Zero : Jurnal Sains, Matematika, dan Terapan
E-ISSN : 2580-5754; P-ISSN : 2580-569X
Volume 10, Number 1, 2026
DOI: 10.30829/zero.v10i1.28726
Page: 265-277

Actuarial Evaluation of Additional Contributions in Early Retirement Programs Using the Spreading Gains and Losses Method

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Article Info

Article history:

Received 25 March 2026

Keywords:

Accelerated retirement;
Actuarial liability;
Pension funding;
Projected Unit Credit;
Spreading Gains and Losses;
Unfunded Actuarial Liability.

ABSTRACT

This study examines the actuarial and funding implications of accelerated retirement in a defined benefit pension scheme by integrating the Projected Unit Credit (PUC) method with the Spreading Gains and Losses approach. While both methods are widely applied in pension valuation, limited empirical evidence evaluates their combined implementation under retirement age acceleration scenarios, particularly in Indonesian public sector schemes. This study addresses that gap using secondary administrative employment data of 87 female civil servants obtained from the Investment and One-Stop Integrated Services Office of Lampung Province (Dinas Penanaman Modal dan Pelayanan Terpadu Satu Pintu Provinsi Lampung), grouped into four entry-age cohorts (22–25 years). The analysis compares normal retirement at age 58 with accelerated retirement at age 50, assuming a 5% annual effective interest rate and 8% biennial salary growth. The results indicate that, at valuation age 45, actuarial liabilities increase by approximately 49.8% under retirement at age 50 relative to age 58. The shorter discounting period and earlier benefit payments outweigh the reduced contribution period, resulting in the emergence of Unfunded Actuarial Liability (UAL). The resulting Past Service Liability (PSL) is amortized over five years, requiring additional contributions ranging from IDR 27.06 million to IDR 82.05 million across entry-age groups. These findings highlight the high sensitivity of pension funding to retirement age assumptions and emphasize the importance of actuarial impact assessments prior to policy implementation. However, the deterministic framework and relatively small sample size limit broader generalization of the results.

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1. INTRODUCTION

According to the Central Statistics Agency (BPS), the increasing proportion of individuals aged above 64 years reflects demographic aging that elevates old-age dependency ratios and poses significant sustainability challenges for defined benefit pension systems[1]. The continued rise in life expectancy and the growing share of elderly individuals intensify financial pressure on pension systems, particularly defined benefit schemes that guarantee post-retirement income. As the ratio of retirees to contributors increases, maintaining pension sustainability becomes increasingly challenging.

A pension program is a legal entity responsible for managing retirement benefits for participants who have completed their working years. In practice, retirement does not always occur at the normal retirement age, as early or accelerated retirement may arise due to health conditions, organizational restructuring, workforce regeneration, or fiscal efficiency policies [2]. In Indonesia, early retirement has been widely implemented as a strategic instrument in public sector workforce management to improve structural efficiency. Similar patterns are observed internationally, particularly in OECD countries, where early retirement incentives have historically increased pension expenditures and triggered reforms aimed at extending retirement ages. Empirical evidence indicates that accelerated retirement tends to increase pension liabilities when not accompanied by adequate actuarial adjustments [3].

From an actuarial perspective, accelerated retirement significantly alters pension funding dynamics. Earlier benefit payments shorten the contribution accumulation period and reduce the discounting horizon, leading to higher actuarial liabilities within a shorter timeframe. This condition increases the risk of asset–liability mismatches and may generate Unfunded Actuarial Liability (UAL), thereby threatening long-term pension sustainability. Consequently, rigorous actuarial evaluation is required to assess the financial implications of retirement age changes. In defined benefit pension schemes, actuarial liabilities are commonly estimated using the Projected Unit Credit (PUC) method, which projects future benefits based on salary growth and allocates them proportionally to years of service [4]. Changes in retirement age represent a critical actuarial assumption that can generate actuarial gains or losses and result in UAL. To mitigate funding volatility, the Spreading Gains and Losses approach is often applied to amortize UAL over multiple periods, ensuring contribution stability and reducing short-term financial shocks [5].

Previous studies have examined these methods separately. Research on the Projected Unit Credit method shows that salary growth assumptions significantly influence pension benefits while studies on the Spreading Gains and Losses approach indicate potential long-term funding risks under certain actuarial return assumptions [6]. However, limited studies have integrated these approaches within the context of accelerated retirement. In particular, there is a lack of empirical analysis that simultaneously evaluates the impact of retirement age changes on actuarial liabilities, UAL formation, and supplementary contribution requirements within a structured amortization framework. Moreover, existing studies largely focus on normal retirement conditions, with limited comparison to accelerated retirement scenarios.

This gap is increasingly relevant given demographic aging, fiscal constraints, and the growing adoption of early retirement policies. Without integrated actuarial analysis, pension funds risk underestimating the financial consequences of retirement age adjustments. Therefore, a comprehensive evaluation that combines liability estimation and funding adjustment mechanisms is essential to support sustainable pension management. Based on these considerations, this study aims to analyze pension funding under accelerated retirement conditions by estimating normal contributions and actuarial liabilities using the Projected Unit Credit (PUC) method, comparing liabilities between normal and accelerated retirement ages, and determining supplementary contributions using the Spreading Gains and Losses approach. By integrating these methods, this research provides an assessment of the financial implications of accelerated retirement policies and offers practical insights for maintaining pension sustainability.

2. RESEARCH METHOD

2.1 Data and Research Procedure

The dataset comprises administrative employment records of civil servants (PNS) of Lampung Province, officially obtained from the Investment and One-Stop Integrated Service Office of Lampung Province (DPMPTSP), Indonesia the data were collected in 2023, and all participants had their official starting date of appointment as civil servant candidates (CPNS) in 2023. The participants are employees with entry ages of 22, 23, 24, and 25 years. Each participant has an initial basic monthly salary of IDR 3,375,300. A total of 37 participants were grouped into four entry-age categories, as presented in Table 1.

Table 1. Research Data

Age Group	Age at Entry (e)	Number of Participants (S_0)	Initial Monthly Salary
A	22	6	Rp3.375.300
B	23	16	Rp3.375.300
C	24	6	Rp3.375.300
D	25	9	Rp3.375.300

A total of 37 participants were classified into four entry-age categories, as shown in Table 1. The sample reflects the availability of complete administrative records within the selected employment category. Although relatively small, the dataset is homogeneous in rank and salary structure, allowing for controlled actuarial comparisons across entry-age groups. However, the limited sample size may constrain broader statistical generalization beyond similar employment classifications.

A formal power analysis is not the primary basis of this study, as it adopts a deterministic actuarial framework. The sample of 37 participants reflects data availability rather than statistical sampling design. Nevertheless, under standard assumptions ($\alpha = 0.05$, moderate effect size), the sample size implies limited statistical power, indicating that the study is exploratory and focused on actuarial modeling rather than statistical inference.

Consistent with prevailing civil service regulations in Indonesia, the statutory retirement age for civil servants (PNS) is 58 years, which is adopted as the normal retirement benchmark in this analysis. In addition, the study evaluates a structured retirement acceleration scenario in which participants submit an early retirement application at age 45, with the effective retirement occurring at age 50. This framework enables a controlled comparison between normal and accelerated retirement within a fixed regulatory setting, with the primary objective of quantifying the additional contribution required to restore funding adequacy under each retirement scenario.

The research procedure adopted to obtain the desired results is structured as follows:

a. Data Collection
b. Construction of the Mortality Table and Development of the Commutation Functions
c. Projection of Future Salaries
d. Calculation of Pension Benefits (B_r) and Present Value of Future Benefits (PVFB)
e. Calculation of Normal Cost (NC)
f. Calculation of Actuarial Liability (AL)
g. Determination of Unfunded Actuarial Liability (UAL)
h. Calculation of Past Service Liability (PSL)
i. Application of the Spreading Gains and Losses Method
j. Conclusion Drawing

2.2 Mortality Table and Commutation Functions

A mortality table provides a systematic representation of the probability of death and survival of individuals at each age. It serves as a fundamental actuarial tool for estimating life expectancy and calculating present values of future benefits in pension and insurance programs [7]. The mortality table used in this study is the Indonesian Mortality Table (TMI) IV 2019 for women. In a mortality table, the symbol $x$denotes the exact age of an individual who has survived to age x. The probability that an individual aged $x$will die before reaching age $x + 1$ is denoted by q_x. Conversely, the probability that an individual aged $x$will survive to age $x + 1$ is denoted by p_x. These two probabilities are complementary and satisfy the following relationship:

$$p_x = 1 - q_x \tag{2.1}$$

The number of individuals who die between age x and $x + 1$ is represented by d_x. This value is obtained by multiplying the probability of death by the number of individuals alive at age x, denoted by l_x. Mathematically, this relationship is expressed as:

$$d_x = q_x l_x \tag{2.2}$$

In constructing a mortality table, the initial number of lives at age 0, denoted by l_0, is commonly assumed to be 100,000 individuals. The number of survivors at age $x + 1$ is then calculated by subtracting the number of deaths during the year from the number of individuals alive at age x, as follows:

$$l_{x+1} = l_x - d_x \tag{2.3}$$

These fundamental actuarial relationships form the basis for deriving commutation functions, life expectancy, and present value calculations required in pension fund valuation. Commutation functions are actuarial tools used to simplify the calculation of present values in life insurance and pension mathematics. These functions are derived from the mortality table and incorporate the effect of discounting based on an assumed interest rate. The commutation function $D_x$represents the present value of a payment of one unit made to each individual who is alive at exact age x. D_x reflects the discounted value of the cohort surviving to age x, combining both time value of money and mortality assumptions. It is defined as follows [8]:

$$D_x = v^x l_x \tag{2.4}$$

The commutation function N_x represents the cumulative sum of $D_x$from age $x$up to the highest attainable age in the mortality table. The function N_x therefore represents the total present value of a series of unit payments made to individuals who survive from age $x$onward. These commutation functions form the basis for calculating actuarial present values of life annuities and pension benefits. It is expressed as:

$$N_x = \sum_{t=0}^{\infty} D_{x+t} \tag{2.5}$$

2.3 Salary Function and Benefit Function

Salary represents the compensation received by employees for work or services that have been and will be provided. It aims to ensure a decent standard of living for workers and is determined based on mutual agreement, statutory regulations, and employment contracts between employers and employees [9]. In this study, the salary of a participant aged x is denoted by s_x. The salary progression follows a periodic increment scheme in which salary increases occur once every two years. At the beginning of employment, participants receive an initial salary denoted by s_e, where $e$represents the age at entry into employment. Salary adjustments do not occur annually but only at the end of every two-year working period. Consequently, during odd years of service, the salary remains equal to that received in the preceding even year. Let u denote the rate of salary increase applied every two years. The salary at age x therefore depends on the number of completed two-year periods of service.

$$s_x = \begin{cases} s_e(1+u)^{\frac{(x-e)-1}{2}}, x-e = 2z+1 \\ s_e(1+u)^{\frac{(x-e)}{2}}, x-e = 2z \end{cases} \tag{2.6}$$

Thus, the exponent represents the number of two-year salary increase periods completed since entry into employment. To determine the final salary prior to retirement at age $r-1$, a similar formulation is applied.

$$s_{r-1} = \begin{cases} s_e(1+u)^{\frac{(r-e-2)}{2}}, r-e-1 = 2z+1 \\ s_e(1+u)^{\frac{(r-e-1)}{2}}, r-e-1 = 2z \end{cases} \tag{2.7}$$

Therefore, the difference $r-e-1$ determines whether the total years of service before retirement constitute an odd or even number, which in turn determines the total number of salary increases received. This salary function is essential for projecting pension benefits, as final salary directly influences the calculation of retirement benefits under defined benefit pension schemes.

The pension benefit at retirement, denoted by B_r, is determined based on the final salary and the total years of service. Let k denote the accrual rate (proportion of salary allocated per year of service). Under the final salary scheme, the retirement benefit is expressed as [5]

$$B_r = k\,(r-e)\,s_{r-1} \tag{2.8}$$

Similarly, for accelerated retirement at age r', the pension benefit is given by

$$B_{r'} = k(r'-e)s_{r'-1} \tag{2.9}$$

These expressions show that pension benefits are directly influenced by three key factors: the length of service $(r-e)$or $(r'-e)$, the projected final salary, and the accrual rate k. Under accelerated retirement, the shorter service period and lower projected final salary generally lead to smaller pension benefits compared to normal retirement. However, because benefits are paid earlier and discounted over a shorter period, the present value of liabilities may increase despite the lower nominal benefit amount.

2.4 Present Value of Future Benefits (PVFB)

The Present Value of Future Benefits (PVFB) represents the actuarial present value of all pension benefits that are expected to be paid to a participant in the future. In defined benefit pension schemes, PVFB forms the fundamental basis for determining normal cost, actuarial liability, and funding requirements. To determine the present value at age $x < r$, the benefit must be discounted back $r-x$years and adjusted for survival probability. Thus, the PVFB at age x is :

$$(PVFB)_x = B_r\, v^{r-x}\; {}_{r-x}p_x\, \ddot{a}_r \tag{2.10}$$

The term v^{r-x} reflects financial discounting from retirement age to valuation age, while ${}_{r-x}p_x$ represents the probability that the participant survives from age x to retirement age r. If retirement occurs earlier at age $r' < r$, the PVFB becomes

$$(PVFB)_x^{ACC} = B_{r'}\, v^{r'-x}\; {}_{r'-x}p_x\, \ddot{a}_r \tag{2.11}$$

Although $B_{r'}$ is generally smaller due to shorter service and lower final salary, the shorter discount period $(r'-x)$and higher survival probability to retirement may result in a larger present value at valuation age. This mechanism explains why accelerated retirement can increase actuarial liabilities despite lower nominal benefits.

2.5 Projected Unit Credit Method

The Projected Unit Credit (PUC) method is an actuarial cost allocation method commonly used in defined benefit pension valuation. Under this method, the total projected pension benefit at retirement is attributed proportionally to each year of service [10]. The method recognizes that pension benefits accrue gradually over the participant's working lifetime and therefore allocates the present value of benefits in proportion to completed service. The Normal Cost at age x, denoted by NC_x, represents the cost of the additional year of service accrued between age $x$and $x+1$. Under the PUC method, it is calculated as the increment in actuarial liability due to one additional year of service. Mathematically,

$$NC_x = \frac{1}{r-e}\,(PVFB)_x \tag{2.12}$$

Thus, the normal cost is constant as a proportion of the projected total benefit accrual but may vary in monetary value due to changes in PVFB over time. In the PUC method, the Actuarial Liability at age x, denoted by AL_x, is defined as the portion of the PVFB attributable to completed service $(x-e)$. Therefore,

$$AL_x = \frac{x-e}{r-e}\,(PVFB)_x \tag{2.13}$$

Accordingly, actuarial liability under accelerated retirement is

$$AL_x^{(acc)} = \frac{x-e}{r'-e}(PVFB)_x^{(acc)}. \tag{2.14}$$

Because $r'-x < r-x$, the discount period becomes shorter, which may increase the present value of benefits and consequently increase actuarial liabilities despite the reduction in total years of service.

2.6 Unfunded Actuarial Liability (UAL)

Unfunded Actuarial Liability (UAL) represents the portion of actuarial liability that is not covered by the available pension fund assets. It is defined as the difference between the total actuarial liability and the accumulated assets of the pension fund at a given valuation date. Let AL_x denote the actuarial liability at age x, and let $F_t$denote the actuarial value of pension fund assets at time t. The Unfunded Actuarial Liability is expressed as

$$UAL_r = AL_r - F_t. \tag{2.15}$$

Under normal retirement conditions, the pension fund is often assumed to be in actuarial equilibrium at retirement age r. In this case, the actuarial liability equals the available assets, resulting in

$$UAL_r = 0 \tag{2.16}$$

or equivalently,

$$AL_r = F_t \tag{2.17}$$

When this condition holds, the pension plan is said to be fully funded. A fully funded status indicates that the accumulated contributions and investment returns are sufficient to finance the present value of accrued pension benefits, ensuring financial stability without deficit or surplus. In the context of accelerated retirement, changes in retirement age may increase actuarial liabilities due to shorter discounting periods and earlier benefit payments.

2.7 Past Service Liability (PSL)

Past Service Liability reflects the obligation for participants' pension rights based on their length of service up to the time of assessment PSL arises due to changes in retirement age, namely when the actuarial liability at retirement is accelerated and is greater than the obligation at normal retirement [8]

$$\begin{aligned} PSL &= UAL_x^{r'} \\ &= AL_x^{r'} - F_t \\ &= AL_x^{r'} - AL_x^{r} \end{aligned} \tag{2.18}$$

Let AL_x^r denote the actuarial liability at age $x$under the assumption of normal retirement age r, and let $AL_x^{r'}$denote the actuarial liability at age x under the assumption of accelerated retirement age r'. Furthermore, let F_t represent the actuarial value of pension fund assets at time t, while $UAL_x^{r'}$ denotes the Unfunded Actuarial Liability calculated under the accelerated retirement assumption.

This formulation shows that PSL represents the incremental actuarial liability resulting from the change in retirement age assumption. If $AL_x^{r'} > AL_x^r$, then $PSL > 0$, indicating that previously accumulated assets are insufficient to finance the newly increased obligation. In such a case, the difference constitutes an actuarial loss attributable to past service. Thus, PSL measures the financial impact of policy changes on accrued benefits and becomes the basis for determining additional funding requirements, typically amortized over future periods through supplementary contributions [11].

2.8 Spreading Gains and Losses Method

The Spreading Gains and Losses method is used to determine the supplementary contribution (SC) arising from gains or losses. Through this method, the amount of gains or losses is not charged all at once in one year, but is spread proportionally over several future periods. In the case of accelerated retirement, there is an increase in liabilities that causes actuarial *losses* for the pension fund. The amount of the loss incurred is equal to the past service liability value shown in Equation (2.18). Supplementary contributions are additional contributions paid by employers that arise due to gains or losses from actuarial liabilities [10]

$$SC = \frac{L_x}{\ddot{a}_{x:\overline{n}|}}$$

$$= \frac{AL_x^{r'} - AL_x^r}{\ddot{a}_{x:\overline{n|}}} \quad (2.19)$$

Here, SC refers to the supplementary contribution, L_x denotes the actuarial loss at age x, and $\ddot{a}_{x:\overline{n|}}$ represent the present value of an n – year temporary life annuity due for individual aged x.

3. RESULT AND ANALYSIS

3.1 Completing the Mortality Table and Comuutation Function

The mortality table is a description of the probability of death and survival of a person based on a certain age. The mortality table is used as the basis for calculating life expectancy in the future[7]. The symbol x in the mortality table indicates that a person has lived for x years. The discount factor is used to calculate the present value of benefits or payments to be received in the future, taking into account the interest rate. The interest rate used is 5%. Based on equation (2.1) – (2.5) , the values of the probability of survival, the number of people alive, the number of people who died and commutation function are obtained in the following table.

Table 3. Mortality Table and Commutation Functions

Age	q_x	p_x	l_x	d_x	D_x	N_x
0	0.00266	0.99734	100,000	266	100.000	2.041.834,00610
1	0.00041	0.99959	99,734	40.89094	94.984,76190	1.941.834,00610
2	0.00031	0.99969	99,693.10906	30.90486	90.424,58872	1.846.849,24419
3	0.00021	0.99976	99,662.20420	23.91893	86.091,95914	1.756.424,65548
…	…	…	…	…	…	…
108	0.50427	0.49573	163.52772	82.46212	0,84168	1,47910
109	0.54477	0.45523	81.06560	44.16210	0,39738	0,63742
110	0.58702	0.41298	36.90349	21.66309	0,17228	0,24005
111	1	0	15.24040	15.24040	0,06776	0,06776

Based on Table 3, The probability of death (q_x) increases with age, while the probability of survival (p_x) decreases correspondingly, reflecting the natural rise in mortality risk at older ages. For example, at age 0, the probability of survival is 0.99734, indicating a very high likelihood of surviving to age 1. In contrast, at age 110, the probability of survival declines significantly to 0.41298. This inverse relationship between $q_x$and $p_x$directly influences the number of survivors (l_x), which decreases progressively from the initial cohort of 100,000 individuals at age 0 to only 15.24040 individuals at age 111. As age increases, $D_x$declines due to the combined effects of decreasing survival probabilities and financial discounting. Similarly, N_x, which represents the cumulative present value of future payments from age $x$onward, decreases substantially at higher ages and approaches zero near the limiting age.

3.2 Calculating the Final Salary Projection

The salary function defined in Equation (2.6) reflects a structured biennial growth mechanism, where salary increases occur only after the completion of each two-year service period. Unlike an annual compound growth model, this approach introduces a stepwise salary progression pattern. As a result, salary levels remain constant during odd years of service and increase only at even service intervals. This structure creates a piecewise exponential function in which the number of completed two-year periods determines the exponent of the growth factor $(1+u)$. Then, the final salary before normal retirement at age 57 and accelerated retirement at age 49 for age groups A, B, C, and D is as follows.

Table 4. Research Data

Age Before Retirement $(r-1)$	Last Salary (s_{r-1})			
	${}_A s_{r-1}$	${}_B s_{r-1}$	${}_C s_{r-1}$	${}_D s_{r-1}$
49	59.480.902,1162	59.480.902,1162	57.748.448,6565	57.748.448,6565
57	66.946.279,3585	66.946.279,3585	64.996.387,7267	64.996.387,7267

Based on Table 4, participants who opt for early retirement at age 49 obtain a lower projected final salary compared to those who continue working until the normal retirement age of 57. For instance, in Group A (entry age 22), the projected annual salary before early retirement is approximately IDR 59,480,902, whereas postponing retirement until age 57 increases the projected final salary to approximately IDR 66,946,279. This difference arises from the biennial compound salary growth mechanism, where salary increases are applied every two years at a fixed rate. A longer employment period allows for more completed increment cycles, thereby increasing the exponent in the compound growth and resulting in a substantially higher final salary. These findings confirm that the length of service plays a decisive role in salary accumulation under a compound growth structure.

The multiplicative effect of repeated salary increments amplifies differences in retirement timing, making even a few additional years of service financially significant. Conversely, early retirement shortens the accumulation period, reduces the number of completed salary increment cycles, and ultimately lowers the final projected salary. From an actuarial perspective, this reduction directly affects the pension benefit under a final salary defined benefit scheme, as the retirement benefit is proportional to the last salary received before retirement.

3.3 Pension Benefits

The calculation of pension benefits is conducted under two retirement scenarios: normal retirement at age 58 (r) and accelerated retirement at age 50 (r'). Since the pension scheme applied in this study follows a final salary defined benefit structure, the amount of pension benefit depends directly on the total years of service and the final salary prior to retirement. The resulting pension benefit values for each entry-age group are presented in Table 6.

Table 5. Retirement Benefits

Age Group	$r' = 50$	$r = 58$
	B_{50}	B_{58}
A	41.636.631,4813	60.251.651,4226
B	40.149.608,9284	58.577.994,4387
C	37.536.491,6267	55.246.929,5677
D	36.092.780,4103	53.622.019,8745

Table 5 confirms that pension benefits under the normal retirement scenario (age 58) are consistently higher than those under accelerated retirement (age 50) across all entry-age groups. This pattern is consistent with the final salary defined benefit formula $B_r = k(r - e)s_{r-1}$, where both the length of service and final salary growth determine the pension amount. These findings align with prior studies showing that final salary growth influences pension benefits under the Projected Unit Credit method [6]. However, unlike research focusing on salary sensitivity or investment return risk [7], this study identifies retirement age acceleration as a key policy variable affecting actuarial liabilities and supplementary contributions within the Spreading Gains and Losses framework. The results indicate that early retirement, despite reducing nominal benefits, increases funding pressure due to earlier payment timing. As the analysis adopts a deterministic defined benefit framework, future research may incorporate stochastic modeling to assess funding resilience under economic uncertainty.

3.4 Present Value of Future Benefit (PVFB)

The present value of future pension benefits (PVFB) for each entry-age group under normal retirement is presented in Table 7. PVFB reflects the actuarial present value of retirement benefits discounted to the participant's current age. The results indicate that younger entry ages produce lower PVFB due to a longer discounting period, while PVFB increases significantly as participants approach retirement. This pattern underscores the need for careful funding management.

Table 6. PFVB Value at Normal Retirement

Age	***Present Value of Future Benefit* (PVFB)**			
(x)	${}^{58}_{A}(PVFB)_x$	${}^{58}_{B}(PVFB)_x$	${}^{58}_{C}(PVFB)_x$	${}^{58}_{D}(PVFB)_x$
22	145.841.440,7278	-	-	-
23	153.179.466,5744	148.924.481,3918	-	-
24	160.889.924,6968	156.420.760,1219	147.525.820,9194	-
25	168.991.878,1913	164.297.659,3527	154.954.796,6156	150.397.302,5975
…	…	…	…	…
45	456.200.702,4476	443.528.460,7129	418.307.008,7167	406.003.861,4015
…	…	…	…	…
56	808.034.702,2318	785.589.293,8365	740.916.393,6322	719.124.734,9959
57	852.905.622,6929	829.213.838,7292	782.060.208,5102	759.058.437,6717
58	900.621.445,3272	875.604.182,9570	825.812.544,2590	801.523.940,0161

Based on Table 6, this increasing trend occurs because the time interval between the valuation age and retirement age becomes shorter, thereby reducing the discount period. A shorter discounting horizon increases the present value factor, which in turn elevates the PVFB. Although the probability of survival to retirement age slightly decreases as age increases, its impact is smaller compared to the reduction in the discount period and the growth in projected pension benefits due to longer service duration. Consequently, the PVFB demonstrates an upward pattern as the participant approaches retirement age.

Furthermore, Table 7 presents the PVFB values at age 45 under the accelerated retirement scenario (retirement at age 50). Under this condition, pension benefits commence earlier, shortening the time between the valuation age and the benefit payment period. As a result, the discounting period is reduced, leading to a higher present value compared to the normal retirement scenario at the same valuation age. In contrast, under normal retirement, benefits begin at a later age, increasing the discount period and lowering the PVFB.

Tabel 7. PFVB Values for Early Retirement at Age 45

Age Group	*Present Value of Future Benefit* (PVFB)
	${}^{50}(PVFB)_{45}$
A	531,500,570.7665
B	512,518,407.5248
C	479,161,402.2167
D	460,732,117.5160

Additionally, survival probability plays a role in determining PVFB. The probability of surviving to the accelerated retirement age (50) is higher than the probability of surviving to the normal retirement age (58), which further increases the PVFB under early retirement. Therefore, although accelerated retirement results in lower nominal pension benefits, the shorter discounting period and higher survival probability can lead to a comparatively larger present value at certain valuation ages. These findings highlight the sensitivity of PVFB to changes in retirement age assumptions and confirm its crucial role in determining actuarial liabilities and funding adequacy.

3.5 Normal Contribution Using the *Projected Unit Credit Method*

The Normal Cost (NC_x) is calculated for each entry-age group from the age at entry into employment up to age 57, which represents the period prior to retirement. The results of these calculations are presented in Table8.

Tabel 8. Normal Cost for Each Age Group

Age	Iuran Cost (NC_x)			
(x)	${}_{A}NC_x$	${}_{B}NC_x$	${}_{C}NC_x$	${}_{D}NC_x$
22	4.051.151,1313	-	-	-
23	4.254.985,1826	4.254.985,1826	-	-
24	4.469.164,5749	4.469.164,5749	4.338.994,7329	-
25	4.694.218,8386	4.694.218,8386	4.557.494,0181	4.557.494,0181
…	…	…	…	…
45	12.672.241,7347	12.672.241,7347	12.303.147,3152	12.303.147,3152
…	…	…	…	…
56	22.445.408,3953	22.445.408,3953	21.791.658,6362	21.791.658,6362
57	23.691.823,9637	23.691.823,9637	23.001.770,8385	23.001.770,8385

Based on Table 8, it can be observed that the normal contribution increases progressively as participants grow older. At the beginning of employment, the normal cost is relatively low; however, it rises gradually as the participant approaches retirement age. This pattern is consistent with the Projected Unit Credit (PUC) method, where the normal cost represents the portion of the Present Value of Future Benefits (PVFB) allocated to each year of service. Under the PUC framework, changes in PVFB directly influence the magnitude of normal contributions. As participants age, the discounting period to retirement becomes shorter, which increases the PVFB. In addition, projected pension benefits also rise due to longer accumulated service and higher projected salaries. These combined effects result in a higher annual normal cost. Therefore, the upward trend in normal contributions reflects the increasing actuarial value of accrued benefits as participants advance toward retirement age.

3.6 Actuarial Liability Using the Projected Unit Credit

The calculation of the actuarial liability using the Projected Unit Credit method is determined based on the length of service completed divided by the total length of service, which is then multiplied by the present value of the total pension benefit ($PVFB_x$), resulting in the following.

Tabel 9. Actuarial Liability for Each Age Group

Age	Actuarial Liability ($\mathbf{AL_x}$)			
(x)	${}^{58}_{A}AL_x$	${}^{58}_{B}AL_x$	${}^{58}_{C}AL_x$	${}^{58}_{D}AL_x$
22	0	-	-	-
23	4.254.985,1826	0	-	-
24	8.938.329,1498	4.469.164,5749	0	-
25	14.082.656,5159	9.388.437,6773	4.557.494,0181	0
26	19.723.213,9392	14.792.410,4544	9.574.375,6987	4.787.187,8493
…	…	…	…	…
45	291.461.559,8970	278.789.318,1624	258.366.093,6192	246.062.946,3040
…	…	…	…	…
56	763.143.885,4411	740.698.477,0458	697.333.076,3597	675.541.417,7235
57	829.213.838,7292	805.522.014,7656	759.058.437,6717	736.056.666,8332

58	900.621.445,3272	875.604.182,9570	825.812.544,2590	801.523.940,0161

At the age of entry, the actuarial liability for each participant is zero, as no service has yet been rendered to generate accrued pension obligations. As participants grow older and accumulate years of service, the actuarial liability increases progressively. This upward trend is consistent with the growth in the Present Value of Future Benefits (PVFB), which rises due to the shortening of the discount period and the increase in projected pension benefits over time. Among the age groups, Group A records the highest actuarial liability at age 58, amounting to IDR 900,621,445.3272, whereas Group D shows the lowest value at the same age, totaling IDR 801,523,939.6256. These differences correspond to variations in PVFB across groups, as higher PVFB values lead directly to higher actuarial liabilities. The variation arises from differences in projected pension benefits within each group. Nevertheless, despite disparities in magnitude, all groups exhibit a consistent pattern: actuarial liability increases with age as service accrues and the time to retirement decreases.

Next, each group at the age of 45 will apply for early retirement at the age of 50. Therefore, actuarial liabilities for each group at the age of 45 are calculated taking into account this change in retirement age. This results in the actuarial liabilities for early retirement for each age group in Table 10 below.

Tabel 10. Actuarial Liability of each group at age 45

Age Group	Actuarial Liability
	${}^{50}(AL)_{45}$
A	436.589.754,5582
B	417.607.591,3165
C	387.014.978,7135
D	368.585.694,0128

The results presented in the table indicate that actuarial liabilities are consistently higher under accelerated retirement compared to normal retirement for all participant groups. For instance, in Group A at age 45, the actuarial liability under normal retirement amounts to IDR 291,461,559.8970, whereas under accelerated retirement it increases to IDR 436,589,754.5582. A similar pattern is observed across the remaining groups, demonstrating that advancing the retirement age assumption significantly increases actuarial obligations.

The higher actuarial liability under accelerated retirement is primarily attributable to the shorter time horizon to retirement. When retirement occurs earlier, pension benefits are discounted over a shorter period, resulting in a higher present value of future benefits. Consequently, the actuarial liability at age 45 increases. Moreover, the benefit payment phase under accelerated retirement effectively begins sooner, which may extend the expected payout duration relative to the valuation date and further contribute to the increase in actuarial liability. Under both retirement scenarios, Group A records the highest actuarial liability, followed sequentially by the other groups. This ordering is consistent with the corresponding PVFB values, as groups with larger projected pension benefits generate higher actuarial liabilities. Across age groups, a decreasing trend in actuarial liabilities is observed as entry age increases. The calculations show that the average difference between groups is approximately 5.48% under both retirement assumptions, indicating a relatively stable proportional variation across scenarios.

3.7 Unfunded Actuarial Liability (UAL) and Past Service Liability (PSL)

Under normal retirement, actuarial liabilities are fully covered by pension fund assets, resulting in a zero Unfunded Actuarial Liability (UAL) across all age groups at age 45. This indicates a fully funded position up to the normal retirement age of 58. In contrast, under accelerated retirement (age 50), positive UAL values emerge for all groups, reflecting a funding shortfall. Earlier benefit payments shorten the discounting period and reduce the time available for asset accumulation, thereby increasing the present value of liabilities and creating a deficit.

Tabel 11. UAL for Each Group at Age 45

Age Group	*Unfunded Actuarial Liability*	
	UAL_{45}^{58}	UAL_{45}^{50}
A	0	Rp145.128.194,6611
B	0	Rp138.818.273,1541
C	0	Rp128.648.885,0943
D	0	Rp122.522.747,7088

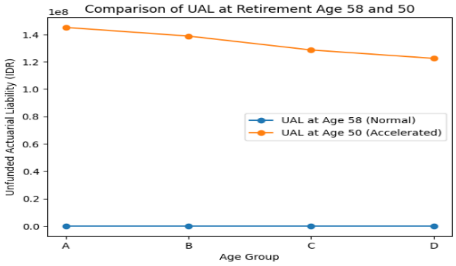

Figure 3.1 Unfunded Actuarial Liability

Specifically, Group A records the highest UAL at age 45 under accelerated retirement, amounting to IDR 145,128,194.6611, indicating the largest funding deficit among the groups. These differences reflect variations in projected pension benefits and actuarial liabilities, where groups with larger accumulated benefits experience greater increases in liability when retirement is accelerated.

Therefore, the UAL arising from early retirement can be classified as a loss-type Past Service Liability (PSL), as the increase in actuarial obligations attributable to past service is not supported by previously accumulated funding.

Table 12. Past Service Liability for Each Group

Past Service Liability $(\boldsymbol{PSL})$	
$_{A}PSL$	IDR 145,128,194.6611
$_{B}PSL$	IDR 138,818,273.1541
$_{C}PSL$	IDR 128,648,885.0943
$_{D}PSL$	IDR 122,522,747.7088

Table 12 presents the PSL values for each age group at age 45. These values are consistent with the earlier UAL calculations, confirming that the funding gap originates entirely from the increase in actuarial liability due to the revised retirement assumption. Furthermore, the results reveal a decreasing trend in PSL values as the entry age increases. On average, each one-year increase in entry age corresponds to an approximate 5.48% reduction in PSL. This pattern reflects the shorter service duration associated with higher entry ages, which leads to lower accumulated benefits and, consequently, smaller additional liabilities when retirement is accelerated. Overall, the findings demonstrate that PSL is highly sensitive to both retirement age and length of service, emphasizing the importance of carefully evaluating policy changes that affect retirement timing.

3.8 Spreading Gains and Losses

Referring to Equation (2.19), the additional contribution required to amortize the actuarial loss is determined using an n-year temporary life annuity factor, where $n = 5$ represents the remaining funding period from age 45 to age 49. Thus, the present value factor for five annual payments at age 45 is 4.52798. This factor is used to determine the Supplementary Contribution (SC) required to amortize the Past Service Liability (PSL) over the remaining working period.

Tabel 13. Supplementary Contribution for Each Group

Supplementary Contribution (SC)	
$_{A}SC$	IDR 32,051,421.3095
$_{B}SC$	IDR 30,657,881.2526
$_{C}SC$	IDR 28,411,981.7434
$_{D}SC$	IDR 27,059,030.2318

The results of the supplementary contribution calculations are presented in Table 13. Group A requires an additional contribution of IDR 32,051,421.3095, which is the highest among all groups. These values reflect the magnitude of the actuarial loss (PSL) in each group. To provide a clearer understanding of the financial implications of retirement acceleration, the following discussion focuses on a comparative analysis between the normal contribution under normal retirement and the normal contribution under early retirement. The analysis uses Group A as a representative case to illustrate how changes in retirement timing affect the required contribution level. By examining the contribution pattern at the same entry age under both retirement scenarios, this section aims to highlight the magnitude of additional funding required when retirement is accelerated relative to the statutory normal retirement age. The comparison serves as an actuarial assessment of the cost consequences arising from retirement timing adjustments.

Table 14. Comparison of Normal Contributions for Age Group A

Age (Period)	**Normal Retirement vs Early Retirement**	
	$^{58}_{A}NC_x$	$^{50}_{A}NC_x$
46	Rp 13.330.782,39	Rp 45.382.224,78
47	Rp 14.026.637,18	Rp 46.078.079,57
48	Rp 14.761.921,45	Rp 46.813.363,85
49	Rp 15.539.332,04	Rp 47.590.774,43
50	Rp 16.361.620,33	Rp 48.413.062,72

Table 14 compares normal contributions (NC) under normal and early retirement alongside annual salary levels for ages 45–50. The results show that NC under early retirement is consistently more than three times higher than under normal retirement across all ages. For instance, at age 45, the contribution increases from IDR 12.67 million to IDR 44.72 million, representing an increase of approximately 253%. When evaluated relative to annual salary, the difference becomes more pronounced. Under normal retirement, the required contribution represents approximately 22–27% of annual salary, indicating a moderate and relatively manageable funding level. In contrast, under early retirement, the contribution ranges from approximately 75% to nearly 80% of annual salary, implying a substantially heavier financial burden.

Although both NC and salary increase with age, the relative burden remains consistently high under early retirement, suggesting that the effect of retirement timing dominates the effect of salary growth. Importantly, similar patterns are observed across other age groups, indicating that the magnitude of the additional contribution required under early retirement is not unique to this cohort but represents a systematic outcome of the model.

From an actuarial perspective, early retirement significantly compresses the contribution period while maintaining benefit obligations, thereby requiring substantially higher contributions to achieve funding adequacy. This finding is consistent with established actuarial principles, which emphasize that shorter contribution horizons and earlier benefit payments increase the present value of pension liabilities [12], [13]. In practical terms, the normal retirement scenario can be considered financially sustainable within a payroll-based funding framework, whereas the early retirement scenario may impose critical funding pressure unless supported by higher contribution rates or additional funding sources, in line with actuarial funding guidelines [14].

Under a 5-year amortization period, the fund is generally restored to a fully funded position under normal retirement. However, under early retirement, residual deficits may persist unless contributions are increased or the amortization period is extended, highlighting potential funding sustainability risks. Accordingly, pension sponsors may need to increase employer contribution rates, potentially up to twice the current payroll level, or extend the amortization horizon to restore funding adequacy. In addition, reallocating a portion of assets (e.g., around 20%) into fixed-income instruments can improve asset–liability matching and reduce funding volatility. Overall, early retirement policies should be supported by continuous actuarial evaluation to ensure long-term sustainability.

3.9 Sensitivity Analysis of Contribution under Interest Rate Scenarios

To assess the robustness of the model, a sensitivity analysis is conducted by varying the discount rate at 3%, 5%, and 8%. This analysis aims to evaluate how changes in interest rate assumptions affect the required normal contributions (NC) under both normal and early retirement scenarios.

Table 15. Normal Contributions under Different Interest Rate Scenarios

Age (Period)	$i = 3\%$		$i = 5\%$		$i = 8\%$	
	${}^{58}_{A}NC_x$	${}^{50}_{A}NC_x$	${}^{58}_{A}NC_x$	${}^{50}_{A}NC_x$	${}^{58}_{A}NC_x$	${}^{50}_{A}NC_x$
46	Rp 13.330.782,39	Rp 45.382.224,78	Rp13.330.782,39	Rp45.382.224,78	Rp7.200.167,02	Rp36.095.143,63
47	Rp 14.026.637,18	Rp 46.078.079,57	Rp14.026.637,18	Rp46.078.079,57	Rp7.792.466,64	Rp36.687.443,24
48	Rp 14.761.921,45	Rp 46.813.363,85	Rp14.761.921,45	Rp46.813.363,85	Rp8.435.265,08	Rp37.330.241,68
49	Rp 15.539.332,04	Rp 47.590.774,43	Rp15.539.332,04	Rp47.590.774,43	Rp9.133.193,26	Rp38.028.169,87
50	Rp 16.361.620,33	Rp 48.413.062,72	Rp16.361.620,33	Rp48.413.062,72	Rp9.891.247,48	Rp38.786.224,09

The results show that the discount rate has a significant inverse relationship with the required contribution level. Lower interest rates (3%) produce substantially higher contributions due to the reduced discounting effect, while higher interest rates (8%) decrease the present value of liabilities and consequently reduce the contribution requirement. This relationship is consistent with established actuarial valuation principles for defined benefit obligations [12], [13]. However, across all interest rate scenarios, the contribution under early retirement remains consistently much higher than under normal retirement. For instance, at age 45, the required contribution under early retirement is more than twice that under normal retirement at 3%, and remains more than five times higher at 8%. This indicates that the effect of retirement timing dominates the effect of interest rate variation, as also emphasized in international pension analyses highlighting the financial impact of early retirement behavior [13].

When expressed relative to annual salary, the funding burden becomes more evident. Under normal retirement, the contribution ranges from approximately 35–45% of annual salary at 3%, decreases to around 22–27% at 5%, and further declines to about 10–15% at 8%. In contrast, under early retirement, the contribution represents approximately 90–100% of annual salary at 3%, around 75–80% at 5%, and remains high at approximately 55–65% even at 8%. Such magnitudes indicate potential funding strain, in line with actuarial funding guidelines that stress the importance of aligning contribution levels with sustainable payroll capacity [14]. These findings suggest that, although higher interest rates may partially alleviate funding pressure, early retirement continues to impose a structurally high contribution burden. The consistency of this pattern across all scenarios indicates that the results are robust to changes in discount rate assumptions. From a policy perspective, this highlights the importance of incorporating interest rate sensitivity and retirement behavior into long-term pension funding strategies to ensure sustainability and solvency [12], [14]

4. CONCLUSION

Based on the findings of this study, retirement age is a critical determinant of pension funding adequacy. The application of the Projected Unit Credit (PUC) method shows that actuarial liabilities under accelerated retirement are substantially higher than under normal retirement across all entry-age groups. At age 45, liabilities increase by approximately 49.8% when retirement is accelerated from age 58 to age 50. This occurs because earlier benefit payments reduce the discounting horizon, thereby increasing the present value of pension obligations. The comparison of normal contributions further highlights the financial impact of retirement timing. In Age Group A, the required contribution increases from approximately IDR 12.67 million under normal retirement to IDR 44.72 million under early retirement, representing an increase of about 253%. Relative to annual salary, contributions under normal retirement remain within a moderate range, whereas under early retirement they approach or exceed one full year of salary. This pattern is consistently observed across other age groups, indicating that early retirement imposes a structurally higher funding burden.

The Spreading Gains and Losses method confirms that this increase generates Unfunded Actuarial Liability (UAL), requiring amortization through supplementary contributions. While alternative strategies such as adjusting contribution rates or extending amortization periods may be considered, early retirement policies must be carefully aligned with funding capacity. Given that this study adopts a deterministic framework, future research incorporating stochastic assumptions is necessary. Overall, sustainable pension policy requires integrating retirement age decisions with long-term funding strategies.

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