024	256.98	9.58	+2.35	+1.38	+3.73	+1.38	+0.37	−0.16	−0.11	+0.51
Total (million USD)	794.24	155.10	120.05	35.54	155.10	35.45	33.21	12.31	51.07	50.95
Share of S3 PF (%)			77	23	100	23	21	8	33	33
Timing only (million USD)			120.05			0.00	−2.09	−22.71	15.93	15.75

Note: PF = perfect foresight (upper bound); naive = random walk; MA = three-month moving average; drift = extrapolated three-month change. Rule (13): threshold rule with ARIMA and $\theta = 2\%$, selected ex post from $\{0{,}2\%, 5\%\}$; totals for $\theta = 0$, 2%, 5% (million USD): ARIMA 50.61, 50.95, 43.95; MA 28.13, 38.84, 35.90; drift 8.33, 27.93, 35.15. Timing only: separate runs with the mix fixed at S0 and the same forecasts and inventory state (Rule (13): actual mix); by year in Table S4. Naive differs from S2 only through the real-time price series.

The dynamic analysis (Figure 3) applies the 2024 plant to 129 months of prices. The mix sits at the same band limits in most months, but DAP rises to the upper band in 20 months, mainly in 2022 and mid-2025, and the shadow price of capacity peaks at USD 232/ton in April 2022. Because capacity is constant and shutdowns and seasonality are ignored, this indicates when capacity is worth most under these assumptions, not when maintenance should be scheduled. Monthly mix adjustment adds USD 32.69 million (2.01%) without price foresight.

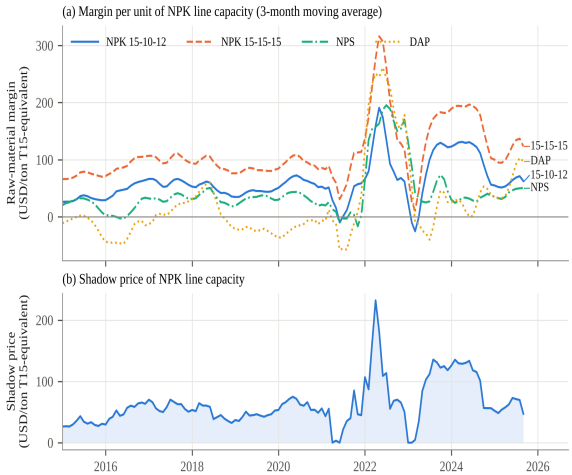


Figure 3. Counterfactual monthly analysis with the 2024 plant (constant capacity, no shutdowns or seasonality): margin per unit of capacity (3-month moving average) and shadow price of NPK line capacity, 2015–2025

3.4 Value Attainable without Perfect Foresight (RQ3)

The naive forecast never buys ahead and captures only the mix gain: USD 35.45 million, 23% of the four-year bound of USD 155.10 million (Table 5). ARIMA captured 33%, MA 21%, and drift 8%. On timing alone, measured with the mix fixed at S0 (Table 5, last row), ARIMA recovered USD 15.93 million, 13% of the S1 bound, whereas MA and drift lost USD 2.09 and 22.71 million, and all three lost in 2023; these values differ by 0.2–0.5 million from the S3 forecast totals minus the S2 total, because timing and mix interact slightly under forecasts. One-step forecast errors of 8.4–12.2% (naive and ARIMA) are as large as the monthly price changes (Table S8). The threshold rule with ARIMA also captured 33%, with a timing part of USD 15.75 million, but its margin $\theta = 2\%$ was selected ex post, so this result is optimistic (Table S4). In 300 simulated years, the S3 bound had a median of USD 16.9 million, S2 was always positive, and the forecast-based plans captured 9–16% of the bound on average, less than the 21% from mix re-optimization alone (Table S11).

3.5 Sensitivity and Robustness

Phosphoric acid is the largest cost driver (46.5% of the S3 procurement cost): a 10% rise in its price in every month lowers the margin by 11.73%, followed by KCl (−5.35%), ammonia (−4.22%), and ZA (−3.29%), and the plan does not change (Figure 4a). Single-month shocks affect timing (Table S7), the S1 gain rose from 0 with flat prices to 5.30% at twice the observed volatility (Figure 4b), and the S2 gain grows linearly with the band width (Figure 4c).

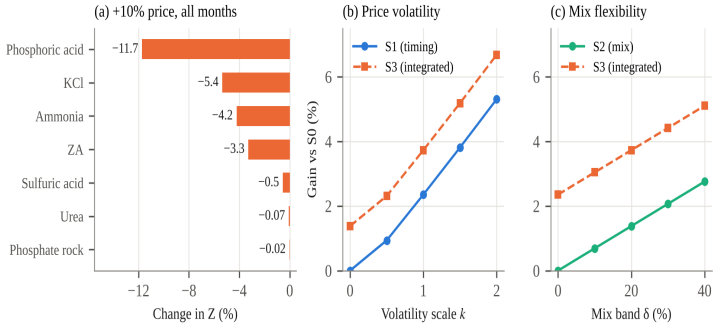


Figure 4. Sensitivity of the 2024 margin: (a) 10% price increase in every month; (b) gains under rescaled price volatility; (c) gains under different mix flexibility bands

Table 6. Robustness of the 2024 gains to price series, operational constraints, and valuation

Variant	Margin S0 (million USD)	S1 ΔZ (million USD)	S2 ΔZ (million USD)	S3 ΔZ (million USD)	S3 ΔZ (%)
Main model (Table 3)	256.98	6.04	3.54	9.58	+3.73
CRU series in every month	256.07	7.35	3.49	10.82	+4.23
Internal supply fixed	256.98	5.59	3.54	9.13	+3.55
Quarterly phosphoric acid contracts	256.98	3.64	3.54	7.18	+2.79
Tank storage net of safety stock	256.98	4.71	3.54	8.24	+3.21
Annual commitment, $\delta_a = 5\%$	256.98	6.04	4.33	10.35	+4.03
NPK 15-10-12 at 80% of reference value	88.44	6.04	2.38	8.41	+9.52
NPK 15-10-12 at 120% of reference value	425.52	6.04	5.71	11.75	+2.76
Combined operational constraints*	256.98	1.37	3.54	4.91	+1.91

*Purchase cap, minimum lot, fixed internal supply, quarterly contracts, and tank storage together. Percentages are relative to the S0 margin of each variant; all variants are in Table S3.

All operational constraints except the minimum lot reduce the timing gain (Table 6), most of all quarterly phosphoric acid contracts (S1 from USD 6.04 to 3.64 million). Fixing the internal supply lowers S1 to USD 5.59 million, so buying the internal share ahead adds USD 0.45 million to the bound. Combined, the constraints leave an S3 bound of USD 4.91 million (1.91%), about half of the main result. The mix gain depends instead on the valuation of the subsidized grade (USD 2.38 or 5.71 million at 80% or 120%). Against the company's actual purchase profile (S0-actual), the 2024 S1 and S3 gains would be 2.83% and 4.21%, so S0 is a slightly favorable baseline (Table S9).

3.6 Discussion and Limitations

The timing bound corresponds to procurement savings of 1.1% (2024) to 7.6% (2021), which brackets the 4.33% of Mandl and Minner [12], and the larger gains in volatile years agree with Devalkar *et al.* [11]. Little of it is attainable because forecast errors are as large as the price changes that timing exploits [7], so the deterministic optimum of such LPs should be reported as a bound and compared with a rolling-horizon policy [17]. The mix gain needs no price forecast, but its band is centred on the annual volumes R_p, so it assumes that these commitments are known in advance; in practice they come from the subsidy allocation and the sales plan set at the start of the year. Revenue from subsidized NPK is set by the regulated retail price, cut by 20% in October 2025 [31], and the subsidy payment, so the valuation of NPK 15-10-12 shifts the mix gain but not the timing gain.

The model is deterministic and retrospective, assumes purchases at the realized monthly price, and represents contracts, lead times, and storage only through the variants above; S0 has a flat profile, and the margin excludes non-material costs. Several of these assumptions overstate the timing gain, which is why it is reported as a bound. In practice, the monthly grade mix should be re-optimized at current prices, and buying ahead should be limited to clear cases, for example by rule (13) with a margin set on past data.

4. CONCLUSION

The integrated multi-period LP was evaluated against perfect-foresight bounds and rolling-horizon policies. On RQ1, integration with perfect foresight would have raised the 2024 raw-material contribution margin by USD 9.58 million (3.73%), and by much more in 2021–2023. On RQ2, timing gave about two-thirds of this bound, and the timing and mix effects were additive. On RQ3, plans using only information available at decision time captured 8–33% of the 2021–2024 bound, mostly through the mix; timing alone recovered at most 13% of its bound and lost value when prices fell. Monthly mix re-optimization, which needs no price forecast, is therefore the robust source of value, whereas procurement timing is valuable mainly in hindsight. Extensions include stochastic purchase timing and contract terms in a mixed-integer model.

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