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A Modified Frequency-Based FLRG Fuzzy Time Series Model for National Rice Production Forecasting

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ABSTRACT

Accurate predictions of national rice production are crucial for food sustainability, yet data fluctuations pose a major challenge. This study aims to improve forecasting accuracy by developing a modified Fuzzy Time Series (FTS) model that simplifies the Fuzzy Logical Relationship Group (FLRG) by retaining only the logical relationships with the highest frequency of occurrence. Monthly Indonesian rice production data from January 2018 to March 2025 were used to test this model. To assess the effectiveness of this modification, the model's performance was compared with Chen's conventional FTS models of orders 1 to 3 through MAD, RMSE, and MAPE. Results indicate that the modified third-order FLRG achieved the best accuracy (MAD = 196,410; RMSE = 271,774; MAPE = 5.46%), while reducing FLRG complexity by 10.84%. This demonstrates that FLRG simplification effectively captures longer seasonal dependencies while reducing computational complexity. Nevertheless, the model's sensitivity to sudden structural changes underscores the need for adaptive or probabilistic FLRG enhancement, with hybrid mechanisms as a potential complement. Overall, the proposed approach provides an efficient decision-support tool for maintaining food supply stability and guiding data-driven agricultural policy in Indonesia.

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1. INTRODUCTION

The impact of imported rice can affect the welfare of local rice farmers, particularly in terms of selling prices, as imported rice is often cheaper than domestically produced rice. Overreliance on rice imports also exposes Indonesia to global food price volatility and disruptions in the international supply chain. Therefore, accurate forecasting of national rice production is essential to support evidence-based food policy, particularly for import planning and maintaining food supply stability [1]. Recent studies have highlighted the growing importance of intelligent forecasting systems, including machine learning and hybrid approaches, to support national food security strategies when dealing with highly volatile agricultural commodities influenced by multiple external drivers [2]. However, although such approaches often demonstrate strong predictive performance, they generally require large training datasets and offer limited interpretability for policy-oriented decision making, motivating the continued use of rule-based forecasting methods.

Among rule-based forecasting approaches, Fuzzy Time Series (FTS), initially introduced by Song and Chissom [3] and later simplified by Chen [4], has emerged as a robust method for modeling agricultural data due to its ability to handle uncertainty, seasonality, and non-stationary behavior. This framework was further enhanced by Tsaur [5], who introduced techniques to refine fuzzy relationships, providing a more structured approach to capturing temporal dependencies. Bilal et al. [6] then demonstrated that FTS-based models can accurately forecast rice production and prices by effectively capturing seasonal agricultural patterns. In a more applied institutional context, Wahyu et al. [7] showed that FTS can be implemented within governmental agricultural systems to support operational crop production planning with improved forecasting reliability. Meanwhile, Lestari and Yurinanda [8] emphasized the suitability of FTS model for economically sensitive time series, highlighting its interpretability and stability when dealing with fluctuating real-world data. From a theoretical perspective, the effectiveness of FTS models is fundamentally grounded in fuzzy set theory, particularly in the construction of linguistic variables and membership functions that enable the representation of uncertainty and imprecision in real-world time series data [9].

Beyond classical univariate formulations, more recent studies have extended FTS into multivariate and multi-rule combination frameworks to further enhance forecasting accuracy. In particular, Huang et al. [10] proposed a multivariate fuzzy time series model that combines multiple fuzzy rules to capture complex dependency structures, demonstrating improved predictive performance on compositional time series data. Nevertheless, such advanced models typically increase rule complexity and computational requirements, which may reduce their transparency and practical usability for policy-driven forecasting applications.

Despite their demonstrated effectiveness, these existing FTS studies [6], [7], [8] primarily rely on first-order or classical FTS formulations. Consequently, they struggle to capture long-term temporal dependencies in highly volatile agricultural time series, limiting their predictive horizon and overall accuracy. A major limitation of these conventional FTS methods is the lack of flexibility in defining Fuzzy Logical Relationship Groups (FLRGs) that specifically handle different variations or sub-patterns within highly volatile time series data. To overcome this, High-order Fuzzy Time Series was developed as an extension of the classical FTS framework, which is an advancement of the Fuzzy Time Series (FTS) method introduced by Chen [11], where forecasting does not only rely on previous historical data, but uses two or more historical data to form a more complex Fuzzy Logic Relation (FLR) [12]. The purpose is to improve forecasting accuracy by incorporating more historical information into the model. However, conventional high-order FTS methods often produce complex Fuzzy Logic Relationship Groups (FLRG) because they cover all possible state transitions [13]. This exhaustive approach generates "noisy" logical relationships that are based on infrequent or coincidental data patterns. This noise directly reduces the model's prediction accuracy and robustness, especially when dealing with volatile agricultural data.

Despite these advancements, a systematic frequency-based filtering of FLRG transitions in high-order FTS has not been adequately explored. This lack of simplification approach, specifically the failure to systematically filter rare or insignificant state transitions constitutes the core research gap in current FTS applications, where the few studies that have introduced advanced FTS approaches, such as higher-order models [11], [12] and multivariate or hybrid frameworks [9], [10], generally retain all transitions in the FLRG without filtering out rare events. Addressing this limitation is crucial, as the resulting rule complexity significantly hampers the model's performance and interpretability when applied to highly volatile real world time series like agricultural data.

To close this gap, this study proposes a novel modification to the FTS framework. A simplified FLRG formation rule is developed to mitigate the issues of noise and complexity in high-order FTS. The proposed model simplifies the FLRG by retaining only the logical relationships with the highest frequency of occurrence, thus filtering out the insignificant noise. This refinement aims to provide a cleaner and more interpretable framework that potentially enhances both forecasting precision and model robustness. Such improvements are relevant for national food security, as increased accuracy is a key requirement for supporting government intervention and data-driven agricultural policy in Indonesia.

To this end, the objectives of this study are to (1) propose a frequency-based FLRG simplification; (2) compare the forecasting accuracy of the proposed approach with conventional Chen FTS models of orders 1–3; (3) quantify the reduction in FLRG complexity; and (4) assess the suitability of the modified FTS model as a decision-support tool for national rice production policy.

2. RESEARCH METHOD

2.1 Fuzzy Time Series (FTS) Method

Fuzzy Time Series (FTS) is a forecasting approach that utilizes the concept of fuzzy sets to estimate the future value based on its historical data patterns [14], [15]. This method converts numerical data into fuzzy linguistic representations, then analyses the relationships between data to make predictions, and converts the prediction results back into numerical form [16]. FTS is highly appropriate for this study as it manages data uncertainty, performs well with limited and non-stationary datasets, and derives forecasting knowledge directly from historical patterns, making it suitable for agricultural forecasting. Unlike most statistical time series models, FTS does not rely on stationarity assumptions, which are rarely satisfied in practical datasets [17].

Definition of FTS

a. Fuzzy Set

For each linguistic value, define a fuzzy set with a specific membership function, which associates each value in the universe of discourse with a membership value between 0 and 1. This change in value is called the fuzzification of historical data [18]. In the fuzzy time series framework, membership functions define the degree to which each data point belongs to a fuzzy set, directly influencing the model's forecasting precision [19]. Generally, this is the process of converting rigid numerical data into linguistic categories (words) to process uncertainty.

b. Fuzzy Logic Relations (FLR)

Fuzzy Logical Relationships (FLR) describe the temporal dependency between consecutive fuzzy sets, establishing the basis for mapping transitions among linguistic states in the time series [20]. FLR is formed after time series data is converted into linguistic values through the fuzzification process. The goal is to model and predict complex and uncertain historical data patterns [21]. FLR captures the dependency between past and future states in linguistic form, making it ideal for predicting the next time value based on historical observations.

c. Fuzzy Logic Relations Group (FLRG)

Fuzzy Logical Relationship Groups (FLRG) are formed by aggregating FLRs that share the same antecedent fuzzy set, providing a grouped representation used in the forecasting inference stage [22]. FLRG serves to organize interrelated FLRs into a single group, where all FLRs that start from the same fuzzy set are grouped together [23]. FLRG functions as the model's knowledge base, it groups all possible outcomes that have historically occurred following a specific condition.

2.2 Present the procedural steps

Chen introduced high-order Fuzzy Time Series in 2002 [11]. The difference between Chen's high-order Fuzzy Time Series lies in the determination of Fuzzy Logic Relations (FLR) [24]. The FTS approach involves partitioning historical data into intervals and transforming them into fuzzy sets to establish temporal dependencies for forecasting [25]. This study follows the foundational FTS framework established by Song and Chissom [3], as further refined in recent literature [26], [27], [28]. The systematic steps are outlined below:

1. Construction of universal sets

The universe of discourse

$$U = [D_{min}\ ; D_{max}] \tag{1}$$

is defined based on the smallest (D_{min}) and the largest (D_{max}) values in historical data.

2. Constructions of intervals

The universal set U is divided into several intervals with equal distances to represent data variability. The number of intervals can be determined using average-based approach, forming linguistic values that represent fuzzy sets within the defined range of U.

$$U = \{u_1, u_2, u_3, .., u_n\} \tag{2}$$

where U denotes the universe of discourse, u_i represents its elements, and i = $1,2,3, ..., n$ refers to the index of each interval.

3. Fuzzification

A fuzzy set is a class or group of objects with a continuum of degrees of membership. Suppose U is the universal set, $U = \{u1\ , u2\ , \dots , un\}$ where u_i are elements of U (the possible values), then the linguistic variable A_i with respect to U can be formulated as follows:

$$A_i = \frac{\mu_{Ai}(u_1)}{u_1} + \frac{\mu_{Ai}(u_2)}{u_2} + \frac{\mu_{Ai}(u_3)}{u_3} + \cdots + \frac{\mu_{Ai}(u_n)}{u_n} \tag{3}$$

$\mu_{Ai}: U \to [0,1]$. If μ_i s membership of A_i then $\mu_{Ai}(u_i)$ is the degree of membership u_i to A_i

4. Establishing the Fuzzy Logic Relations (FLR)

Determine the FLR according to the time,

- If $F(t-1)$ fuzzification into A_i and $F(t)$ into A_j, then formed FLR $A_i \to A_j$
- A_i is the LHS (left hand side) which represents the previous condition
- A_j is the RHS (right hand side) which represents the next condition

5. Establishing the Fuzzy Logic Relations Group (FLRG).

That is, grouping several fuzzy logic relations (FLR) that have the same Left Hand Side (LHS) but different Right Hand Sides (RHS) into a single group. For example, $A_i \to A_j, A_i \to A_m, A_i \to A_m,\ A_i \to A_m,\ A_i \to A_n$ can be grouped into $A_i \to A_j, 3\ A_m, A_n$.

6. **Defuzzification**

If $F(t-1) = A_i$ resulting FLRG $(A_i \rightarrow A_{j1}, A_{j2}, \dots, A_{jk})$ than $F(t) = A_{j1}, A_{j2}, \dots, A_{jk}$, the defuzzification $\boldsymbol{\hat{y}(t)}$:

$$\hat{y}(t) = \frac{\sum_p^k m_{jp}}{\boldsymbol{k}} \tag{4}$$

where $\boldsymbol{m_{jp}}$ denotes the median value of $\boldsymbol{A_{jp}}$

2.3 High-Order Fuzzy Time Series Chen

The setup of FLR in Chen's high-order Fuzzy Time Series method considers two or more data points from the past according to the order used. If using order three, then in determining FLR, three historical data points will be considered. For example, in the Chen Fuzzy Time Series method using an order of two, if three consecutive fuzzy sets $A_i(t-2)$ and $A_j(t-1)$ and $A_k(t)$ can be formed as FLR $A_i, A_j \rightarrow A_k$. Hence, the resulting FLRG also changes. For example, in FLR, we obtain $A_1, A_1 \rightarrow A_1$; $A_1, A_1 \rightarrow A_2$; $A_1, A_1 \rightarrow A_3$ The resulting FLRG is $A_1, A_1 \rightarrow A_1, A_2, A_3$ [29]. High-order fuzzy time series models extend the conventional first-order structure by incorporating multiple past observations to establish more comprehensive fuzzy logical relationships, thereby enhancing the model's capability to capture long-term temporal dependencies and improving forecasting accuracy [30].

2.4 Prediction Value Accuracy

The accuracy of forecasting results is a measure of the degree of difference between actual results and forecast results [31]. In time series forecasting, the difference between actual observations and the forecast estimate is referred to as the residual [32]. Good accuracy has a low level of difference between the actual results and the forecast results, but if the level of difference between the actual results and the forecast results increases, the accuracy will become worse [33].

Accuracy measures used in this study:

1. **MAD**

Mean Absolute Deviation (MAD) accuracy value Used to calculate the absolute average value of a prediction error, the following is the calculation formula:

$$MAD = \frac{1}{n} \sum_{t=1}^{n} |A_t - F_t| \tag{5}$$

2. **RMSE**

Root Mean Square Error (RMSE) It is a measure of error that measures the difference between the actual value and the value predicted by the forecasting model. A lower RMSE value indicates higher forecast accuracy, the following is the calculation formula:

$$RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} |A_t - F_t|^2} \tag{6}$$

3. **MAPE**

Mean Absolute Percentage Error (MAPE) accuracy value is a calculation of the error in a prediction to find the average absolute percentage error, using the formula:

$$MAPE = \frac{1}{\mathrm{n}} \sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right| \times 100\% \tag{7}$$

A_t denotes the actual value, F_t represents the predicted value, *n* refers the number of observations in the sample.

2.5 Proposed Frequency-Based FLRG Modification

The proposed frequency-based modification focuses on the formation of the Fuzzy Logical Relationship Group (FLRG) rather than on altering the membership functions. Specifically, a new rule is introduced for constructing a simplified FLRG, where, for any given current state, the successor state(s) are determined based on the highest transition frequency observed in the original FLRG. In cases where multiple successor states share the same maximum transition frequency, a deterministic aggregation rule is applied to produce a unique successor value and avoid arbitrary rule selection.

The following algorithm describes the FLRG simplification process for a given antecedent fuzzy set, A_i:

1. **Form Standard FLRGs:**
   - Generate all possible Fuzzy Logical Relations (FLRs) from the fuzzified training data based on the chosen order (*k*).
   - Group these FLRs into standard FLRGs based on their antecedent. For a given antecedent A_i, the initial group is $A_i \rightarrow \{A_{j1}, A_{j2}, \dots, A_{jm}\}$ where A_j represents all observed subsequent states.
2. **Count Transition Frequency:**
   - For the specific antecedent A_i, calculate the frequency of each subsequent state A_j occurring.

3. **Identify Maximum Frequency Transition:**
    - Identify the next state, A_{max}, that occurred with the highest frequency count among all subsequent states $\{A_{j1}, A_{j2}, \dots, A_{jm}\}$.
4. **Simplify and Retain (The Modification):**
    - Retain only the identified transition $A_i \rightarrow A_{max}$. All other transitions (those with lower or equal frequency) are discarded.
5. **Replace FLRG:**
    - Replace the original multi-transition FLRG (which included all A_j) with the new, simplified FLRG that contains only the most frequent transition:
      Original $A_i \rightarrow \{A_{j1}, A_{j2}, \dots, A_{jm}\}$ is replaced by Simplified $A_i \rightarrow \{A_{max}\}$
    - If two or more successor fuzzy sets have the same highest transition frequency (tie condition), a deterministic aggregation rule is applied. Let $A_{j1}, A_{j2}, \dots, A_{jm}$ denote successor fuzzy sets whose frequencies equal the maximum frequency F_{max}. Instead of selecting a single successor arbitrarily, the model aggregates these fuzzy sets by computing the average of their representative values. This aggregated value is then used as the unique successor for the corresponding antecedent state.

In conventional high-order FTS models, the FLRG rule space grows rapidly with increasing model order due to the expansion of possible antecedent state combinations. The proposed frequency-based filtering is designed to control this growth by retaining only dominant transitions within each FLRG. The resulting reduction in rule-space complexity across different model orders is quantitatively evaluated in Section 3.4 (Table 5).

2.6 Justification for FTS Model order (*k*)

The FTS model order (*k*) determines the number of preceding time periods used for forecasting. This study tested and compared FTS models up to **order 3** (*k*=1, 2, and 3) for both the conventional and the proposed modified methodologies.

The selection of this range (up to order 3) is based on two primary considerations:

1. **Computational Efficiency and Complexity:** Preliminary analysis showed that increasing the model order beyond *k*=3 provides negligible accuracy improvement while substantially enlarging FLRG complexity. As the primary objective of this study is complexity reduction, testing higher orders was not pursued to avoid unnecessary computational burden.
2. **Capturing Historical Dependence:** FTS order 3 is sufficient to capture monthly dependencies over a quarter-year period, which aligns well with the known seasonal and short-term trends present in national rice production data.

The best model order is selected through empirical evaluation, with the configuration showing the lowest MAD, RMSE, and MAPE considered optimal, ensuring that complexity is determined based on evidence rather than assumption.

2.7 Research Flowchart

The overall procedure of this study, starting from data input to final forecasting, is illustrated in the Figure 1.

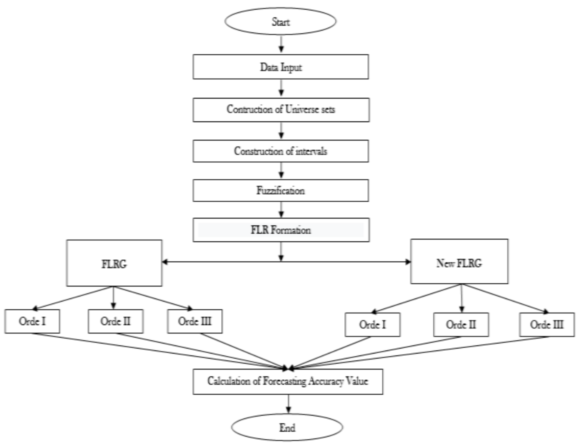

Figure 1. Research Flowchart

Figure 1 summarizes the overall forecasting process, from data preparation to performance evaluation. The key novelty lies in the modified FLRG formation stage, in which only the highest-frequency relationships are retained, and the New FLRG approach is applied to the orders (1-3) model.

3. RESULT AND ANALYSIS

3.1 Data Description

The data used in this study consist of monthly national rice production figures from January 2018 to March 2025 (87 observations), obtained from the official online database of Badan Pusat Statistik (BPS), specifically the 'Luas Panen dan Produksi Padi Menurut Bulan' series (Code: 5403001.01). The selected series corresponds to finalized monthly statistics, with no reported re-benchmarking or historical revisions for the study period.

Prior to analysis, the raw data were reviewed to ensure completeness and consistency, confirming that no missing observations were present and that data imputation was unnecessary. No normalization, detrending, or scale transformation was applied. This choice is justified by the Fuzzy Time Series (FTS) framework, in which fuzzification through interval partitioning and membership functions inherently accommodates magnitude differences in the original data without compromising membership resolution.

It should be noted that the production series may be affected by external factors such as agricultural policy changes or climate-related anomalies that are not explicitly modeled. These factors may introduce structural changes in the data; however, this study focuses on evaluating the proposed frequency-based FLRG simplification under real-world conditions rather than modeling all exogenous influences.

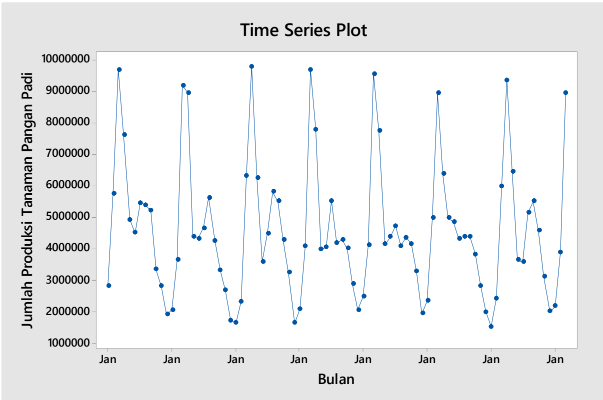

Figure 2. Time Series Plot of Indonesian rice food crops in 2018-2025

Figure 2 illustrates the strong seasonal cycle of Indonesia's monthly rice production, which typically reaches its lowest levels around January and peaks near April, with year-to-year variability in both peak intensity and trough depth. These recurrent peaks correspond to major harvest periods, whereas the troughs reflect planting or off-season intervals. Such volatility in production increases forecasting uncertainty, highlighting the need for models capable of capturing long-term dependencies. Consequently, higher-order FTS models particularly order 3 are more effective due to their extended memory of previous states. These observations also support the application of the modified FLRG approach in addressing unstable seasonal fluctuations.

3.2 Application of Fuzzy Time Series (Conventional Model)

The application of the FTS model commences with the definition of the universe of discourse, U. These initial steps ensure that the data structure aligns with the characteristics of the FTS framework.

1. Formation of the Universal Set U

The intervals formed were obtained by the universe of talks $U = [1{,}516{,}040;\ 9{,}768{,}002]$. Its by taking D_{min} as the smallest value from the data and D_{max} as the largest value from the research data. The next step is to determine data intervals.

2. Constructions of intervals

The universe of discourse was partitioned into 16 equal sub-intervals, $(u_1, u_2, u_3, \dots, u_{16})$, as the basis for interval construction. The midpoint of each interval was subsequently calculated to serve as the representative value for fuzzification, providing the linguistic framework for the FTS model (Table 1).

These median values function as reference points that enable each monthly observation to be mapped to its most appropriate linguistic term. This structured mapping facilitates the identification of temporal production patterns, which are essential for the subsequent formation of fuzzy logical relationships.

Table 1. Interval & Median Value (m)

Interval	Lower Limit	Upper Limit	Median Value (m)
u_1	1,516,038	2,043,038	1,779,538
u_2	2,043,038	2,570,038	2,306,538
u_3	2,570,038	3,097,038	2,833,538
⋮	⋮	⋮	⋮
u_{16}	9,421,038	9,948,038	9,684,538

Then the analysis is continued by determining the fuzzy set obtained defined as follows:

$$A_1 = \{\frac{1}{u_1} + \frac{0,5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \cdots + \frac{0}{u_{16}}\}$$
$$A_2 = \{\frac{0,5}{u_1} + \frac{1}{u_2} + \frac{0,5}{u_3} + \frac{0}{u_4} + \cdots + \frac{0}{u_{16}}\}$$
$$\vdots$$
$$A_{16} = \{\frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \cdots + \frac{0,5}{u_{15}} + \frac{1}{u_{16}}\}$$

Each fuzzy set A_i represents the linguistic state associated with the corresponding interval and illustrates the degree to which a given production value belongs to adjacent intervals through gradual membership transition (e.g., coefficient 0.5). This overlapping structure enables smoother representation of boundary values and improves the model's ability to capture transitional behavior between states. Following the fuzzification stage, the time-ordered sequence of fuzzy states is used to construct FLRs, which form the basis for subsequent forecasting.

3. Determining FLR

The subsequent step is data fuzzification, where each numerical data point is converted into its corresponding linguistic value. For instance, the production value recorded in January 2018 (2,783,961 tons) falls within interval U_3, and is therefore fuzzified as A_3. This process is applied to all subsequent observations, as shown in Table 2.

Table 2. Determining Fuzzification & FLR

Period	Data	Fuzzification	FLR
1	2783961	A_3	-
2	5738544	A_9	$A_3 \to A_9$
3	9678183	A_{16}	$A_9 \to A_{16}$
⋮	⋮	⋮	⋮
87	8931919	A_{15}	$A_5 \to A_{15}$

Table 2 illustrates the transformation of raw production data into fuzzy categories and how these categories evolve over time. The "Fuzzification" column assigns each monthly production value to its respective linguistic term (A_1, A_2, ..., A_{16}) based on its interval midpoint. Meanwhile, the "FLR" column captures the transitions from one fuzzy state to the next, reflecting how production levels progress between months—for example, moving from a lower to a higher-level during harvest periods or decreasing during planting phases. These fuzzy transitions provide valuable insight into production dynamics and form the foundation for identifying forecasting patterns through FLRG construction in later stages

4. Determining FLRG

At this stage, the Fuzzy Logical Relationships (FLRs) identified previously are aggregated into Fuzzy Logical Relationship Groups (FLRGs). This grouping process maps all observed transitions from a specific "Current Stage" (antecedent) to its corresponding "Next Stage" (consequent).

For example, the production value in January 2018 was mapped to A_3, while the value in February 2018 corresponded to A_9, resulting in the Fuzzy Logical Relationship (FLR) $A_3 \to A_9$. This procedure was applied sequentially to all observations through March 2025 and repeated for each FTS configuration (order 1, 2, and 3) to capture various levels of temporal dependence.

Table 3. Formation of FLRG order 1

Curent Stage	Next Stage	FLRG order 1
A_1	$A_1, A_5, A_1, A_2, A_2, A_2, A_2, A_1, A_2, A_2$	$A_1 \to 3A_1, 6A_2, A_5$
A_2	$A_{10}, A_5, A_5, A_7, A_9, A_5$	$A_2 \to 3A_5, A_7, A_9, A_{10}$
A_3	A_9, A_1, A_1, A_1, A_1	$A_3 \to 4A_1, A_9$
⋮	⋮	⋮
A_{16}	$A_{12}, A_9\ A_{12}, A_{12},$	$A_{16} \to A_9, 3A_{12}$

Table 3 illustrates the grouping of FLRs into FLRGs based on their antecedent states. For instance, the transition for A_1 is $A_1 \to 3A_1, 6A_2, A_5$, this means that there are 3, 6, and 1 weight assigned to each respective state. Under this conventional framework, all identified relationships are retained, which significantly increases computational complexity, especially in higher-order models. Due to space considerations, Table 3 presents the FLRG order 1 as a representative illustration, while the complete listings for all orders are documented in Appendix A (Tables A1–A3). The subsequent analysis in Table 5 will provide a comparison with the proposed modified methods.

5. Defuzzification

Defuzzification is performed to convert the fuzzy forecasting results into numerical production values using Equation (4). This process enables direct comparison between predicted and actual rice production for each FTS model order.

For the FTS order 1 model, the forecasting relies on a single historical observation $F(t-1)$, resulting in limited contextual awareness of seasonal production dynamics. As a consequence, the model exhibits relatively large forecasting errors, particularly during periods of rapid production increase at the onset of major harvest cycles. This limitation reflects the restricted memory inherent in first-order formulations.

The FTS order 2 model incorporates two preceding observations $F(t-2)$ and $F(t-1)$, allowing it to better capture general seasonal trends. Initial forecasts show improved accuracy compared to the first-order model; however, the model remains sensitive to abrupt production shifts, leading to increased deviations in certain periods.

By utilizing three historical observations $F(t-3), F(t-2)$ and $F(t-1)$, the FTS order 3 model captures longer temporal dependencies and produces more context-aware forecasts. Overall, this model demonstrates more stable and accurate predictions across the time series, indicating that higher-order memory improves forecasting robustness in the presence of seasonal volatility.

These results indicate a clear improvement in forecasting performance as the model order increases, motivating further evaluation using quantitative accuracy metrics and complexity comparisons in the subsequent analysis.

3.3 Proposed Frequency-Based FLRG Modification

To address the limitations of conventional FTS model—particularly the excessive number of fuzzy logical relationships (FLRGs), which often introduce noise and reduce interpretability—this study proposes a modified FLRG formation method. The modification simplifies the rule structure by retaining only the most frequently occurring transitions within each FLRG, thereby eliminating low-frequency or irregular patterns that contribute minimally to the forecasting process. This approach is expected to reduce computational complexity, produce a more compact rule base, and enhance forecasting stability, particularly in time series with strong seasonal dependencies such as rice production.

The proposed modification is applied during the FLRG construction stage. After the historical data are fuzzified into linguistic terms, standard FLRG are first generated by grouping all observed fuzzy logical relationships sharing the same antecedent. For each antecedent state, the frequency of each successor state is then calculated. The modified FLRG is formed by retaining only the successor with the highest observed transition frequency, while all other successors are discarded. As a result, each antecedent is associated with a dominant successor, yielding a simplified and deterministic FLRG structure.

For example, a conventional FLRG such as $A_1 \rightarrow 3\,A_1, 6A_2, A_5$ is simplified into a modified FLRG $A_1 \rightarrow A_2$, where A_2 represents the most frequently observed successor. A representative example of the modified FLRG structure is provided in Table 4, while the complete FLRG listings for all model orders are presented in Appendix B (Tables B1-B3).

Table 4. Formation of New FLRG order 1

Current stage	FLRG orde 1	New FLRG order 1
A_1	$A_1 \rightarrow 3A_1, 6A_2, A_5$	A_2
A_2	$A_2 \rightarrow 3A_5, A_7, A_9, A_{10}$	A_5
A_3	$A_3 \rightarrow 4A_1, A_9$	A_1
⋮	⋮	⋮
A_{16}	$A_{16} \rightarrow A_9, 3A_{12}$	A_{12}

3.4 Reduction in FLRG Complexity

To assess the structural efficiency of the proposed modification, this subsection compares the number of Fuzzy Logical Relationship Groups (FLRGs) produced by the conventional and modified approaches. Since FLRG complexity influences interpretability, computational demand, and forecasting stability, measuring the reduction in rules provides a clear indication of the effectiveness of the simplification strategy. The comparison across all model orders is summarized in Table 5.

Tabel 5. Summary of FLRG Complexity Reduction Across Model orders

Model	FLRG (Conventional)	New FLRG (Modified)	Reduction
FTS order 1	80	47	41.25%
FTS order 2	84	73	13.09%
FTS order 3	83	74	10.84%

As shown in Table 5, the most substantial simplification occurs in the order 1 model, where the number of rules decreases from 80 to 47 (41.25%). This indicates that many low-frequency transitions are effectively consolidated through the frequency-based modification. For the order 2 and 3 models, reductions of 13.09% (84 → 73) and 10.84% (83 → 74) are observed, respectively. Although the proportional reduction becomes smaller at higher orders—reflecting the more structured nature of long-memory FLRGs—the modified approach consistently eliminates irrelevant transitions and refines the rule base.

From a complexity perspective, conventional high-order FTS models experience rapid growth in the number of FLRGs due to the combinatorial expansion of historical state combinations. The proposed frequency-based filtering limits this growth by retaining only dominant transitions within each FLRG, resulting in a more compact, scalable, and computationally efficient model structure, as reflected in Table 5.

3.5 Model Accuracy Evaluation

Following model implementation, forecasting accuracy was evaluated using MAD, RMSE, and MAPE, where lower values indicate better alignment between predicted and observed rice production. To provide a compact and comprehensive comparison, the overall forecasting performance of the conventional and modified FTS models across different orders is summarized in Table 6.

Table 6. Comparison of Forecasting Models with MAD, RMSE, and MAPE

Method	MAD	RMSE	MAPE
FTS order 1	1,118,187	1,634,284	25.46%
Modified FTS order 1	1,040,791	1,631,106	22.67%
FTS order 2	366,974	591,460	10.50%
Modified FTS order 2	365,085	515,509	9.32%
FTS order 3	197,614	305,017	5.77%
Modified FTS order 3	196,410	271,774	5.46%

As shown in Table 6, the proposed frequency-based FLRG modification consistently improves forecasting accuracy across all model orders. In the first-order configuration, MAPE decreases from 25.46% to 22.67% after modification. More substantial improvements are observed in higher-order models, with MAPE reduced from 10.50% to 9.32% for order 2 and from 5.77% to 5.46% for order 3. These results indicate that the modified FTS model achieves lower forecasting error while maintaining stable performance as model order increases.

The visual comparisons presented in Figures 3–5 further support these findings. Across all orders, the modified FTS model produces smoother prediction curves and follows the seasonal production pattern more closely than the conventional approach. The improvement becomes increasingly evident at higher orders, reflecting the combined benefit of longer historical memory and a simplified FLRG structure.

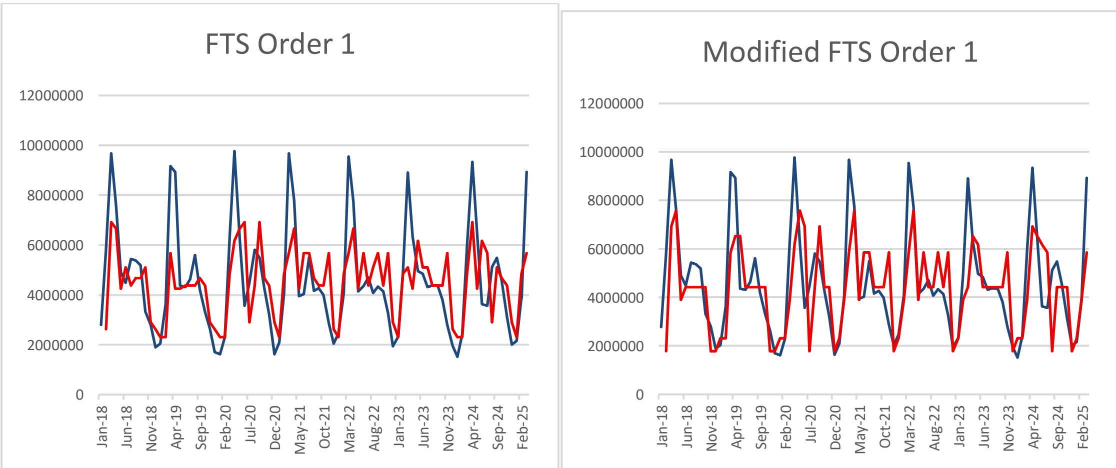

Figure 3. Forecasting Accuracy of the Conventional and Modified FTS order 1

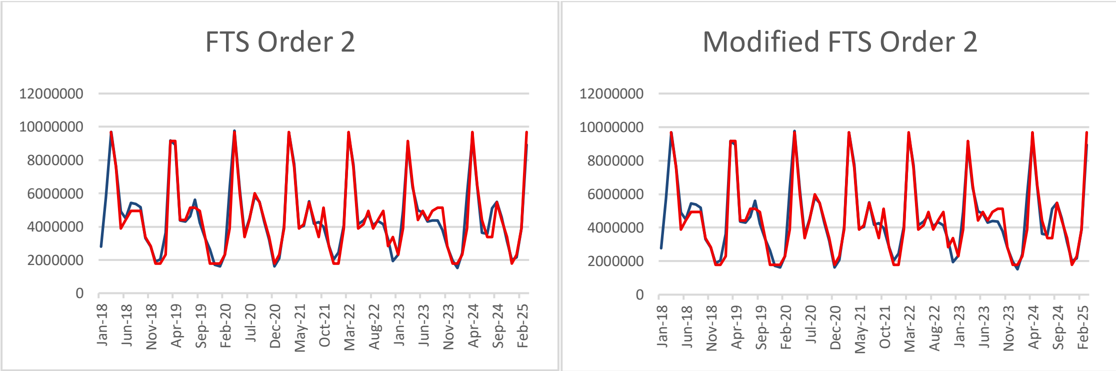

Figure 4. Forecasting Accuracy of the Conventional and Modified FTS order 2

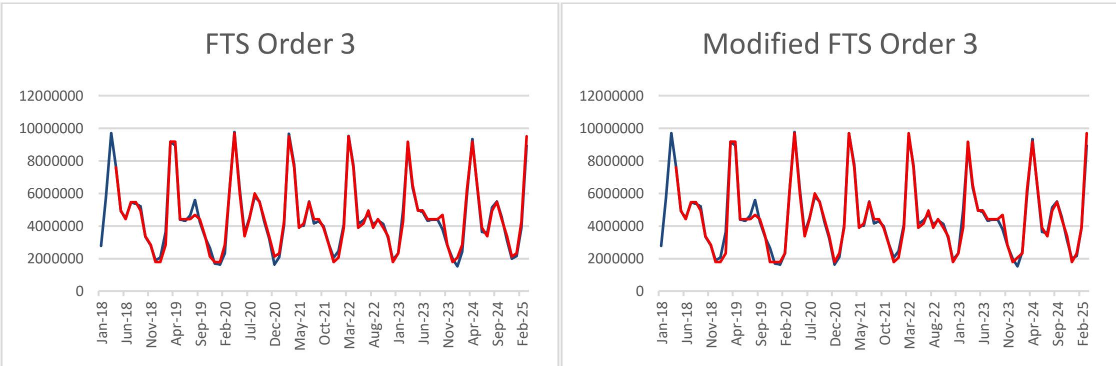

Figure 5. Forecasting Accuracy of the Conventional and Modified FTS order 3

Although MAD, RMSE, and MAPE were used as standard accuracy metrics—and MAPE is particularly relevant for policy interpretation—this study did not apply cross-validation or out-of-sample testing. Monthly rice production is a temporally ordered and strongly seasonal time series; random cross-validation would disrupt the chronological structure and distort seasonal dependencies captured by the FTS model. Similarly, allocating a limited portion of the data for out-of-sample testing would reduce the number of complete seasonal cycles available for training. Therefore, the evaluation focuses on in-sample forecasting performance across different model orders. Future studies may incorporate rolling-window or expanding-window validation to further assess long-term forecasting stability.

The observed improvement in performance, as indicated by the reduction in MAPE, should be interpreted in the context of Indonesia's strongly seasonal rice production system. In conventional high-order FTS models, seasonal anomalies often generate rare logical transitions that introduce noise and reduce forecasting stability. By retaining only the most frequent FLRs, the proposed model filters out these infrequent transitions and yields a cleaner representation of dominant seasonal patterns. This structural refinement enhances robustness against data volatility and improves forecasting accuracy, particularly around seasonal turning points.

3.6 Final Forecasting Outcome

Based on the accuracy evaluation, the modified FTS order 3 model was identified as the superior method, achieving the lowest MAD (196,410), RMSE (271,774) and MAPE (5.46%). This model was selected to forecast future production values. The forecasts for April–August 2025 (Table 7) are purely model-based extrapolations intended for policy planning support and have not yet been empirically verified against official BPS data.

Table 7. Final Forecasting Outcome

No	Period	FLRG	Prediction
1	April 2025	$A_2, A_5, A_{16} \rightarrow A_{12}$	7,576,538 tons
2	May 2025	$A_5, A_{16}, A_{12} \rightarrow A_5$	3,887,538 tons
3	June 2025	$A_{16}, A_{12}, A_5 \rightarrow A_6$	4,414,538 tons
4	July 2025	$A_{12}, A_5, A_6 \rightarrow A_7$	4,941,538 tons
5	August 2025	$A_5, A_6, A_7 \rightarrow A_5$	3,887,538 tons

The final forecasting outcome indicates a projected decline in national rice production from May to August 2025, providing an early warning signal for policymakers and supply regulators. Such information enables anticipatory planning for supply stabilization measures, including the adjustment of buffer stock policies and import scheduling, to avoid sudden price spikes or reactive import decisions, as previously observed in early 2025. Using historical production patterns, the proposed model allows short-term forecasting up to five months ahead, supporting proactive rather than reactive policy formulation.

Improved forecasting accuracy plays a strategic role in minimizing the risks of import oversupply or undersupply. By offering a more reliable and timely projection of future production trends, the modified FTS model supports better-aligned import and logistics decisions, helping ensure that national rice availability remains consistent with anticipated demand. In this sense, enhanced forecasting transforms import planning from a retrospective adjustment process into a forward-looking decision-support mechanism.

From a methodological perspective, the contribution of this study is positioned as complementary to existing FTS variants, including frequency-based partitioning approaches, optimization-driven methods such as PSO-based FTS, and hybrid FTS–machine learning models. Rather than introducing additional model complexity, the proposed frequency-based FLRG modification emphasizes structural simplification, interpretability, and robustness, making it particularly suitable for operational and policy-oriented forecasting environments where transparency and computational efficiency are essential.

The observed improvement in forecasting performance, as reflected by the consistent MAPE reduction within the same model order, particularly in the third-order configuration (5.77% → 5.46%), has meaningful practical implications. At peak production levels, even a small percentage reduction in forecasting error can translate into substantial absolute differences in estimated rice volume, potentially on the order of hundreds of thousands of tons. Such accuracy gains can reduce the risk of misestimating import requirements or buffer stock levels, thereby supporting more reliable national food security planning.

Despite these advantages, the proposed frequency-based simplification introduces an inherent trade-off between model parsimony and sensitivity to rare or extreme events. By prioritizing frequently occurring logical transitions, the model may underrepresent infrequent but critical structural changes in production caused by exceptional policy shifts or extreme climatic events. Consequently, the proposed approach is most appropriate for baseline seasonal planning and medium-term supply forecasting, while extreme-event risk management may require complementary modeling frameworks. This limitation also provides a clear direction for future research aimed at integrating rare-event awareness into simplified FTS structures.

4. CONCLUSION

Based on the testing and analysis of forecasting results using the Fuzzy Time Series model with modified frequency-based FLRG, several key conclusions can be drawn. First, prioritizing the highest-frequency logical relationships significantly enhances the forecasting accuracy of national rice production. The proposed model consistently outperforms the conventional Chen FTS across all tested orders, achieving its best performance with MAPE reduced to 5.46% for modified FTS order 3. These results demonstrate not only numerical gains but also improved capability in capturing seasonal lag structures inherent in rice production cycles. Second, the modified frequency-based FLRG FTS model effectively reduces rule complexity, as evidenced by the decrease in the number of FLRG relationships across all model orders. This simplification produces a more compact and interpretable rule base, improves computational efficiency, and mitigates the risk of overfitting by eliminating infrequent and unstable transitions. Consequently, this study contributes theoretically to Fuzzy Time Series development by showing that structural simplification can coexist with improved predictive performance, particularly in high-order configurations. Furthermore, the enhanced model is particularly suitable for time series characterized by strong seasonality and long historical dependencies, such as agricultural production data. The improved forecasting stability and interpretability indicate that the model demonstrates potential suitability as a transparent and computationally efficient decision-support tool for baseline seasonal planning in national rice production policy. Beyond methodological advancements, this improved reliability may support policymakers in anticipating seasonal supply trends, helping to inform import planning and buffer-stock management without overreacting to short-term fluctuations. Finally, this study acknowledges several limitations. The frequency-based FLRG simplification, while enhancing robustness against noise, introduces a potential trade-off by reducing the model's sensitivity to sudden, rare structural breaks in production. Based on these constraints, future research may focus on (1) rolling-window or expanding-window validation to assess long-term stability, (2) applications to other agricultural commodities, and (3) integration with probabilistic or scenario-based approaches to better address extreme events.

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