Zero : Jurnal Sains, Matematika, dan Terapan
E-ISSN : 2580-5754; P-ISSN : 2580-569X
Volume 10, Number 1, 2026
DOI: 10.30829/zero.v10i1.28497
Page: 190-201

Mathematical Model of Zakat and Low-Tax Policy Effects on Economic Population

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Article Info

Article history:

Accepted 25 March 2026

Keywords:

Islamic economics;
Low-tax policy;
Mathematical modelling;
Population dynamics;
Zakat.

ABSTRACT

Many Islamic countries implement low-tax policies, but these often fail to reduce inequality and poverty. Zakat has a function to redistribute wealth, but previous studies fail to explore its dynamic interaction with tax systems. This study builds a mathematical model that describes the combined effects of zakat and low-tax policies on economic population dynamics. We develop a system of differential equations that represents wealthy, middle, and poor populations, then analyze it qualitatively by finding equilibrium points and their stability. Analysis reveals a single asymptotically stable equilibrium point, indicating that zakat effectively reduces the wealth gap, even under a low-tax setting. We also established a threshold condition for the middle-to-poor population ratio, guiding policymakers on designing effective poverty interventions. This study lays a theoretical groundwork that incorporates Islamic financial institutions into poverty reduction strategies.

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1. INTRODUCTION

Poverty and disparity are among the major economic problems in most countries in the world [1]. Although the World Bank predicts extreme poverty may diminish, the gap between the richest and the poorest 20% will persist [1] Government revenue, primarily from taxation, funds public expenditures that drive economic growth [2]. Implementation of tax policy has always been a dilemma for the government. High tax rates can stifle economic activity and lead to a larger informal economy [3], but low rates limit the government's ability to provide more informal jobs and fund public services that improve living standards [4] and exacerbate the economic inequality [5]. Many Muslim-majority nations implement low tax rates, making alternative poverty reduction tools important.

Islamic fiscal instruments like zakat may offer a complementary approach. Zakat differs from taxation; its main objective is to alleviate poverty, and its expenditures are strictly defined, whereas tax revenue may be utilized for various general purposes. Zakat functions as both a spiritual obligation and an economic redistribution mechanism [6], and promotes human welfare through equitable wealth distribution [7]. Indonesia, which has the largest Muslim population in the world, has a zakat potential of 0.6-1.75% of its GDP [8]. This amount is sufficient to reduce unemployment, enhance investment, and eradicate excessive wealth inequality [9].

Analyzing the interaction between zakat and taxation requires analytical tools that are capable of capturing their dynamic relationship. This study employs a system of ordinary differential equations (ODEs), as this approach enables analytical derivation of equilibrium points and stability conditions, yields threshold conditions for policy guidance, and naturally captures population flows between economic strata [10][11][12][13]. This mathematical framework is directly relevant to the research questions.

Alternative approaches provide distinct benefits, although are less appropriate for these objectives. Agent-based models that require simulation-based analysis can describe the heterogeneous agent behavior and interactions [14], but cannot provide the behavior of equilibrium conditions. In parallel, econometric models using historical data [15][16][17] can give empirical estimation and hypothesis testing, but they are not designed for deriving a theoretical model of parameter influence on population transitions over time. Both complement rather than substitute for theoretical modeling. Conversely, ODE models assume population homogeneity and determinism, which was acceptable for establishing theoretical foundations, but requires extension for empirical application.

The study of complex systems in economics increasingly relies on such mathematical frameworks to capture emergent behavior [18]. The use of mathematical and system dynamics in previous models also failed to capture its interdependent relationship with the tax system since they explored zakat's role in isolation [19][20][21][22]. In practice, zakat and tax systems interact; for example, zakat contributions may lower tax liabilities in some countries [23], and both mechanisms influence economic activity and demographic shifts.

To bridge this gap, we construct a mathematical framework that captures the joint influence of zakat and taxation on the dynamics of economic populations. Specifically, the model employs a system of differential equations, providing a formal theoretical basis for examining how Islamic fiscal instruments may offset constraints in tax capacity. Adopting a dynamical systems approach enables a rigorous investigation of stability properties and long-run behavior, which is crucial for addressing the central research question: (1) How can the interaction between zakat mechanisms and low-tax policies be formally modeled within an economic population framework? (2) Is there an equilibrium state, and what conditions govern it? (3) How do zakat effectiveness and tax deviation affect poverty level? (4) Is there a threshold condition that determines poverty dynamics for policy intervention?

2. RESEARCH METHOD

This study is theoretical research, where the materials are gathered from scientific resources related to the topic. The stage of this study follows the mathematical modelling framework. The first step is to identify the problem by reading relevant literature related to population dynamics, economic factors, and taxation systems [10][24]. The next step is to construct the model in the form of a system of three ordinary differential equations. In this step, we define four variables (three dependent variables: wealthy, middle-class, and poor population; and time as an independent variable), six parameters, and make some assumptions [25]. Complexity theory provides foundational support for such compartmental approaches to socioeconomic modeling [18].

The proposed model is analyzed qualitatively by determining fixed points and their stability [12][13][26]. The interpretation step consists of explaining the dynamic behaviour and sensitivity of the model affected by two controllable/policy-sensitive parameters (related to zakat effectiveness and tax deviation). The parameter that represents economic growth has been framed specifically around growth data from Organisation of Islamic Cooperation (OIC) member states, thereby establishing a stylized OIC baseline for the analysis. Furthermore, the parameter of business failure rates is enriched with context from the Indonesian economy.

The model was validated using multiple approaches: checking the consistency of the metrics, verifying the behaviour of the model around the boundaries, conducting numerical simulations using parameter values estimated from real sources and assumed from a feasible range, and performing sensitivity analysis to test robustness. We use a Monte Carlo simulation to assess the model's robustness under parameter uncertainty, generating random parameter combinations from uniform distributions within feasible ranges. We visualize the dynamical behaviour using Maple software. The last step of the modelling process is drawing conclusions and reporting findings.

2.1 Parameter Estimation and Data Sources

Parameter values were estimated from empirical sources (Table 1). We choose ε and μ from feasible ranges based on economic growth literature [27][28][29]. Remaining parameters were calibrated from OIC country data [30][31][32][33][34][35][36][37][38].

Table 1. Parameter Values for Simulation

Parameter	Value	Range	Data Sources and Estimation Rationale
μ	0.02/year	0.008 to 0.022	Based on average global population growth over the last century [29]. We take a high baseline because of the characteristics of Islamic nations.
ε	0.05/year	-0.017 to 0.058	Based on the average economic growth of OIC members for 2013-2022 [30]. We take 5% because it is more common to achieve.

γ	0.09/year	0 to 0.4	Based on global business failure statistics for established businesses [31]. For country-specific calibration, we estimate using empirical data from Indonesia. The manufacturing sectors indicate the closure of at least 60 textile manufacturers between 2023 and 2024, with 60,000 workers laid off from 50 companies in early 2025 alone [32]. Agricultural challenges affect over 55% of farmers, with 34% facing capital constraints [33]. Based on these indicators, we adopt γ= 0.09/year.
α	0.01/year	0.0045 to 0.0414	Estimated from studies on zakat's economic impact [34][35] and collection efficiency data from BAZNAS [36] and the World Zakat Forum [37]. The baseline represents a moderate effectiveness.
δ	-0.002/year	$\|\delta\| \ll \gamma, \varepsilon\gamma$	Assumed based on tax elasticity literature [39], empirical evidence of modest tax impacts on transition rates [28][40], and cross-country comparisons of mobility [28]. The small negative value represents a mildly low-tax regime (tax rate below the optimal threshold).

3. RESULT AND ANALYSIS

The initial model

The total population is categorized into three economic groups: wealthy individuals (W), poor individuals (X), and the middle-income group that falls between the two extremes (N). These three groups are assumed to be homogeneous. Newborn individuals enter the middle-income category. Transitions between the wealthy and poor groups cannot occur directly; movement must pass through the middle group. This structural assumption is supported by stratification economics literature [41][42], which demonstrates that middle classes serve as buffers in wealth dynamics, and direct extreme transitions are empirically rare in stable economies

Economic growth, taxation effects, and business failures influence changes in each category. The overall population size is assumed constant, with birth and death rates equal (represented by μ) and migration effects neglected. The rate of change of each population affected by economic growth (represented by ε) is equal for all populations. Some research shows that growth can reduce poverty [25][26], but other research shows that growth also increases poverty [27]. The rate of business failures (represented by γ) is equal for every compartment. The rate of change affected by zakat (represented by α) reduces business failure for the wealthy population. While these studies use different metrics of effectiveness, they collectively suggest that zakat's measurable economic influence falls within this magnitude. Islamic Development Bank data from a comprehensive overview of zakat collection efficiency across OIC countries confirms the approximate range [37][38]. The rate of change from a lower economic population into a higher one is slightly affected by a factor resulting from the difference between the tax level and the threshold value. We use δ to represent the factor, where its value is positive if the tax level is higher than the threshold and negative if the tax level is lower than the threshold.

Zakat payments and tax reductions are assumed to lower the likelihood of business failure among the wealthy, while zakat receipts improve the economic condition of the poor. All transition rates associated with economic growth, taxation, zakat, and business failure are treated as constant parameters. Additionally, a threshold tax level is assumed to exist, representing an optimal taxation point [43]. Modern developments in zakat administration, including digital and blockchain-based systems, have the potential to significantly enhance collection efficiency and targeting accuracy [44][45][46]. The integration of zakat with tax systems continues to evolve as an important policy consideration in Islamic economies [47].

From the assumptions, we build the first initial model as follows:

a. The rate of change of the wealthy population depends on the number of middle-income populations that become rich, affected by economic growth and taxation $(\varepsilon + 2\delta)N$, and the number of wealthy populations that become middle population because of business failure, death $(\gamma - \alpha + \delta + \mu)W$.
b. The rate of change of the poor population depends on the number of middle-income populations that become poor because of business failure $(\gamma - \delta)N$ and the number of poor populations that become middle-income population because of economic growth, taxation, zakat, and death $-(\varepsilon + \alpha + \delta + \mu)X$.
c. The rate of change of the middle-income population depends on: the number of new-born μT where T respresents the total population, the number of rich population that becomes middle-income population because of business failure $(\gamma - \alpha + \delta)W$, the number of poor population that become middle-income population because of economic growth, taxation, and zakat $(\varepsilon + \alpha + \delta)X$, and the decreasing number of middle population because of economic growth, taxation, and death $(\delta + \varepsilon + \mu)N$

The initial suggested model is:

$$\frac{dW}{dt} = (\varepsilon + 2\delta)N - (\gamma + \delta - \alpha + \mu)W$$
$$\frac{dX}{dt} = (\gamma - \delta)N - (\varepsilon + \alpha + \delta + \mu)X$$
$$\frac{dN}{dt} = \mu T + (\varepsilon + \alpha + \delta)X + (\gamma - \alpha + \delta)W - (\delta + \varepsilon + \mu)N \quad (1)$$

The proportion model

Since the population is constant, we can simplify the model into just two equations. Using these new scaled variables

$$R = \frac{W}{T}, P = \frac{X}{T}, M = \frac{N}{T},$$

we have $M = 1 - R - P$. Our concern is two variables R (proportion of rich population) and P (proportion of poor population). So, the new model is:

$$\frac{dR}{dt} = (\varepsilon + 2\delta)(1 - R - P) - (\gamma - \alpha + \delta + \mu)R$$
$$\frac{dP}{dt} = (\gamma - \delta)(1 - R - P) - (\varepsilon + \alpha + \delta + \mu)P \quad (2)$$

In this model, all variables are feasible on $[0,1]$. We validated the model at the boundaries and found that all derivatives $\frac{dP}{dt}, \frac{dR}{dt}$ are nonnegative when the value of the variable is zero, and all $\frac{dP}{dt}, \frac{dR}{dt}$ are less than zero when the value of the variable is one. We can conclude that the model is well-defined, and we can use the model.

Model analysis

The system (2) has one fixed point (R_0, P_0) were

$$R_0 = \frac{2\alpha^2 + 2\alpha\delta + 3\alpha\varepsilon + 2\alpha\mu + \delta\varepsilon + \varepsilon^2 + \varepsilon\mu}{\alpha^2 + \alpha(3\delta + 2\varepsilon + 2\mu) + \delta(\gamma + 2\varepsilon + \mu) + \varepsilon^2 + \varepsilon(\gamma + 2\mu) + (\gamma + \mu)^2}$$
$$P_0 = \frac{(\gamma - \delta)(\gamma - \alpha + \delta + \mu)}{\alpha^2 + \alpha(3\delta + 2\varepsilon + 2\mu) + \delta(\gamma + 2\varepsilon + \mu) + \varepsilon^2 + \varepsilon(\gamma + 2\mu) + (\gamma + \mu)^2}$$

The existence conditions of the fixed point

For the equilibrium point (R_0, P_0), which represents long-term stable proportions of wealth and poor population, to be biologically and economically meaningful, it must satisfy these conditions

$$R_0 \geq 0, P_0 \geq 0, \text{ and } R_0 + P_0 \leq 1$$

The conditions for the existence of (R_0, P_0), because our assumption before that δ is small in magnitude (typically $|\delta| \ll \varepsilon, \gamma$), are

$$\delta \geq -\frac{2\alpha^2 + 3\alpha\varepsilon + 2\alpha\mu + \varepsilon^2 + \varepsilon\mu}{2\alpha + \varepsilon} \text{ for non-negativity of } R_0$$
$$\delta \geq \alpha - \gamma - \mu \text{ for non-negativity of } P_0.$$

And for $R_0 + P_0 \leq 1$,

$$(2\alpha^2 + 2\alpha\delta + 3\alpha\varepsilon + 2\alpha\mu + \delta\varepsilon + \varepsilon^2 + \varepsilon\mu) + (\gamma - \delta)(\gamma - \alpha + \delta + \mu) \leq$$
$$\alpha^2 + \alpha(3\delta + 2\varepsilon + 2\mu) + \delta(\gamma + 2\varepsilon + \mu) + \varepsilon^2 + \varepsilon(\gamma + 2\mu) + (\gamma + \mu)^2$$

Linearizing the system around the fixed point, we have the Jacobian matrix:

$$\begin{bmatrix} -(\gamma - \alpha + 3\delta + \varepsilon + \mu) & -(2\delta + \varepsilon) \\ -(\gamma - \delta) & -(\gamma + \alpha + \varepsilon + \mu) \end{bmatrix}$$

The eigenvalues of the matrix are

$$-\frac{1}{2}(2\gamma + 3\delta + 2\varepsilon + 2\mu) \pm \frac{1}{2}\sqrt{\delta^2 + 4\alpha^2 + 4\varepsilon\gamma + 8\delta\gamma - 12\alpha\delta - 4\delta\varepsilon}$$

That can be written as

$$-A \pm \sqrt{(-A)^2 - B} \quad (3)$$

were

$$A = \frac{1}{2}(2\gamma + 3\delta + 2\varepsilon + 2\mu)$$
$$B = (\gamma + \mu)(\gamma + \alpha + \varepsilon + \mu) + (2\delta + \varepsilon)(2\delta + \mu - \alpha)$$

Since the magnitude of the first term $|-A|$ dominates the value under the radical sign $\sqrt{(-A)^2 - B}$ in equation 3, we have that both eigenvalues are negative. Therefore, the fixed point is asymptotically stable.

If we consider poverty reduction efforts, then the threshold value of dR is when

$$\frac{M}{P} = \frac{\varepsilon + \alpha + \delta + \mu}{\gamma - \delta} \quad (4)$$

This means that the proportion of the poor will decrease when the ratio between the middle-income and poor population is below $\frac{\varepsilon+\alpha+\delta+\mu}{\gamma-\delta}$ and will increase when the ratio is above $\frac{\varepsilon+\alpha+\delta+\mu}{\gamma-\delta}$.

Sensitivity analysis

Given that economic parameters such as the rate of business failure and the effectiveness of zakat collection can vary significantly between nations, it is crucial to verify the robustness of the model. We performed a sensitivity analysis to observe how changes in key parameters—specifically the zakat effectiveness rate (α) and the tax deviation parameter (ε)—affect the equilibrium proportion of the poor population (P_0). We use the elasticity, the percentage change in equilibrium poverty (P_0) divided by the percentage change in a parameter, to evaluate the poverty-reducing potency of zakat.

Sensitivity to Zakat Effectiveness

We simulate α values from 0.01 to 0.04, based on documented zakat performance across OIC countries [37, 38], while keeping other parameters constant ($\gamma = 0.09,\ \delta = -0.002, \varepsilon = 0.05, \mu = 0.02$). Table 2 shows negative elasticities, which means that improving zakat effectiveness reduces poverty. A 1% gain in zakat efficiency may reduce the equilibrium poverty P_0 by 0.168% to 0.578%. As the effectiveness of zakat arises, its poverty-reducing power accelerates.

The other findings are the strong monotonic inverse relationship of zakat effectiveness with the equilibrium poverty proportion P_0 and a positive relationship with the equilibrium wealthy proportion R_0. The increasing of α from 0.01 to 0.04 reduces P_0 significantly from 40.76% to 22.63% and increases R_0 from 24.68% to 50.8%. This confirms the dual role of zakat to uplift the poor while also protecting the wealth from economic failures.

Table 2. Simulation of the parameter α on the model

α	Empirical Benchmark	R_0	P_0	E_α
0.010	Indonesia (current estimate) [46]	0.2468	0.4076	-
0.015	Pakistan (moderate efficiency) [37]	0.2898	0.3734	-0.168
0.020	Malaysia (tax-integrated) [23]	0.3333	0.3407	-0.262
0.030	Sudan (high efficiency) [37]	0.4211	0.2803	-0.355
0.040	Theoretical maximum OIC	0.5080	0.2263	-0.578

The analysis provides strong quantitative evidence for the poverty-reducing potency of zakat. It suggests that policymakers should not just aim for any zakat system, but should strive for highly efficient and well-integrated systems (like the tax-integrated model of Malaysia) because the marginal benefit of improvements increases at higher levels of effectiveness [23][46][37].

Sensitivity to Tax Deviation

We simulate δ values from -0.01 to -0.002, while keeping all other parameters constant ($\alpha = 0.01, \gamma = 0.09,\ \varepsilon = 0.05, \mu = 0.02$). One reason for using that range of δ is that any major policy effects on OECD members rarely exceed 1-2% tax-to-GDP ratios [28], so it supports our assumption of $|\delta| \leq 0.01$ in the model. The using of negative range is consistent with most of OIC members' tax policies with the baseline of -0.02 represents a mildly low-tax regime (like Indonesia and Malaysia), and the lower bound -0.01 represents extremely low-regime tax (such as Gulf countries).

Table 3. Simulation of the parameter δ on the model

δ	R_0	P_0	E_δ
-0.010	0.2426	0.4455	-
-0.008	0.2437	0.4360	0.107
-0.006	0.2448	0.4265	0.087
-0.004	0.2458	0.4170	0.067
-0.002	0.2468	0.4076	0.045

Table 3 shows the positive elasticities, which means the decrease in tax deviation to its optimal threshold will reduce the equilibrium poverty P_0. The elasticity decreases when δ aproaches zero. The impact of tax policy on poverty reduction is better when the tax regime is extremely low.

Numerical simulation

To back up the analysis, we perform a numerical simulation using some estimated parameters. Using this set of parameters, as shown in Table 1, we obtain the fixed point (R_0, P_0) = (0.2468354430, 0.4075949367), which represents the proportion of rich and poor is respectively around 24.68% and 40.76%. This represents the steady –state distribution of the country. It is important to emphasize that this result is an illustrative model output, not a prediction of actual poverty rates. The high poverty proportion reflects the model's structural assumptions and parameter choices, and would require substantial policy intervention (i.e., parameter changes) to shift toward a more equitable distribution. The phase portrait in Figure 1 and the trajectory of both rich and poor proportions in Figure 2 show that the behaviour of the dynamical system is consistent with our previous analysis.

DISCUSSION

The purpose of this study is to describe how a low-tax policy, when combined with zakat, impacts economic population dynamics. The interpretation of the model provides several recommended economic policies.

The stability of the steady-state distribution

From the qualitative analysis, the system has a single asymptotically stable equilibrium point, which implies the proposed Islamic fiscal system is converges to a stable steady-state distribution. Zakat functions as a stabilizer when the economic system is perturbed by market forces (business failures). Zakat also prevents situations such as economic collapse into extreme wealth disparity. This finding aligns with the mathematical properties of stable dynamical systems [12][13].

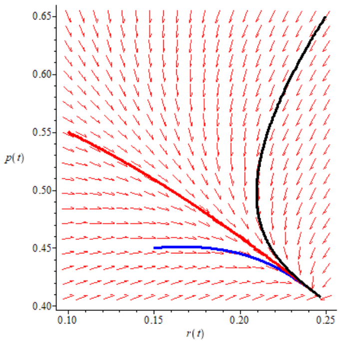

Figure 1. Phase portrait for initial value $R(0)$= 0.1,$P(0)$=0.55, $R(0)$= 0.15,$P(0)$=0.45, and $R(0)$= 0.25,$P(0)$=0.65

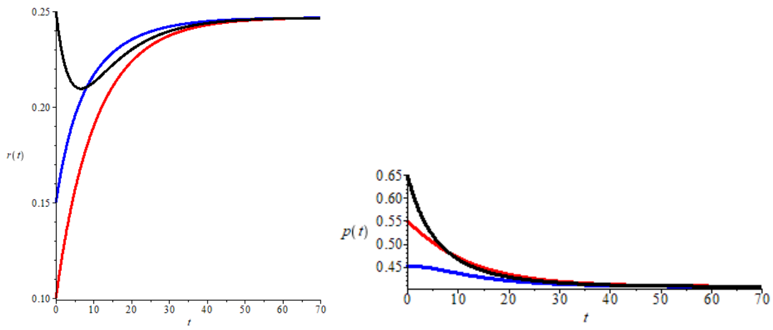

Figure 2. Trajectory of R (left) and P (right) over 70 years in case low tax for initial value $R(0)$= 0.1,$P(0)$=0.55, $R(0)$= 0.15,$P(0)$=0.45, and $R(0)$= 0.25,$P(0)$=0.65

Zakat as a complement to taxation

The model shows that a low-tax policy works for reducing poverty when zakat is operational and function as complemntary of tax. Empirical data from Malaysia shows that the integration of zakat into the tax system increased zakat compliance while maintaining tax revenues [23]. This supports our finding that both mechanisms together produce better outcomes than either in isolation. The sensitivity analysis shows that increasing zakat effectiveness reduces equilibrium poverty significantly, but in reality, the magnitude of zakat is quite small compared to actual fiscal capacities. Zakat potential in OIC countries ranges from 1.8-4.34% of GDP [48], while tax-to-GDP ratios in these same countries average 18-25% [30]. Even at maximum efficiency, zakat cannot replace the revenue scale required for public goods provision that taxation funds. For context, Indonesia's 2023 education budget allocation alone (approximately 3% of GDP) exceeding the upper bound of zakat potential.

The model assumes a linear relationship between zakat effectiveness (α) and poverty reduction, but empirical evidences indicate diminishing returns. Countries with high collection efficiency (Malaysia, Sudan) show slower marginal per efficiency improvement [46][37][38]. This suggests the relationship between α and poverty reduction may be nonlinear in practice, a limitation of our current linear formulation.
The other hole in our model is the absence of public goods (education, infrastructure, and healthcare) that cannot be funded by zakat and only by taxation. Public education funding significantly affects intergenerational mobility; a mechanism our compartmental model cannot capture without extension [49]. It strengthens our conclusion that zakat can supplement limited tax capacity for direct transfers, but cannot replace the broader fiscal role.

Threshold conditions and empirical validation

The analysis formulated a threshold condition for the ratio of the middle-income to poor population (equation 4). This finding of the importance of the middle-income class in poverty reduction resonates with previous studies [50]. The threshold provides policymakers with practical insight: simple economic growth (ε) is not enough when the demographic structure is above the threshold. The policies must target the rate of transition (α) to satisfy the condition for poverty reduction.
We tested this threshold condition against observed ratios in two OIC countries, Indonesia and Malaysia, using parameter estimates derived from 2024 national statistics [51][52]. Empirical data show that actual M/P ratios are 1.99 (Indonesia) and 1.5 (Malaysia). These are above our calculated threshold ratios of 0.85 (Indonesia) and 0.96 (Malaysia). It means the model anticipates poverty reduction under the current demographic structure. The policy intervention suggested by this model may be implemented when the economic situation appears favorable, as it is better for the government's anticipation. But the threshold condition calculated using the approximate parameter still requires more rigorous econometric testing.

Comparison with existing models

While the other previous models isolated zakat in analysis, this model differently integrates the tax rate as a dynamic variable. A systematic comparison between the current and existing models in the literature is provided in Table 4.

Table 4. Comparison of Key Features in Modeling Approaches

Feature	Subhan [19]	Yusuf [20]	Pakdemirrli [21]	Setiawan et al.[22]	Indarningsih et al. [15]	Prawoto & Basuki [16]	Robbana et al. [17]	Muis et al. [14]	Current Model
Methodology	Differential equations	Modified mathematical model	Analytical mathematical model	System dynamics	VECM	Econometric regression	Panel econometrics	Artificial Neural Networks	Differential equations
Tax Policy Integration	-	-	-	-	-	-	-	-	Explicit (δ parameter)
Population Stratification	Three classes	Macroeconomic aggregates	Two classes (rich/poor)	Socioeconomic groups	Macroeconomic	Macroeconomic	Macroeconomic	Macroeconomic	Three classes with transition constraints
Analysis	Stability analysis	-	Fixed point analysis	Simulation-based	Cointegration	Cointegration	Growth models	Pattern recognition	Asymptotic stability proven
Threshold	-	-	-	-	-	-	-	-	M/P ratio threshold

Policy Parameters	Zakat management	Zakat collection	Zakat distribution	Zakat distribution	ZIS variables	Zakat variable	Zakat institution	Zakat variable	Both zakat (α) and tax deviation (δ)
Empirical Validation	Simulation only	Conceptual	Conceptual	Indonesian data	Indonesian data	Indonesian data	OIC panel	Indonesian data	Parameter-based simulation
Dynamic Behavior	Population flows	Growth effects	Poverty dynamics	Causal loops	Time series	Time series	Panel dynamics	Time series	Transition rates between classes
Low-Tax Regime Analysis	-	-	-	-	-	-	-		Primary focus

There are three key distinctions between the current model and previous ones: explicit integration of zakat and tax policy, rule of zakat as poverty reducer is analyzed separately from tax system, and introduction of novel theoretical contributions such as the threshold condition for poverty reduction, dual role of zakat in preventing business failure of the wealthy and upgrading the poor, the explicit modeling of tax deviation from its threshold providing the analysis of low-tax regimes, and demonstrations of zakat ability to substitute high tax policy under specific parameter conditions. Unlike the predictions of the World Bank that economic growth solely may reduce extreme poverty, this model offers a different view: growth combined with zakat is required to ensure the gap between rich and poor does not destabilize the system.

Limitations and Future Research

While this study establishes a theoretical framework for integrated fiscal policy, several modeling constraints influence the scope of its conclusions. The assumption of constant population with equal birth/death rates ignores migration and demographic transitions [29]. Many OIC countries experience rapid urbanization and youth bulges, affecting economic mobility. The use of homogeneous compartments disregards individual variation in tax liability, zakat eligibility, and business success. Also, a deterministic approach cannot capture stochastic shocks (financial crises, natural disasters, or sudden policy changes), which often produce discrete jumps [42] rather than continuous adjustment.

To confirm the robustness of the model, our Monte Carlo simulations show P_0 remains feasible in 97.3% of random parameter combinations. The 90% confidence interval spans 0.242–0.534, which is wide enough to caution against point predictions. Correlation analysis identifies γ (+0.72) and α (-0.58) as dominant poverty drivers. But these results assume parameter independence, unlikely in reality. Zakat effectiveness may correlate negatively with business failure rates if better-managed economies have both lower γ and higher α.

On empirical comparison, our baseline equilibrium of 40.8% poor exceeds Indonesia's 2024 poverty rate (≈19.9% [53]). Several factors explain this discrepancy. First, our "poor" category includes vulnerable populations near the poverty line, not just those below official thresholds. Second, the model excludes government social programs beyond zakat cash transfers, subsidies, and public employment [54]. Third, our generic OIC parameters differ from Indonesia-specific values. Recalibrating with Indonesian data would lower equilibrium poverty, but the threshold condition and stability properties remain qualitatively valid.

Practical implementation remains a final hurdle. Even optimal parameter combinations face real-world obstacles. BAZNAS data indicate that only 37% of collected zakat reaches intended beneficiaries [36]. Corruption, administrative inefficiency, and elite capture erode zakat's impact in weak-governance settings [17]. Political resistance from wealthy taxpayers may block tax adjustments toward optimal levels. Emerging technologies offer potential solutions [44][45][46], but institutional reform remains fundamental.

Future research should focus on extending this framework to strengthen the model. Adding stochastic shocks would capture random events like financial crises, while relaxing the constant-population assumption could incorporate migration and demographic differences across classes. Modeling heterogeneity within economic groups would reveal distributional effects hidden by averages, and including a government sector would clarify how taxation-funded public goods complement zakat's role. Empirical testing using OIC panel data could estimate actual parameter values and validate the threshold condition against real-world behavioral responses to policy incentives.

4. CONCLUSION

We develop a mathematical model demonstrating how zakat and low-tax policies can work together to stabilize economies and alleviate poverty, even when tax revenue is constrained. Our analysis reveals that the system maintains a single asymptotically stable equilibrium, confirming that zakat functions as an automatic stabilizer that prevents extreme wealth disparity during market fluctuations.

A central feature of the model is a threshold condition that determines poverty trajectories. The poverty consistently declines when the ratio of the middle-to-poor population remains below $(\varepsilon + \alpha + \delta + \mu)/(\gamma - \delta)$. This provides policymakers with a clear diagnostic tool for evaluating fiscal interventions. Sensitivity analysis demonstrates strong poverty-reduction elasticities to zakat effectiveness (ranging from -0.17 to -0.58), while tax adjustments have a more moderate impact (0.045–0.107) that diminishes as rates approach optimal levels. The model formalizes zakat's dual role: providing a safety net for the poor while simultaneously protecting the wealthy from business failure.

This research offers three contributions: integrating zakat and taxation within a dynamical framework with analytically proven stability; deriving an actionable threshold condition; and providing a quantitative comparison of policy sensitivity. While the current model establishes a theoretical base for integrated Islamic fiscal policy, future research incorporating stochastic shocks, demographic dynamics, and empirical calibration would enhance these findings. For Muslim-majority countries, these results suggest a viable path for leveraging unique institutional heritage to achieve economic stability.

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