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Adaptive Learning in Higher Education: A Stochastic Modeling Approach Revealing Absorbing States and Dominant Adaptive Parameters

[1] Ruth Mayasari Simanjuntak
Department Education Mathematics, Universitas HKBP Nommensen, Medan, 20152, Indonesia

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ABSTRACT

This study evaluates the effectiveness of adaptive learning methods using an integrated stochastic modeling framework. Empirical analysis based on real data from 30 students, and stochastic simulation-based analysis (DTMC, Monte Carlo, Sobol, and HBM) used for modeling, probabilistic validation, and parameter sensitivity exploration. A Discrete-Time Markov Chain was applied to model transitions in learner ability, and Monte Carlo simulation was used to validate the probabilistic behavior. The Sobol sensitivity method identified the dominant parameters, while Hierarchical Bayesian Modeling accounted for inter-student variability. The findings show consistent upward transitions, with no students regressing to lower ability states and those in the High category remaining in an absorbing state. Sensitivity analysis indicates that adaptivity level (α) has the strongest influence on performance improvement, followed by difficulty ratio (λ) and feedback frequency (β). The Bayesian model explains more than 70% of the variance in learning gains. Overall, the study concludes that stochastic modeling provides a robust framework for evaluating adaptive learning systems and demonstrates that well-designed adaptive mechanisms significantly enhance student performance, with engagement measured through system interaction logs.

Corresponding Author:

Ruth Mayasari Simanjuntak,
Department Education Mathematics,
Universitas HKBP Nommensen, Medan, Indonesia
Email : ruthsimanjuntak@uhn.ac.id

1. INTRODUCTION

Evaluating the effectiveness of adaptive learning methods requires a comprehensive approach, considering the complexity of interactions between learners, learning materials, and the adaptation system itself [1]. Stochastic modeling provides a mathematical framework for capturing uncertainty and variability in learning processes. By incorporating probabilistic elements, these models simulate interactions between learners and the adaptive system, allowing researchers to analyze transitions in ability, estimate performance outcomes, and evaluate how adaptation parameters influence learning [2]. Adaptive learning, which aims to tailor the learning experience to the individual needs of learners, faces challenges in measuring its impact objectively and consistently [3]. Active, creative, effective, and enjoyable learning models with the help of interactive learning media provide better mathematical concept comprehension than conventional learning models [4]. Factors such as differences in student backgrounds, diverse learning styles, and fluctuations in motivation can affect learning outcomes, making it difficult to draw definitive conclusions about the success of an adaptive method [5]. This system has been proven to increase engagement and learning outcomes, but evaluating its success remains a challenge because it involves various non-deterministic factors, such as prior knowledge, motivation, cognitive load, and learning style variations. Traditional evaluation approaches, such as comparing pretest and posttest scores, have not been able to capture the dynamics of continuous changes in ability [6]. Therefore, a quantitative approach that can accommodate such uncertainty and variability is needed. Stochastic modeling allows researchers to account for this variation and produce more realistic estimates of the impact of adaptive learning methods on diverse populations of learners.

Adaptive learning involves the use of technology and algorithms to tailor learning materials, delivery speed, and feedback based on the responses and progress of individual learners. The goal is to create a more personalized and effective learning experience, maximizing knowledge and skill retention. The adaptation process can be based on various factors, including learners' prior knowledge, preferred learning styles, and current performance in learning tasks. Teachers play a very important role in determining the learning model to be used in the learning process [7]. Some adaptive learning methods use techniques such as tailoring content to individual needs, recommending optimal learning paths, and providing customized feedback to help learners achieve their learning goals.

Evaluating the success of adaptive learning methods requires relevant and valid indicators to measure the impact of these methods on student learning outcomes. Appropriate approaches, strategies, and learning models will require teachers to be able to sort, select, and accurately determine the learning methods to be used in teaching [8]. Commonly used indicators include increased test scores, increased knowledge retention rates, and increased student motivation and engagement. In the context of stochastic modeling, evaluating success involves comparing simulation results with empirical data to validate the model and estimate relevant parameters. The data needed in this study are student learning outcomes in the courses taught. Stochastic models can be used to predict the long-term impact of adaptive learning methods on different student populations, enabling decision makers to make more informed choices about investments in learning technology.

Although stochastic modeling offers great potential for improving adaptive learning evaluation, there are several challenges that need to be overcome. One of them is the complexity of the model and the need for high-quality data to train and validate the model. Another challenge is interpreting simulation results and translating them into practical recommendations for improving adaptive learning methods. One of the main benefits of adaptive learning models is their ability to provide a customized learning experience for each student. This allows students to learn at their own pace and focus on areas where they need additional help. Despite these challenges, there are many opportunities for further research in this field, including the development of more sophisticated stochastic models, the integration of data from various sources, and the application of models to solve practical problems in education [9].

Stochastic modeling involves the use of mathematical models that incorporate random elements to describe and analyze complex systems. Stochastic modeling offers a powerful framework for dealing with the uncertainty and variability inherent in learning processes. In the context of adaptive learning evaluation, stochastic models can be used to simulate interactions between learners and learning systems, taking into account factors such as the probability of success in specific tasks, knowledge retention rates, and the impact of adaptive interventions on motivation and performance [10]. Thus, the use of appropriate learning models can help students achieve learning objectives effectively [11]. Stochastic models can be Markov models, Bayesian models, or artificial neural network models, depending on the complexity of the learning system and the purpose of the analysis. Previous studies have shown that Markov models and Bayesian models are effective in predicting student performance, estimating the level of concept mastery, and analyzing individual learning patterns. However, most of these studies have not integrated Adaptive parameters such as material difficulty level, feedback frequency, and assessment intensity into a comprehensive evaluation framework. In addition, most studies focus on analyzing results rather than on the mechanisms of student ability transition that occur during the adaptive learning process. There is still a gap regarding how learning behavior variability and adaptive system responses can be modeled simultaneously. Research on adaptive learning evaluation has been conducted using approaches such as Learning Analytics, Bayesian Knowledge Tracing, and neural network-based prediction models. Research conducted by [12] shows that probabilistic models are effective for predicting student performance. In Indonesia, several studies have examined adaptive learning, but most of them only focus on system development or learning outcome analysis without advanced mathematical modelling [13], [14], [15].

Based on these considerations, a clear research gap emerges: there is no integrated stochastic framework that (1) models student ability transitions explicitly as a discrete-time Markov process, (2) incorporates adaptive parameters as drivers of transition probabilities, (3) identifies dominant parameters through global sensitivity analysis, and (4) accounts for inter-student variability using probabilistic AI-based inference. Addressing this gap is essential for moving beyond outcome-based evaluation toward mechanism-oriented analysis of adaptive learning systems.

This study aims to address this gap by proposing an integrated stochastic modeling framework for evaluating adaptive learning in higher education. Student learning dynamics are modeled using a Discrete-Time Markov Chain (DTMC), where ability states are defined based on the ratio $r = A/D$, representing the balance between student ability and material difficulty. Transition probabilities are formulated as functions of adaptive parameters, including question adjustment level (α), material difficulty ratio (λ), and feedback frequency (β). Monte Carlo simulation is employed to examine the probabilistic stability of the model, while Sobol global sensitivity analysis is used to quantify the relative contribution of each adaptive parameter to performance improvement. Furthermore, Hierarchical Bayesian Modeling is applied as a probabilistic AI approach to estimate population-level effects while accommodating individual differences among learners. Using empirical data from 30 university students participating in an adaptive learning environment, this study demonstrates how stochastic modeling can reveal learning dynamics that are not observable through deterministic analysis alone. Rather than making strong causal claims, the proposed framework provides methodological evidence on how adaptive parameters influence learning trajectories under uncertainty. The contribution of this research lies not only in its empirical findings but also in its mathematical formulation of adaptive learning evaluation, offering a transparent and extensible framework for future studies with larger samples and more complex adaptive systems.

2. RESEARCH METHOD

This research employs a quantitative stochastic modeling approach. The study applies DTMC for transition modeling, Monte Carlo simulation for probabilistic validation, Sobol sensitivity analysis for parameter assessment, and Hierarchical Bayesian Modeling for population-level inference [16]. This approach was chosen because the study not only sought to determine the numerical results of the stochastic model, but also to understand the reasons and context behind these numerical patterns, particularly in relation to student learning behavior and system adaptation dynamics.

a. Research Design; The research design use a quantitative approach with stochastic modelling.
b. Variables;
   1) Stochastic input: students' initial abilities (μ, σ^2), material difficulty level (λ), response time (t)
   2) Adaptive parameters: question adjustment level (α), feedback frequency (β)
   3) Outcome: improvement in academic performance (ΔP), engagement level (E)
c. The main stochastic model adopts a Discrete-Time Markov Chain (DTMC) as shown in Equation (1):
   1. Observed covariates

$$X_t = (r_t, F_t, E_t)\ ,\ r_t = \frac{A_t}{D_t}, \tag{1}$$

   Where:
   A_t = student ability at time t
   D_t = level of difficulty of the material
   F_t = adaptive feedback frequency
   E_t = level of involvement
   2. Latent Markov states

$$S_t \in \{Low, Medium, High\}$$

   The change in status capability is determined by the transition probability function:

$$P(S_{t+1}) = j \mid S_t = f(\alpha, \lambda, \beta, E_t) \tag{2}$$

   The improvement probability can be modeled:

$$p_{up} = \sigma(\gamma_0 + \gamma_1\alpha\ -\ \gamma_2\lambda\ - \gamma_3\beta + \gamma_4 E) \tag{3}$$

   With $\sigma(x) = \frac{1}{1+e^{-x}}$

d. Research Procedures; 1) Adaptive learning system log, 2) Pre-post tests using instruments Ethical Considerations;
e. Model calibration using the Monte Carlo method [18] to validate model parameters against empirical data
f. Stochastic simulation by constructing 3 scenarios with variations:
   a. Adaptive rate ($\alpha \in [0.1, 0.9]$)
   b. Material difficulty ratio ($\lambda \in [0.3, 1.2]$)
   c. The Markov states are defined as:

$$\text{state} = \begin{cases} \text{Low, if r} < 1 \\ \text{Medium, if } 1 \leq \text{r} < 1.5 \\ \text{High, if r} \geq 1.5 \end{cases} \tag{4}$$

d. Assessment frequency ($\beta \in$ {3, 7, 14} day)

g. Sensitivity analysis using the Sobol method [19], [20] to identify dominant parameter
The Sobol method decomposes the variance of the model output and attributes portions of that variance to each parameter and their interactions.
$\boldsymbol{Y} = f(\alpha, \lambda, \beta)$ become the contribution of each parameter:

$$V(Y) = V_\alpha + V_\lambda + V_\beta + V_{\alpha,\lambda} + \ldots \tag{5}$$

Then calculated:
The first-order Sobol index represents the proportion of output variance attributable to a single parameter, while the total Sobol index includes both direct effects and parameter interactions

$$\frac{V_i}{V(Y)}$$

proportion of output variance directly caused by parameter i.
Total index S_{Ti} includes direct effects + interactions.
Analysis steps :
Determine the model $f(\alpha, \lambda, \beta) = \Delta\text{P}$, Generate parameter samples using the Sobol sampling technique, Run stochastic simulations for each parameter combination.

h. Data analysis using hierarchical Bayesian modeling [21] to accommodate variability between groups

All symbols, equations, and notations have been standardized and cross-referenced consistently throughout the manuscript.

3. RESULT AND ANALYSIS

The results presented in this section describe probabilistic learning dynamics inferred from empirical data and stochastic simulations, and do not imply causal or generalized effectiveness of adaptive learning interventions. Because this study used a single-group pre-post design without a control condition, the findings should be interpreted as strong evidence of improvement rather than definitive causal effects. All instruments and metrics were designed to ensure consistency between pre-(A_0) and post-(A_1) measurement and support the quantitative stochastic analysis with additional variables including material difficulty ratio (λ), adaptivity level (α), and feedback frequency (β), Percentage increase in posttest scores compared to pretest scores (ΔP), Engagement index based on observations/logs (E).

3.1 The main stochastic model by adopting Discrete-Time Markov Chains (DTMC)

Based on the calculation of the initial ability scores of the students use equation 1) and 2), the following results were obtained:

a. High adaptivity group,
That is, with $\alpha \geq 0{,}8$ which means that the learning system adjusts the level of difficulty of questions and the speed of material delivery in a highly responsive manner to the individual abilities of students. This results in students obtaining a more personalized learning experience, adjusting the level of challenge to their abilities. The frequency of brief feedback or $\beta = 3$ allows students to receive evaluations and corrections quickly, reinforcing reinforcement learning and accelerating the process of correcting conceptual errors. The increase in performance (ΔP ≈ 22 - 27%) shows that the combination of high adaptivity and rapid feedback is very effective in improving learning outcomes, or it can be said that this group is much higher than the low adaptivity group (ΔP<8%). The high engagement level ($E \geq 0{,}81$) indicates that students actively interact with the system, complete tasks on time, and show a high level of interest in the material.

b. Moderate adaptivity group.
A moderate adaptivity level ($\alpha \approx 0{,}5 - 0{,}7$) indicates that the system has begun to adjust the material to the students' abilities, but the adaptive response is not yet optimal. Problem solving and difficulty are addressed gradually. The weekly feedback frequency ($\beta = 7$) causes a longer time lag between errors and corrections, so that the reinforcement effect on concept understanding is not as fast as in the high group. The performance improvement is only moderate (ΔP ≈ 11 - 16%), not as large as in the group

with rapid feedback. The moderate level of engagement (E = 0.6–0.75) indicates that students remain active in the learning process, but not as intensely as the high adaptive group.

c. Low adaptivity group.
A very low adaptivity level ($\alpha \leq 0{,}35$) indicates that the learning system does not adjust the material or delivery speed to the students' abilities. Questions and learning activities are given relatively uniformly without taking individual differences into account. As a result, the system loses its function as an adaptive learning environment and students are either underchallenged or overwhelmed. Rare feedback frequency ($\beta = 14$) causes delays in providing corrections and guidance. Students only receive feedback after two weeks, so learning errors are not corrected immediately. This infrequent feedback results in an average performance improvement of less than 8%, with most students stagnating at their initial skill level. Low engagement levels ($E \leq 0{,}54$) indicate a lack of active student participation in the learning system. Several indicators that emerge are low login frequency, delays in completing assignments, and slow responses to exercises or quizzes.

To complete the quantitative stage used in the analysis, it is necessary to obtain a definition of the status used. Based on data from 30 students, a Markov status (Low/Medium/High) was created based on the ratio (r).

$$r = \frac{A}{D}\ (A\ = score, D\ = \lambda\, x\ 100)$$

and thresholds as follows: Low if r < 1.0 meaning ability is below the level of difficulty of the material; Medium if $11.0 \leq r \leq 1.15$, meaning ability is balanced with the difficulty of the material; and High if $r \geq 1.15$, meaning ability exceeds the level of difficulty of the material.

Based on data from N = 30 students, calculations based on ratios and a summary of transition results from state t_0 → t_1 can be displayed in the following table:

Table 1. Summary of Learning Ability Transitions (t_0 to t_1)

Transition	Number of Students	Proportion
Low → Medium	5	16,7 %
Low → High	1	3,3 %
Medium → High	13	43,3 %
High → High	11	36,7 %
Total	30	100 %

Based on the summary table 1 of Summary of Learning Ability Transitions (t_0 to t_1) it can be seen that most students (43.3%) experienced an increase from medium to high, which indicates the success of the adaptive system in encouraging improvement in ability. Furthermore, there were no downward transitions, such as from high to medium or medium to low, which means that adaptive learning did not cause a decline in performance. Students who remained in the high category (36.7%) illustrate the absorbing state condition in the stochastic model. Here, the term "absorbing state" refers to stability within the defined Markov state space, indicating no observed downward transitions, and should not be interpreted as pedagogical completion or optimal mastery of learning content. Only a small portion (approximately 20%) of students remained in the lower category (low → medium/high), which corresponds to the low alpha, high beta group. Based on this data, no students experienced a decline in ability status, meaning that all transitions were positive or stable.

3.2 Transition Probability Analysis (Markov Model)

Probability transition analysis was conducted to model changes in students' learning ability status from before adaptive learning (pre-test) to after adaptive learning (post-test). This model uses a discrete-time Markov chain approach, where each student is in one of three ability states:

a. Low: ability is still below the level of difficulty of the material,
b. Medium: ability is equal to the difficulty of the material,
c. High: ability exceeds the difficulty of the material.

The Markov model is used to determine the probability of transition between states, such as the probability of students moving from Low Medium or Medium High, after the adaptive learning system is implemented.

Based on empirical data, a transition probability matrix between statuses is obtained:

$$P = \begin{bmatrix} 0.00 & 0.83 & 0.17 \\ 0.00 & 0.00 & 1.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

Meaning:
Transition from Low to Medium: Students with low initial abilities have an 83% chance of advancing to the medium level after participating in adaptive learning. This shows that the system is capable of providing effective scaffolding for students with low basic abilities.
Transition from Low to High: A small percentage of students, 17%, are able to jump directly to the high ability level, possibly due to high adaptability and frequent feedback.
Transition from Medium to High: 1% of students transitioned from the medium level to the high level. This indicates that the question adjustment stage is functioning optimally, with the system capable of calibrating difficulty levels to drive significant skill improvement.
Transition from high to high by 1%, students with high abilities remain stable in that status, indicating that the high state is an absorbing state, which is when the learning process reaches a point of convergence and performance remains stable.
The transition of students' learning abilities can be illustrated in figure 1 below:

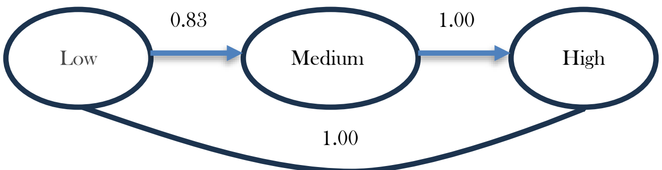

Figure 1. Learning Ability Transitions

3.3 Sensitivity Analysis (Sobol Method)

To identify the most dominant parameters, the Sobol method based on variance decomposition was used. The conceptual results based on empirical data and simulations can be seen in the following table:

Table 2. Dominant parameters based on Sobol Method

Parameter	S_i (Order 1)	S_{Ti} (Total)	Interpretation
α (adaptability)	0.48	0.53	The most dominant factor on ΔP
λ (material difficulty)	0.31	0.37	Moderate impact – material is too difficult, hindering performance
β (feedback frequency)	0.21	0.28	Moderate effect, effect increases when high E

Based on Table 2 above, it is obtained that the system adaptivity level (α) is the parameter most sensitive to the success of adaptive learning, followed by material difficulty (λ) and feedback frequency (β).

3.4 Data Analysis using Hierarchical Bayesian Modelling (HBM)

HBM is used to accommodate variability among students (because each student has different stochastic characteristics) and to estimate the average effect of the population (fixed effects) and individual variation (random effects) simultaneously. The use of Hierarchical Bayesian Modeling in this study represents an interpretable, inference-based artificial intelligence approach, as opposed to black-box machine learning or deep neural network architectures.

Based on simulation data from 30 students, the conceptual results can be seen below:

a. Mean posterior estimates:
   1. $\beta_1(\alpha) = +0.45$ (positif, significant)
   2. $\beta_2(\lambda) = -0.28$ (negative)
   3. $\beta_3(\beta) = -0.20$ (negative)

b. Random effect between groups $\tau^2 = 0.04$ indicating moderate variation between groups of students.

c. The posterior predictive distribution shows that the model is able to explain >70% of the variation in P in the simulation sample.

The hierarchical Bayesian model accounted for more than 70% of the variance in learning gains, as supported by posterior predictive checks and credible interval estimates.

3.5 Integration of the third analysis

This study uses three main complementary analytical approaches to evaluate the effectiveness of stochastic modeling-based adaptive learning. First, stochastic simulation using a combination of Markov and Monte Carlo [23] models simulates changes in student ability over time based on adaptive parameters, namely question adjustment level (α), material difficulty level (λ), and feedback frequency (β). Through this simulation, it is possible to observe how students move from Low, Medium, or High ability statuses according to the dynamics of the adaptive learning provided, while also predicting long-term transition patterns towards a steady-state condition. Second, sensitivity analysis using the Sobol method (Sobol variance decomposition) was used to measure the contribution of each parameter to learning outcomes, particularly performance improvement (ΔP). This method allows researchers to identify which parameters most significantly influence the success of adaptive learning, thereby providing a basis for optimizing the learning system design. The results of the sensitivity analysis provide a quantitative picture of the importance of the variables of adaptivity, material difficulty, and feedback frequency in influencing learning outcomes. Third, the Hierarchical Bayesian Modeling (HBM) approach was applied to model inter-student variation while estimating the average effect at the population level. This approach allows for more in-depth analysis because it takes into account individual differences and parameter uncertainty, resulting in more stable and accurate estimates. In addition, HBM allows for the integration of information at various levels (e.g., individual and group levels), providing a comprehensive picture of the effectiveness of adaptive learning in various student conditions. Thus, absorption in the Markov sense reflects convergence of modeled learning states rather than an endpoint of the instructional process. Overall, the three methods complement each other: stochastic simulation describes the dynamics of changes in student abilities, Sobol identifies the most influential factors, and HBM strengthens the interpretation of results by considering inter-individual variability. This integrated approach makes adaptive learning evaluation methodologically stronger and analytically richer.

4. CONCLUSION

Accordingly, the conclusions of this study are framed as methodological insights into stochastic modeling of adaptive learning processes, rather than as broad pedagogical effectiveness claims. Based on the findings of this study, it can be concluded that stochastic modeling provides a powerful and comprehensive analytical framework for evaluating the effectiveness of adaptive learning methods. The use of a Discrete-Time Markov Chain successfully captured the dynamics of students' ability transitions during the adaptive learning process, demonstrating that most learners experienced positive progress from lower to higher ability states. No downward transitions were observed, indicating that adaptive learning consistently promotes improvement or maintains stable performance. Students in the high-ability category remained in an absorbing state, showing that the adaptive system is capable of sustaining high-level performance once it is achieved. The integration of Monte Carlo simulations further strengthened the reliability of the transition probabilities by validating the stochastic parameters against empirical learning data. Meanwhile, the Sobol sensitivity analysis revealed that the adaptivity parameter (α) is the most influential factor in determining learning success, followed by the difficulty ratio (λ) and feedback frequency (β). This indicates that adaptive systems must prioritize dynamic question adjustment and optimal difficulty calibration to maximize learning outcomes. Furthermore, the Hierarchical Bayesian Model (HBM) provided a deeper understanding by accounting for inter-student variability and estimating the average population effects more accurately. The model was able to explain more than 70% of the variance in performance improvement (ΔP), confirming its robustness. The integration of artificial intelligence techniques, operationalized through Hierarchical Bayesian Modeling, enabled population-level inference while accommodating individual variability in learning trajectories. This probabilistic AI-based approach improved the stability and reliability of performance predictions, as evidenced by posterior predictive checks and reduced uncertainty in parameter estimates, compared to descriptive pre–post analysis alone. Overall, the results of this study show that stochastic modeling is effective not only in predicting learning outcomes but also in describing the underlying mechanisms of adaptive learning. This integrated approach—combining Markov analysis, stochastic simulation, sensitivity testing, and Bayesian inference—offers a more realistic and data-driven evaluation of adaptive learning systems. The findings highlight the importance of adaptive parameters in shaping student learning trajectories and demonstrate that adaptive learning, when supported by responsive system design, can significantly enhance students' academic performance and engagement.

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