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Crank-Nicolson Finite Difference Pricing of European Call Options under the Black-Scholes Model

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ABSTRACT

This study aims to determine the price of European call options using the Crank-Nicolson finite difference method in the Black-Scholes model with stock data from XYZ Company for the period January 2025 to December 2025. Annual volatility is calculated based on historical closing price data, while numerical option prices are obtained through the Crank-Nicolson finite difference scheme and compared it with the Black-Scholes analytical solution as a reference. The results show that the Crank-Nicolson method produces a call option price of 596.08, while the Black-Scholes analytical solution gives a value of 612.50. The relative difference between the two methods is 2.68%, which indicates a good level of accuracy for the numerical method used. These findings indicate that the Crank-Nicolson finite difference method is capable of providing a stable and accurate numerical approach to determining the price of European call options. In practical terms, the results of this study contribute to the application of numerical-based option pricing models in emerging markets, particularly in conditions of dynamic volatility, where analytical approaches may have limitations in implementation

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1. INTRODUCTION

Investment plays a very important role in supporting the economy, both domestically and globally, and continues to evolve in line with increasing financial market integration [1]. In general, investment can be defined as the allocation of resources in the present with the expectation of future returns [2]. With the rapid development of capital markets, particularly in developing countries such as Indonesia, increased stock trading activity has been accompanied by high price volatility, which poses significant risks for investors in their investment decisions.

Stocks as a primary investment instrument have characteristics of uncertain price fluctuations. Therefore, complementary instruments are needed to manage and mitigate these risks, one of which is through derivative instruments in the form of stock options [4], [5]. Options are derivative contracts that give the holder the right (without obligation) to buy or sell an asset at a predetermined price and time [2]. Based on their rights, options are classified into call options and put options, while based on their execution time, options are divided into

European options and American options. European options can only be exercised at maturity, so they have different valuation characteristics compared to American options [4].

Determining fair and accurate option prices requires mathematical models capable of representing asset price dynamics in the market. The Black–Scholes model, introduced by Black and Scholes in 1973, is one of the fundamental models in option pricing theory, particularly for European-style options [6]. This model links option values to market parameters such as the underlying asset price, risk-free interest rate, volatility, and time to maturity. However, the analytical solution of the Black–Scholes model is not always easy to apply to complex cases involving non-ideal market conditions or more common option forms.

To overcome these limitations, various numerical approaches have been developed to solve the Black–Scholes equation, including the binomial method, Monte Carlo simulation, and finite difference methods [7], [8]. Finite difference methods, especially the Crank–Nicolson scheme, are known to have advantages in terms of stability and accuracy because they combine implicit and explicit approaches in a balanced manner [9]. This method is widely used in mathematical financial modeling to solve partial differential equations, including in the valuation of European options.

Although the Crank–Nicolson difference method has been widely discussed in international literature as a stable and accurate approach to solving Black–Scholes equations [7]–[9], its application to Indonesian stock data is still relatively limited and generally lacks quantitative analysis of its accuracy and numerical convergence. Most domestic studies focus more on analytical solutions or model applications without in-depth evaluation of the performance of numerical schemes on actual market data.

To the author's knowledge, studies that specifically compare the results of the Crank–Nicolson method with the Black–Scholes analytical solution using Indonesian stock data and evaluate the difference in results quantitatively are still not widely reported in the national literature. Therefore, the main contribution of this study lies in the empirical evaluation of the performance of the Crank–Nicolson method in the context of the Indonesian capital market, including an analysis of its accuracy relative to analytical solutions. This approach is expected to enrich the mathematical finance literature in emerging markets and provide a more contextual numerical basis for further research.

Based on the above description, this study aims to apply the finite difference method with the Crank–Nicolson scheme to the Black–Scholes equation in determining the price of European call options. This study uses XYZ Company stock data as the object of research, considering its volatility and liquidity characteristics that are representative of banking stocks on the Indonesia Stock Exchange. The results of this study are expected to contribute theoretically to the development of numerical methods in mathematical financial models, as well as practically as a reliable alternative approach to option valuation in facing the dynamics of the Indonesian capital market.

2. RESEARCH METHOD

This research uses a numerical method approach to determine the price of European call options based on the Black-Scholes model. This approach is carried out by applying the finite difference method to solve the Black-Scholes partial differential equation discretely in space (stock price) and time. The basic assumption of the model is that the stock price S(t) moves according to the Geometric Brownian Motion (GBM) process:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

where S_t represents the stock price at time t, μ is the expected return, σ is the assumed constant volatility, and W_t is the standard Wiener process. By applying Itô's lemma to the function $V(S,t)$ the stochastic expansion is obtained:

$$dV - \left(\frac{\partial V}{\partial t} + \mu S \frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right) dt + \sigma S \frac{\partial V}{\partial S} dW_t. \tag{2}$$

Next, a replication portfolio is constructed to eliminate stochastic uncertainty by taking a long position in one option and a short position in Δ units of stock, where $\Delta = \frac{\partial V}{\partial S}$. This combination eliminates the stochastic component dW_t, so that the portfolio becomes risk-free and should provide a return equal to the risk-free rate *r*. From this no-arbitrage argument, we obtain the classic Black-Scholes PDE equation for European call options [6]:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{3}$$

The Black-Scholes model is based on several key assumptions. First, the price of the underlying asset follows a continuous log-normal distribution with constant volatility σ. Second, the market is assumed to be free of arbitrage, so that any risk-free portfolio must yield a return equal to the risk-free interest rate *r*. Third, there are no transaction costs, taxes, or dividend payments during the option's lifetime. Fourth, the risk-free interest rate is assumed to be constant over time. With these assumptions, a structured mathematical relationship is formed between the option price, the underlying asset price, and market parameters [4]. However, in real-world applications, these assumptions are often relaxed, rendering analytical solutions invalid and necessitating the use of numerical approaches, such as the finite difference method [10]. For a European call option with strike price K and maturity T, the terminal condition (payoff at $t = T$) is expressed as

$$V(S,T) = max(S - K, 0). \quad (4)$$

In the numerical solution, natural boundary conditions are used on the price domain $S \in [0, \infty)$. When the stock price is very small, approaching zero $(S \to 0)$, the call option value also approaches zero, so that $V(0,t) = 0$, for every $t \in [0,T]$. Conversely, when the stock price is very large $(S \to \infty)$, the call option value approaches its intrinsic value, which is the difference between the stock price and the discounted value of the strike price:

$$V(S,t) \approx S - Ke^{-r(T-t)}. \quad (5)$$

In numerical implementation, the upper bound S_{max} is chosen to be sufficiently large, for example, several times the initial stock price S_0 or the strike price K, to approximate this asymptotic behavior. With these terminal and boundary conditions, the Black-Scholes equation can be solved both analytically and numerically. With these terminal and boundary conditions, the Black–Scholes equation can be solved numerically using the finite difference method.

The Crank–Nicolson method is an implicit finite difference scheme obtained by averaging the explicit and implicit Euler schemes. Conceptually, this method calculates the solution value at the next time step by considering information at the current and next time steps simultaneously. This approach produces a system of linear equations at each time step, but offers the advantages of unconditional stability and second-order accuracy in both space and time. These properties make the Crank–Nicolson method more stable than explicit schemes, which depend on certain stability conditions, and more accurate than Euler's implicit scheme, which is only first-order.

The discretization process (finite difference method) is performed by dividing the stock price domain $[0, S_{max}]$ into M grid points with distance $\Delta S = \frac{S_{max}}{M}$, and the time domain $[0,T]$ is divided into N backward steps with distance $\Delta S = \frac{T}{N}$. A two-dimensional grid is formed so that each point (S_i, t_j) contains an approximation of the option value V_i^j. The first and second derivatives are discretized using the central difference formula, for example, the second derivative $\frac{\partial^2 V}{\partial S^2}$ is approximated by:

$$\frac{V_{i+1}^j - 2V_i^j + V_{i-1}^j}{(\Delta S)^2}, \quad (6)$$

while the formula for the first derivative uses:

$$\frac{V_{i+1}^j - V_{i-1}^j}{2\Delta S}. \quad (7)$$

Substituting all of these approaches into the Black–Scholes PDE yields a system of tridiagonal linear equations that can be solved using the Thomas algorithm. Terminal conditions are applied at the final time layer $t = T$, namely $V(S,T) = max(S - K, 0)$, and boundary conditions are applied for $S = 0$ and $S = S_{max}$. The backward process is performed from time $t = T$ to $t = 0$ until the option price $V(S_0, 0)$ is obtained.

In its implementation, grid parameters are explicitly set to ensure the stability and accuracy of numerical solutions. The number of spatial grids M and time grids N are selected as $M = 50, 100$, and 200, with N adjusted proportionally. Thus, the spatial step size ΔS and time step size Δt can be calculated directly from the domain boundaries of stock prices and maturity dates. The use of multiple grid resolution levels allows for the evaluation of the sensitivity of the numerical solution to grid refinement.

In this study, numerical simulations were conducted by setting a number of parameters used in the Black–Scholes model. The main parameters used include the initial stock price S_0 , strike price K, maturity date T, risk-free interest rate r, and stock price volatility σ. These parameters are fundamental components in determining option prices and are used in both analytical and numerical approaches. The selection of parameter values was done by considering realistic market conditions and referring to previous studies so that the simulation results could be accurately compared and validated.

The selection of model parameters in this study also considers the characteristics of emerging markets, where stock price volatility tends to be more dynamic and market efficiency has not yet been fully achieved. Emerging markets often face challenges such as information asymmetry, uneven liquidity, and sensitivity to global macroeconomic factors. In this context, a numerical approach to the Black–Scholes equation is relevant because it allows for the evaluation of the stability and sensitivity of option prices to changes in market parameters. Thus, the methodology used is not only mathematical in nature, but also designed to represent real market dynamics more flexibly.

The stock data used in this study comes from XYZ, a company with large market capitalization and high liquidity in the national banking sector. Stocks with these characteristics were selected in order to obtain a more stable and representative volatility estimate, so that the resulting model parameters can reflect market conditions more consistently. High liquidity also reduces distortions caused by low trading frequency, which can affect the accuracy of return and volatility calculations.

In addition to model parameters, numerical parameters in the form of space and time step sizes are also determined. The stock price step size ΔS is determined based on the upper limit of the stock price S_{max} and the number of space grids M, while the time step size Δt is determined based on the maturity time T and the number of time grids N. The selection of ΔS and Δt directly affects the stability and accuracy of the finite difference method. Grids that are too coarse can cause large numerical errors, while grids that are too fine increase the computational load. Therefore, the grid size is chosen in a balanced manner to ensure that the numerical solution remains stable and computationally efficient in the context of processing historical stock data with active trading frequency and relatively consistent volatility estimates.

In the numerical simulation, three grid configurations were used, namely:

i. $M = 50,\ N = 50$,
ii. $M = 100,\ N = 100$, and
iii. $M = 200,\ N = 200$,

The value of S_{max} is set to $3K$ to ensure that the upper bound of the domain adequately represents the asymptotic behavior of the solution. This parameter is chosen based on common practice in the Black–Scholes numerical literature.

Theoretically, the Crank–Nicolson method has a truncation error of order $O((\Delta S)^2 + (\Delta t)^2)$, so refining the grid will reduce the error quadratically. However, increasing the number of spatial grids M and temporal grids N also increases the size of the linear equation system that must be solved at each time step. The computational complexity of this method scales as $O(MN)$, so there is a trade-off between accuracy and computational efficiency. Therefore, in this study, tests were conducted on several grid configurations to obtain a balance between a low error rate and efficient computation time.

To assess the convergence of the numerical solution, a comparison was made between the results of the Crank–Nicolson finite difference method and the Black–Scholes analytical solution for several grid configurations. Numerical error was calculated at the initial stock price S_0 using absolute error and relative error. The results of this comparison are presented in tabular form to show that the error decreases as the number of grids increases, indicating that the numerical solution is convergent.

The numerical calculation procedure begins by applying terminal conditions at maturity $t = T$, which is the payoff function of a European call option. This payoff value is set at all stock price grid points and becomes the basis for backward calculation to the initial time. Next, an iterative process is carried out using the finite difference method, where the option value at each time step is calculated based on the value at the next time step according to the discretization scheme used.

Boundary conditions are also applied at each iteration step to ensure that the numerical solution remains consistent with the properties of European call options. The boundary condition at zero stock price states that the option value is also zero, while the boundary condition at maximum stock price represents the asymptotic behavior of the option value when the stock price is very large. By applying these boundary conditions consistently, the numerical solution obtained remains stable and does not deviate from the theoretical behavior of the Black–Scholes model.

The calculation process is performed backward in time from $t = T$ to $t = 0$. After the entire time grid is completed, the option price at the initial time is obtained, namely $V(S_0, 0)$, which is a numerical estimate of the European call option price. To evaluate the accuracy of the finite difference method used, the numerical results are then compared with the Black–Scholes analytical solution. This comparison is done to assess the level of conformity of the numerical results with the analytical values and to observe the effect of grid size on solution convergence.

Algorithm Crank–Nicolson for European Call Option:
Define model parameters S_0, K, T, r, σ
Set grid parameters M, N
Compute ΔS = Smax/M, Δt = T/N
Initialize payoff V_i^N = max(S_i − K, 0)
For j = N−1 down to 0 do
Construct tridiagonal matrix A
Solve A V^j = V^{j+1} using Thomas algorithm
Apply boundary conditions
End for
Return V(S0, 0)

The selection of the Crank–Nicolson method in this study was based on considerations of stability and numerical efficiency. Compared to the explicit finite difference method, the Crank–Nicolson scheme does not require a specific stability limit between Δt and ΔS, making it more flexible in grid selection. Meanwhile, compared to the implicit Euler method, Crank–Nicolson has a higher level of accuracy because it is second order. When compared to the binomial tree method or Monte Carlo simulation, the finite difference approach more directly represents the structure of the Black–Scholes partial differential equation, thus providing a smoother approximation of the continuous solution for one-dimensional European options with relatively efficient computational costs

3. RESULT AND ANALYSIS

3.1 Research Data

In this study, European call option pricing calculations were also applied to historical real stock data to demonstrate the implementation of the finite difference method in real market situations, with data sourced from Yahoo Finance via the yfinance library for the period January 2025 to December 2025. The data used is the adjusted closing price in daily format only on active trading days, so that some of the effects of corporate actions such as stock splits and dividends are implicitly reflected in the price. The dataset obtained does not contain missing values in the adjusted closing price column during the observation period, so no additional imputation process is required. The data is used directly for logarithmic return calculations and volatility estimates. However, the Black–Scholes model used in this study does not explicitly include dividends in its mathematical formulation, so the option pricing results still have limitations when applied to stocks that pay dividends regularly. The data used is presented in Table 1.

Table 1. Stock Price Data and Logarithmic Returns

t	Date	S_t (IDR)	S_{t-1} (IDR)	r_t
1	January 3, 2025	3780	3834	−0,0144
2	January 6, 2025	3734	3780	−0,0121
3	January 7, 2025	3670	3734	−0,0172
⋮	⋮	⋮	⋮	⋮
235	December 29, 2025	3643	3633	0,0026
236	December 30, 2025	3660	3643	0,0046

Table 1 only shows some initial and final observations from the entire dataset for readability reasons. All daily data during the research period was still used in the parameter estimation process and numerical option price calculations.
A summary of the logarithmic return statistics is presented in Table 2.

Table 2. Summary Statistics of Logarithmic Return

Statistics	Values
Number of observations	236
Average daily return	0,00052
Standard deviation of daily returns	0,02310
Minimum return	-0,0684
Maximum return	0,0742

This study uses a single stock as a case study to test the implementation of the developed numerical method. Although this approach is adequate for model validation purposes, the use of a wider range of stocks or different sectors could provide a more comprehensive picture of the method's performance across various volatility characteristics. Furthermore, comparisons with other numerical methods such as the binomial method or Monte Carlo simulation could be further developments to strengthen the evaluation of the Crank–Nicolson method's performance in the context of the Indonesian capital market.

3.2 Estimation of Model Parameters from Real Data

Based on the historical stock price data of PT XYZ above, the parameters required in the Black Scholes model are determined as follows:

a. Initial Stock Price

The initial stock price of S_0 is taken as the adjusted closing price on the last day of the observation period, which is December 31, 2025. Based on the data used, the initial stock price is obtained as follows: $S_0 = 3660$

b. Estimated Volatility (σ)
Stock volatility is calculated from daily logarithmic return data formulated as:

$$r_i = \ln\left(\frac{P_i}{P_{i-1}}\right). \quad (8)$$

Next, annual volatility is calculated by multiplying the standard deviation of daily returns by the factor $\sqrt{252}$, resulting in:

$$\sigma = \sqrt{252} \times \mathrm{std}(r_i). \quad (9)$$

Based on these calculations, the estimated annual volatility of the stock is $\sigma = 0{,}3660$. This value indicates that the stock price has a relatively high level of fluctuation during the observation period. Since volatility is a key parameter in both the Black–Scholes model and the Crank–Nicolson scheme, the accuracy of its estimation greatly affects the results of option pricing.

c. Risk-Free Interest Rate
The risk-free interest rate (r) is determined based on the Bank Indonesia reference interest rate for the research period, which is 5% per annum. This value is used as an approximation of the risk-free rate of return in the Black-Scholes model.

d. Strike Price and Maturity
The strike price (K) is taken to be equal to the initial stock price S_0, so that the option is at-the-money. The option maturity is set at T = 1 year

3.3 Numerical Model Validation

After the model parameters are estimated, the European call option price is calculated using two approaches:

i. Finite Difference Method (Numerical)
The Crank-Nicolson finite difference method is applied to solve the Black-Scholes partial differential equation as described in Chapter 2. By inserting the estimated parameters $S_0, K, r, \sigma,$ and T, the European call option price is numerically obtained as $V_{CN} = 596{,}08$.

ii. Black-Scholes Formula (Analytical) The call option price is also calculated using the classic Black–Scholes analytical formula as a comparison. Based on the same parameters, the analytical option price is obtained as $V_{BS} = 612{,}50$. A comparison of the option price calculation results using the numerical method and the analytical solution is presented in Table 3 below:

Table 3. Comparison of European Call Option Prices

Method	Option Price $V(S_0, 0)$	Difference from Analytical (%)
Difference to Numerical	596,08	2,68
Black-Scholes Analytical	612,50	0,00

The relative difference of 2.68% between the results of the Crank–Nicolson method and the Black–Scholes analytical solution is mainly due to the spatial and temporal discretization errors inherent in numerical methods. The selection of the price grid size and time step are important factors that affect the accuracy of numerical solutions. Theoretically, refining the grid with smaller steps will improve the accuracy of the results, but on the other hand, it will increase the computational cost.

A relative error value of 2.68% indicates that the Crank–Nicolson numerical approach is capable of producing price estimates that are very close to the analytical solution. In the context of market practice, this difference is relatively small compared to daily stock price fluctuations, so this method can be considered reliable enough for option valuation purposes in real conditions, especially when analytical solutions are not available.

Table 4. Grid Variations

Grid $M = N$	Relative Error (%)
50	2,5859
100	2,6618
200	2,6775

To evaluate numerical stability, grid size variations were performed with three configurations, namely 50, 100, and 200. The calculation results show that changes in discretization do not significantly affect relative error. This indicates that the numerical solution has reached a stable condition with respect to grid variations (grid-independent solution). Thus, increasing the grid resolution does not substantially improve the accuracy of the analytical solution.

Theoretically, the Crank–Nicolson scheme has an accuracy order of two with respect to spatial and temporal discretization. In the implementation of this study, the reduction in relative error was not very significant when the grid was refined. This condition was likely influenced by other factors such as the maximum price domain cutoff (S_{max}) and the effects of numerical rounding, so that discretization errors were not the only source of deviation from the analytical solution.

Sensitivity analysis was conducted to evaluate the effect of changes in key parameters in the Black–Scholes model on the option prices generated by the Crank–Nicolson method. The parameters analyzed included volatility, risk-free interest rate, and strike price. In general, an increase in volatility caused an increase in the price of call options due to the increased chance of the option being in-the-money at maturity. Meanwhile, an increase in the risk-free interest rate tends to increase the price of call options due to a decrease in the present value of the strike price. Conversely, an increase in the strike price has a downward effect on the price of call options. The results of this analysis show that the Crank–Nicolson method is able to capture the sensitivity of option prices to changes in model parameters in a manner consistent with the theoretical characteristics of the Black–Scholes model, thereby reinforcing the reliability of the numerical method used.

3.4 Interpretation of Results

The calculation results show that the option price obtained from the Crank-Nicolson finite difference method approximates the Black–Scholes analytical solution. The difference between the two methods is shown by an absolute error of 16.43 and a relative error of 2.68%. The low relative error value indicates that the Crank-Nicolson method has a good level of accuracy. However, these error values reflect a compromise between accuracy and computational efficiency, which depends on the selection of the price grid size and time step in the numerical scheme. The differences that still appear between the numerical and analytical solutions are mainly due to the discretization process of the stock price and time domains, as well as the limitations of the maximum price domain used in the numerical calculations. However, these results are consistent with the literature, which states that finite difference methods, particularly the Crank–Nicolson scheme, are stable and converge to the exact Black–Scholes solution. With a relative error rate below 3%, the Crank–Nicolson method can be considered feasible and reliable for determining the price of European call options using real stock data in the Indonesian capital market.

From a computational perspective, increasing the number of spatial and temporal grids to improve numerical accuracy will have a direct impact on the complexity of the calculations. At each time step, the resulting system of linear equations must be solved iteratively, so larger matrix sizes require more computing time. Therefore, there is a trade-off between the accuracy of the results and computational efficiency. In this study, increasing the number of grids did not result in a significant reduction in error, so the selection of grid configuration took into account the balance between numerical stability and computational efficiency.

Although the Crank–Nicolson method shows results consistent with the Black–Scholes analytical solution, it should be noted that this model assumes constant volatility throughout the contract period. In practice, stock market volatility in Indonesia tends to be dynamic and influenced by macroeconomic conditions and global sentiment. If volatility changes significantly during the observation period, the estimated option price results may deviate more significantly than models with constant volatility.

Therefore, developing models that take into account stochastic volatility or models with time-varying parameters may be a more realistic alternative. In addition, incorporating dividend payments into stocks can also improve the accuracy of option price estimates, especially for stocks that regularly distribute dividends to investors.

From a numerical perspective, although the results show a small relative error, further error analysis such as the effect of the stock price domain truncation boundary and sensitivity to the selection of the maximum stock price value (S_{max}) needs to be considered. In the context of the Indonesian market as an emerging market with relatively high volatility, the selection of model parameters and domain boundaries are important factors that can affect the accuracy of estimation results.

The results obtained in this study serve as methodological validation of the application of the Crank–Nicolson scheme to real stock data in the Indonesian capital market. Because the testing was conducted on a single stock and a single numerical approach, the conclusions drawn are limited to the context of this case study. Therefore, these findings are more appropriately viewed as proof of concept regarding the feasibility of implementing the method, rather than as a comprehensive generalization of all stock characteristics in emerging markets.

The novelty of this research does not lie in the development of a new numerical method, but rather in the application and evaluation of the Crank–Nicolson method on Indonesian stock data as an emerging market. The Indonesian stock market has different characteristics compared to developed markets, such as a relatively high level of volatility and potential market inefficiencies. Therefore, this study makes a practical contribution by showing how established numerical methods can be applied effectively in the context of emerging markets.

4. CONCLUSION

This study discusses the determination of European call option prices using the Black–Scholes model with a numerical approach using the Crank–Nicolson finite difference method and compares it with the Black–Scholes analytical solution based on real stock data in the Indonesian capital market. Based on historical stock price data from January 2025, to December 2025, the model parameters used include the initial stock price $S_0 = 3660$, annual volatility $\sigma = 0.3660$, risk-free interest rate of 5% per annum, at-the-money strike price, and a one-year maturity.

The calculation results show that the Crank–Nicolson method produces a European call option price of 596.08, while the Black–Scholes analytical solution gives a value of 612.50. A comparison of the two approaches produces a relative error of 2.68%, indicating that the Crank–Nicolson method has a good level of accuracy and is capable of approximating the analytical solution.

The differences in values that still appear between numerical and analytical solutions is mainly due to the process of discretizing the stock price and time domains in the numerical method, as well as the limitations of the maximum price domain used in the calculation. Nevertheless, the results of this study confirm that the Crank–Nicolson method is stable and convergent, making it suitable for use as an alternative in determining stock option prices, especially when analytical solutions are difficult to obtain or the model is expanded to more complex conditions.

With a low relative error rate, this study shows that the finite difference-based numerical approach can be effectively applied to determine the price of European call options using real stock data from the Indonesian capital market. Further research can develop models that take into account non-constant volatility or other types of options to obtain more comprehensive results.

In practical terms, these findings imply that numerical methods can serve as analytical tools for investors and market participants in estimating the fair value of options, especially in emerging markets where derivative instruments are still limited. In addition, this approach also contributes to the development of applied mathematics in finance, particularly in the implementation of numerical methods for solving financial partial differential equations. The uniqueness of this study lies in the application of the Crank–Nicolson method using real stock data in the Indonesian capital market as a representation of emerging markets, which have different volatility and market dynamics compared to developed markets. This implementation shows that numerical methods that are commonly tested in developed markets remain relevant and adaptive when applied to emerging market environments.

Although this study is limited to a single stock and the classic Black–Scholes assumptions, the results obtained show that the Crank–Nicolson method is quite reliable in the context of the Indonesian capital market. Therefore, further research can expand the scope by considering more than one stock, incorporating dividends or stochastic volatility, and comparing the performance of numerical methods under various market conditions to strengthen the generalization and contribution of the research. Thus, the conclusions obtained in this study are contextual and serve as an initial validation of the method's application in emerging markets, so that broader generalizations require additional testing on more diverse data and methods.

For further research, model development can be carried out by considering dividend payment factors, stochastic volatility, or models with time-varying parameters to better reflect dynamic market conditions. In addition, the application of adaptive discretization schemes or extensions to other types of options can be a relevant research direction to improve the accuracy and flexibility of numerical methods in real-world applications.

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