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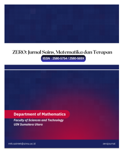

Constructing Efficient Frontiers in Cryptocurrency Market Using Long-Run GARCH Volatility

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ABSTRACT

Persistent volatility makes it difficult to optimize cryptocurrency portfolios. In order to increase portfolio stability in cryptocurrency markets, this study attempts to build optimal frontiers using long-run GARCH (1,1) volatility estimations. The daily prices of Solana (SOL), Ethereum (ETH), and Bitcoin (BTC) are examined from January 2023 to December 2024. GARCH-based risks are used in portfolio optimization when stationarity has been verified and conditional volatility has been modeled. According to the findings, SOL is the most volatile (43.56%), BTC is the least volatile (25.54%), while ETH shows great volatility persistence but a lower risk-adjusted return. While the smallest variance portfolio focuses on BTC, the maximum Sharpe ratio portfolio prefers both SOL and BTC (Sharpe ratio = 3.692). The GARCH method yields more realistic and stable efficient frontiers when compared to conventional variance-based techniques. Our findings suggest that investors can develop more secure cryptocurrency portfolios under structural risk by using dynamic models.

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1. INTRODUCTION

The cryptocurrencies market has changed and expanded the financial market, dominated by Bitcoin (BTC) and Ethereum (ETH). Its features distinct from the other traditional assets, namely stocks and bonds in terms of its volatility, non-normal returns, and limited historical data [1], [2], [3], [4]. These present a challenge for traditional portfolio optimization method, especially when the variance is not constant and not considered normal [5], [6], [7]. Due to this limitation, the crypto assets may not be reflected in traditional portfolio theory, which relying on regular risk-return with normality assumptions. As a result, there is a growing interest in modifying sophisticated econometric models to more accurately capture the stochastic nature of cryptocurrency markets [8], [9], [10], [11].

A key innovation in risk modelling comes from the use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, as introduced by Bollerslev in 1986, which allow for time-varying volatility [12]. These models are particularly effective for financial return series exhibiting volatility clustering—periods of high and low volatility—common in cryptocurrency returns. However, this study highlights the unconditional volatility component, which represents the long-run average variance of returns, whereas the majority of GARCH model implementations concentrate on conditional volatility predictions [13]. This is proven by its robustness, especially when building short-term or efficient portfolios. According to Siaw et al. (2015), who argued that it is suitable for equilibrium-based frameworks, unconditional volatility can be a reliable input for portfolio optimization models [14].

The Efficient Frontier in MPT idea is to either minimize risk for a given return or maximize expected return for a given risk [15]. Its design always relied on sample-based variances and covariances, but these estimates are volatile to shocks in crypto markets. Efficient frontiers can be smoother by replacing models with GARCH-implied long-run volatility. Previous research by Samunderu et al. (2021), Pedersen et al. (2023), and Ampadu et al. (2024) has demonstrated that GARCH-based risk measures improve portfolio performance in non-normal conditions [16], [17], [18].

The Capital Market Line (CML) complements the Efficient Frontier by defining the optimal risk–return trade-off when a risk-free asset is included [19], [20], [21]. Although the existence of a true risk-free asset in crypto markets is debated, instruments like stablecoins or Treasury-bill equivalents provide a conceptual benchmark. Studies by Brière et al. (2015) and Corbet et al. (2019) demonstrate that integrating cryptocurrencies shifts both the EF and CML, reflecting nonlinear risk characteristics [22], [23], [24], [25].

Despite growing literature on crypto portfolio optimization, a key gap remains: few studies apply unconditional GARCH-based long-run volatility in constructing efficient frontiers. Existing research often relies on short-term conditional volatility forecasts or sample statistics, which are highly reactive to transient market shocks and fail to represent persistent structural risk.

This study objective is to construct cryptocurrency efficient frontier using unconditional GARCH volatility, based on Sharpe-optimal allocations, and compare them against traditional sample-based approaches. By analyzing BTC, ETH, and SOL from 2023–2024, the research demonstrates how risk based estimation affect stability in crypto portfolio optimization.

2. RESEARCH METHOD

The idea of this research is to construct optimal cryptocurrency portfolio which adjusts the risk based on GARCH unconditional volatility. This could fill the gap in the regular risk modelling and portfolio optimization [26], [27], [28], [29]. The risk will be estimated reliably by using an advanced models, it is critical to consider risk profile of portfolio managers in emerging digital markets whether it is individual or institutional.

The sample is focused on three major cryptocurrencies, namely BTC, ETH, and SOL; BTC were chosen as the benchmark of cryptocurrency by its largest market capitalization. ETH as the pioneer of smart contract which represent the utilities and market dynamic. SOL, although it is still considered as new cryptocurrency, is considered as the highest growth altcoin with volatility characteristics. Together, these three assets provide a representative cross-section of the cryptocurrency ecosystem while ensuring data reliability and sufficient depth for econometric modeling.

Nonetheless, this sample restricts the generalizability of the findings. Future work could be enhanced by considering stablecoins, utility tokens, and other types of digital assets, this could broaden the scope of portfolio in the cryptocurrency market.

Daily closing price data for both assets is collected from Yahoo Finance, covering the period from January 1, 2023 to December 31, 2024. The price series is transformed into continuously compounded log returns using the transformation

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t denotes the closing price on day t. Stationarity of the return series is confirmed using the Augmented Dickey-Fuller (ADF) test to ensure the appropriateness of time series modelling and ARCH-LM test to ensure the existence of ARCH effect.

To account for time-varying volatility in cryptocurrency returns, each asset is modelled using a univariate GARCH(1,1) specification. Although the GARCH(1,1) model provides a practical framework for capturing volatility clustering, it assumes constant variance parameters over time and symmetric reaction to shocks. It does not directly capture leverage effects or time-varying correlations. In addition, this study measures risk primarily via volatility, common in traditional finance but potentially insufficient for highly speculative markets. Important real-world factors (liquidity risk, transaction costs, tail risks, and portfolio constraints) are not modeled here but merit attention in further studies.

The return process is defined as

$$r_t = \mu + \epsilon_t, \epsilon_t \sim N(0, \sigma_t^2), \tag{2}$$

and the conditional variance evolves according to

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{3}$$

where $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$. The parameters are estimated via Maximum Likelihood Estimation (MLE). To obtain a stable, long-run measure of volatility for portfolio construction, the unconditional variance is computed as

$$\sigma^2_{\text{unconditional}} = \frac{\omega}{1-\alpha-\beta}, \tag{4}$$

provided that the stationarity condition $\alpha + \beta < 1$ holds. The unconditional volatility is then calculated as

$$\sigma_{\text{unconditional}} = \sqrt{\left(\frac{\omega}{1-\alpha-\beta}\right)}, \tag{5}$$

Expected returns μ for the assets are computed as the arithmetic mean of the historical log returns:

$$\mu = \frac{1}{T}\sum_{t=1}^{T} r_t, \tag{6}$$

where T is the total number of observations. The correlation between BTC and ETH, denoted by ρ, is computed using the Pearson correlation coefficient from the entire return series. This correlation serves as an estimate of the unconditional correlation required for portfolio variance computation.

Let $w \in [0,1]$ be the portfolio weight allocated to each asset where N is the number of assets in the portfolio. For tractability and comparability with much of the literature, the optimization in this study does not impose real-world constraints such as minimum/maximum weights, diversification thresholds, or liquidity screens. Incorporating such constraints would make future analysis more applicable to genuine investment practice and regulatory contexts.

The expected return of the portfolio is given by the weighted average:

$$\mu_p = \sum_{i=1}^{N} \mu_i w_i, \tag{7}$$

The portfolio variance is computed as

$$\sigma_p^2 = w^T \Sigma w, \tag{8}$$

where Σ is the covariance matrix consists from each assets and the portfolio volatility is simply the square root of σ_p^2. By varying w from 0 to 1 in small increments, a set of risk-return combinations (σ_p, μ_p) is obtained to form the Efficient Frontier Curve (EFC).

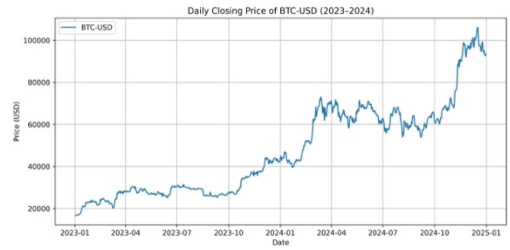
In constructing the Capital Market Line (CML), the study assumes a constant risk-free rate r_f, estimated from average short-term U.S. Treasury yields over the same period. The Sharpe ratio for each portfolio is then calculated as

$$\text{Sharpe} = \frac{\mu_p - r_f}{\sigma_p}. \tag{9}$$

The portfolio with the maximum Sharpe ratio is identified as the tangent portfolio. The CML is plotted as a line passing through the risk-free point (0,r_f) and the tangent portfolio point ($\sigma_{tangent}$,$\mu_{tangent}$).

3. RESULT AND ANALYSIS

The volatility of cryptocurrency market is reflected on Figure 1, it can be seen on the historical closing price movement between January 1, 2023, and December 31, 2024 for these three sample assets of BTC, ETH, and SOL. This shows the differences in price and volatility dynamics in the cryptocurrency market. BTC could maintain the highest price, reflecting its dominant market capitalization and perception of best investment in cryptocurrency market. ETH, while also relatively in an uptrend, demonstrates higher fluctuations. On the other hand, SOL exhibits extreme price fluctuation, indicating a higher market sensitivity and speculative activity.



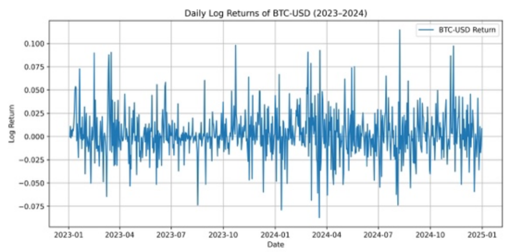
a. BTC Daily Closing Price b. ETH Daily Closing Price

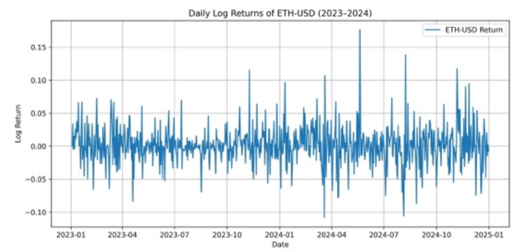
c. SOL Daily Closing Price

Figure 1. Daily Closing Price Plot

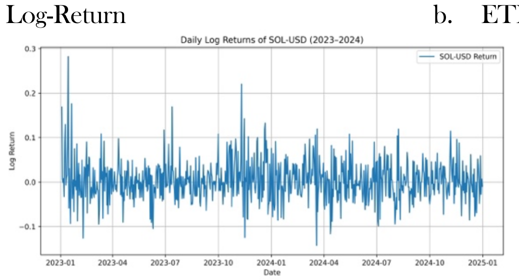
These patterns vary with risk-return profiles among assets we examined. BTC and ETH have a smoother movements with periodic corrections, whereas SOL is had a volatile movement with appreciation and decline. These differences increase the conviction of the importance in considering volatility model in portfolio construction, as asset prices alone do not sufficiently capture the shock of investment risk. The price evolution also define the market regimes during the sample period, including price recovery from the drawdown itself and the parts where investor increased thir interest across the crypto market or ecosystem.

The daily log returns of sample assets over the examined period of time (2023 to 2024) are displayed in Figure 2. These series show fluctuations around a zero mean, this is a common feature of asset returns in efficient markets. However, the frequency of these fluctuations vary across assets. the widest return swings happen in SOL, consistent with its earlier-observed price volatility, while BTC and ETH display relatively narrow but still frequent deviations, reflecting their more established market presence.





a. BTC Log-Return b. ETH Log-Return



c. SOL Log-Return

Figure 2. Daily Log-Return Plot

The return series show a visible patterns of volatility clustering, meaning that period of high volatility followed by another high volatility, or vice versa. This time dependence in series variation justifies the application of GARCH-family models, which are designed to capture time-varying volatility in financial time series. The observed heteroskedasticity reinforces the need for model-based risk estimates rather than static historical measures when optimizing portfolios in cryptocurrency markets. These return dynamics form the empirical foundation for subsequent tests of stationarity, ARCH effects, and volatility forecasting.

To assess the return stationarity, the Augmented Dickey-Fuller (ADF) test was conducted. Table 1 reports the results of the ADF and ARCH-LM tests applied to the log return series of our sample data. The ADF test confirms that all return series are stationary at the 1% significance level, with test statistics well below the critical thresholds and p-values effectively at zero. This made the assumption valid that the return series are mean-reverting and do not contain a unit root, which is necessary for time series models like GARCH.

In addition, the ARCH-LM test the significance of ARCH effects across three assets. The p-values for each series are below the 5% threshold, it confirms the existence of ARCH effect. This finding supports the usage of volatility model GARCH (1,1), as it implies that current variance is affected by past shocks. The confirmation of both stationarity and heteroskedasticity ensures the eligibility of the model and justifies the use of GARCH-derived long-run volatilities for portfolio optimization.

Table 1. ADF and ARCH-LM Test Result

Asset	ADF Statistic	ADF P-Value	ADF Stationary	ARCH LM Statistic	ARCH-LM P-Value	ARCH Effect
BTC	-18.13	0.00	Yes	18.36	0.00	Yes
ETH	-27.98	0.00	Yes	11.57	0.04	Yes
SOL	-26.97	0.00	Yes	26.87	0.00	Yes

Each return series was fit with a univariate GARCH (1,1) model, capturing the time-varying volatility through historical squared returns and lagged variances. The estimated parameters of the GARCH (1,1) presented in Table 2, the models are fitted to the return series of the sample data. All estimated parameters are statistically significant and satisfy the stationarity condition, where the sum of $\alpha + \beta < 1$, indicating the models volatility processes are mean-reverting over time. The use of unconditional variance as a long-run measure of risk is also validated, especially for the portfolio construction.

Table 2. GARCH Parameters

Asset	ω	α	β	$\alpha + \beta$	GARCH Valid
BTC	1.0687	0.0942**	0.7419***	0.8361	Yes
ETH	0.0539	0.0165	0.9780***	0.9945	Yes
SOL	0.7948**	0.0475***	0.9106***	0.9581	Yes

Significance levels are indicated by *** ($p < 0.01$), ** ($p < 0.05$), and * ($p < 0.10$).

Among the samples, ETH exhibits the highest volatility persistence, with a combined $\alpha + \beta = 0.9945$, suggesting that the volatility decay more slowly. BTC follows closely with a persistence of 0.8361, while SOL exhibits a slightly lower persistence of 0.9581. Interestingly, SOL's relatively higher α value implies greater sensitivity to past shock, while ETH's high β suggests more persistent past volatility on the long time horizon. These differences in volatility dynamics highlight the heterogeneity in how each asset responds to new information and justify a differentiated approach to risk modelling within the portfolio optimization process.

Table 3. Long run variance and volatility

Asset	Long Run Variance	Long Run Volatility
BTC	0.0652	0.2554
ETH	0.0988	0.3144
SOL	0.1897	0.4356

Table 3 reports the long-run variance and volatility for these assets, as estimated from the GARCH (1,1) models. It represents the unconditional volatility for each asset, meaning the level of GARCH volatility reverting over the long term. The term "mean-reverting" indicates that, after periods of high or low, it is expected that the variance to approach this one level over time. The estimated risk is less sensitive to recent market shocks, and more suitable for longer-term investment portfolio. The reported unconditional variances and volatilities offer a stable risk exposures in each cryptocurrency.

In result, SOL is the most volatile in the long-run with volatility of 43.56%, second is ETH at 31.44%, and BTC at 25.54%. These values align with the fluctuations in Figure 2 and confirm that SOL is the asset that is considered as high risk within our sample. On the other hand, BTC act as the stabilizer and liquid cryptocurrency, comes with the lowest volatility, showing its ability as the top asset in the crypto market. The relative shocks of long-run volatility suggest that optimal portfolio construction uses SOL as diversification of risk-return.

The correlations among these three assets were calculated using the Pearson correlation of the log return. BTC and ETH demonstrated the highest correlation, it shows that they have similar role in the crypto ecosystem.

While SOL's correlation with other was lower, it is in line with the idea of diversification, it could be a benefit in multi-asset crypto portfolios.

The unconditional correlation and covariance matrix calculated from the log return of BTC, ETH, and SOL is presented in Table 4. BTC and ETH shows a strong relationship as expected from the market, with a correlation of approximately 0.94, reflecting their similar role in crypto market. While SOL acts differently as a diversifier due to its unique risk, it has relatively lower correlation with other anchor asset.

The portfolio risk calculation needs a covariance matrix, it's construction based on both unconditional long-run volatilities and sample correlations. It captures the risk of each asset and their joint movement with the other as well. Although the covariance is positive for whole sample assets, for BTC-ETH and BTC-SOL is smaller than that between ETH and SOL, meaning that BTC still serve as the anchor while the risk adjusted return coming from ETH and SOL. These relationships are essential in constructing the efficient frontier and determining optimal asset allocations under the mean-variance framework.

Table 4. Correlation and Covariance Matrix

Asset	BTC	ETH	SOL
BTC	0.0652	0.0651	0.0751
ETH	0.0651	0.0988	0.0884
SOL	0.0751	0.0884	0.1897

The portfolio optimization of BTC, ETH, and SOL using GARCH annualized volatilities and historical expected returns leads to the construction of EFC and CML in Figure 3. To align the risk and return, the expected returns for all assets must be annualized with 365 market days. From then, it has a consistent feature with the long-run volatility under the common investment time horizon. This methodology is important since investor need to make a risk-adjusted performance measure such as the Sharpe ratio.

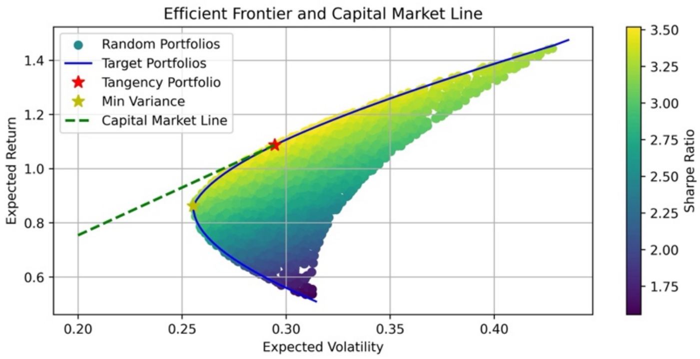


Figure 3. Efficient Frontier Curve of Cryptocurrencies Portfolio

A method of searching the most optimal portfolio among all possible combinations of asset weights is implemented, the constraint is that weights sum to one meaning a fully invested portfolio. For each weight, the annualized portfolio return, variance, volatility, and Sharpe ratio were computed. From this method, three type of portfolios were identified, namely the maximum Sharpe ratio portfolio (tangent), the minimum variance portfolio, and the maximum return portfolio. The EFC illustrates the optimal risk-return tradeoff under the risk model based calculation. An inefficient portfolio are the portfolio that lies under the curve, meaning that with the same level of risk, it yields a lower return.

The straight line from the risk-free rate (5%) to the tangency point on the EFC is the CML, it corresponds to the portfolio with the highest Sharpe ratio. This tangent portfolio allocates 63.1% to BTC and 36.9% to SOL, with no allocation to ETH. It achieves an annualized return of 108.7% and a volatility of 29.5%, resulting in a Sharpe ratio of 3.692. This finding indicates that ETH contributes less to risk-adjusted performance under the given estimation framework, although it still lies on the second market capitalization.

On the other hand, the minimum variance portfolio is dominated by 99.7% BTC with a small portion on ETH (0.3%) and none to SOL. This portfolio yields a lower return of 86.2% and volatility of 25.5% , yielding a Sharpe ratio of 3.3, which is a good number above 3. All of these portfolios performance are displayed in Table 5. These results demonstrate the value of GARCH-based volatility inputs in identifying optimal portfolios and highlight how different objectives (risk minimization vs. Sharpe maximization) result in materially different asset allocations. The placement of these portfolios on the EFC and CML illustrates their relative efficiency and theoretical attractiveness for investors with varying risk preferences.

Table 5. Weighted Portfolio Performance

Portfolio	Weight		Return	Risk	Sharpe Ratio
Optimal	BTC	0.631	1.087	0.295	3.692
	ETH	0.00			
	SOL	0.369			
Minimum Variance	BTC	0.997	0.862	0.255	3.376
	ETH	0.003			
	SOL	0.00			

Our findings suggest two key factors of ETH's contribution limitation in the portfolio. First, it posseses a higher correlation with BTC (≈0.94), this could reduce the diversification effect. Assets with higher joint movement have a lesser effect towards risk reduction, it makes investor have a lesser favor to put it into the efficient frontier composition especially in traditional mean-variance model. Second, it's risk–return profile implies a lower risk premium relative to its volatility. Even if its long-term volatility (31.44%) is lower than SOL's (43.56%), the predicted return is insufficiently high to offset the risk. This results in a lower contribution to the Sharpe ratio when compared to SOL, which gives a bigger return, and BTC, which offers stability. Because of this, the optimization process automatically reduces its weight, giving preference to assets that offer more return per unit of risk or stronger diversity.

By its statistical performance, each investor has a different risk profile and investment style, which is considered in the portfolio setting. For risk-averse investors, BTC should dominate the portfolio due to its lower volatility of 25.5%, it should be maintaining the Sharpe ratio above 3.37 while still considered as the minimum variance portfolio. It points out that BTC act as an superior asset within a cryptocurrency portfolio, providing stability in an otherwise volatile market which aligned with the research from Yae & Tian (2024) and Danilo & Thelissaint (2025) [30], [31].

On the other hand, risk-tolerant investors could maximize the Sharpe ratio with portfolio that includes 36.9% to SOL and the rest to BTC (63.1%). This portfolio undertake higher volatility (29.5%), but it has risk-adjusted performance (Sharpe ratio of 3.692). While SOL alone possess a high volatility, including it with other assets in a portfolio will enchance its stability.

In terms of long-term strategies, the use of unconditional GARCH volatility provides a more stable foundation for portfolio construction by mitigating the influence of short-term shocks. This stability is particularly valuable for institutional investors and fund managers who design strategic asset allocations rather than engaging in frequent rebalancing. By contrast, traditional sample variance approaches are more susceptible to recent market turbulence and may lead to overreaction in portfolio weights. The results therefore suggest that adopting model-based volatility measures can improve resilience in crypto portfolio management, aligning portfolio design with the persistence and structural nature of digital asset risks.

4. CONCLUSION

This research shows that the GARCH volatility model improves the portfolio optimization in random market such as cryptocurrency, specifically in terms of handling its stability. The estimates using this conditional volatilities for the crypto examined (BTC, ETH, and SOL) were produced, and the efficient frontiers are more stable and reliable compared to measures of sample based. Our findings show that BTC serves as a portfolio stabilizer, on the other hand, SOL that has higher volatility could act as the diversifier. Sharpe ratios exceeding 3 highlighting the practical value of volatility modeling in improving long-term portfolio resilience.

These results may show the advantages of handling volatility in crypto markets using GARCH model, but limitations still remain. The model's still unable to treat the difference between upside and downside movement and correlations are not always in a static form. Future works can consider the multivariate and asymmetric specifications, such as DCC-GARCH, GJR-GARCH, or EGARCH, as well as simulation-based and machine learning approaches. It is also possible to expand the asset sample to include stablecoins, utility tokens, and other digital assets would enhance the relevance of this approach.

5. REFERENCES

[1] X. Zhang, Z. Chen, and S. Wang, "A study of the impact of cryptocurrency price volatility on the stock and gold markets," *Financ Res Lett*, vol. 69, p. 106114, Nov. 2024, doi: 10.1016/J.FRL.2024.106114.

[2] F. Aslam, B. A. Memon, A. I. Hunjra, and E. Bouri, "The dynamics of market efficiency of major cryptocurrencies," *Global Finance Journal*, vol. 58, p. 100899, Nov. 2023, doi: 10.1016/J.GFJ.2023.100899.

[3] N. Naifar, "Mapping Systemic Tail Risk in Crypto Markets: DeFi, Stablecoins, and Infrastructure Tokens," *Journal of Risk and Financial Management*, vol. 18, no. 6, Jun. 2025, doi: 10.3390/jrfm18060329.

[4] M. M. Karim, M. H. Ali, L. Yarovaya, M. H. Uddin, and S. Hammoudeh, "Return-volatility relationships in cryptocurrency markets: Evidence from asymmetric quantiles and non-linear ARDL approach," *International Review of Financial Analysis*, vol. 90, p. 102894, Nov. 2023, doi: 10.1016/J.IRFA.2023.102894.

[5] A. Brynjarsson and K. Ágústsdóttir, "Cryptocurrencies in optimized portfolios: a portfolio theory approach The portfolio performance-enhancing capabilities of cryptocurrencies," 2022.

[6] J. Mednis and S. Palmquist, "PORTFOLIO OPTIMIZATION WITH CRYPTO ASSETS Analyzing the Impact of Investors' Subjective Views on Portfolio Risk," 2022.

[7] S. G. Chortane and K. Naoui, "The End of Mean-Variance? Tsallis Entropy Revolutionises Portfolio Optimisation in Cryptocurrencies," *Journal of Risk and Financial Management*, vol. 18, no. 2, Feb. 2025, doi: 10.3390/jrfm18020077.

[8] E. Wati, R. Dacesta Barus, N. Aini, B. Nugroho, and R. Fauzi, "Modeling bitcoin price by using Euler-Maruyama method," 2024. [Online]. Available: http://e-journal.president.ac.id/presunivojs/index.php/JAFRM/index38

[9] G. Zournatzidou, D. Farazakis, I. Mallidis, and C. Floros, "Stochastic Patterns of Bitcoin Volatility: Evidence across Measures," *Mathematics*, vol. 12, no. 11, Jun. 2024, doi: 10.3390/math12111719.

[10] C. Magazzino, T. Gattone, and F. Horky, "Economic and financial development as determinants of crypto adoption," *International Review of Financial Analysis*, vol. 103, p. 104217, Jul. 2025, doi: 10.1016/J.IRFA.2025.104217.

[11] D. T. A. Nguyen and K. C. Chan, "Cryptocurrency trading: A systematic mapping study," *International Journal of Information Management Data Insights*, vol. 4, no. 2, p. 100240, Nov. 2024, doi: 10.1016/J.JJIMEI.2024.100240.

[12] T. Bollerslev, "Generalized autoregressive conditional heteroskedasticity," *J Econom*, vol. 31, no. 3, pp. 307–327, Apr. 1986, doi: 10.1016/0304-4076(86)90063-1.

[13] Y. Zhu, S. I. Taasim, and A. Daud, "Volatility Modeling and Tail Risk Estimation of Financial Assets: Evidence from Gold, Oil, Bitcoin, and Stocks for Selected Markets," *Risks*, vol. 13, no. 7, Jul. 2025, doi: 10.3390/risks13070138.

[14] R. O. Siaw, E. Dei Ofosu-Hene, and T. Evans, "ELK ASIA PACIFIC JOURNAL OF FINANCE AND RISK MANAGEMENT INVESTMENT PORTFOLIO OPTIMIZATION WITH GARCH MODELS," vol. 8, 2015, doi: 10.16962/EAPJFRM/issn.

[15] G. M. Castro, C. Klöckl, P. Regner, J. Schmidt, and A. O. Pereira, "Improvements to Modern Portfolio Theory based models applied to electricity systems," *Energy Econ*, vol. 111, p. 106047, Jul. 2022, doi: 10.1016/J.ENECO.2022.106047.

[16] E. Samunderu and Y. T. Murahwa, "Return Based Risk Measures for Non-Normally Distributed Returns: An Alternative Modelling Approach," *Journal of Risk and Financial Management*, vol. 14, no. 11, Nov. 2021, doi: 10.3390/jrfm14110540.

[17] S. Hetland, R. S. Pedersen, and A. Rahbek, "Dynamic conditional eigenvalue GARCH," *J Econom*, vol. 237, no. 2, p. 105175, Dec. 2023, doi: 10.1016/J.JECONOM.2021.09.003.

[18] S. Ampadu, E. T. Mensah, E. N. Aidoo, A. Boateng, and D. Maposa, "A comparative study of error distributions in the GARCH model through a Monte Carlo simulation approach," *Sci Afr*, vol. 23, p. e01988, Mar. 2024, doi: 10.1016/J.SCIAF.2023.E01988.

[19] S. Jamaldin and A. Mithu, "Capital Asset Pricing Model (CAPM) Analysis of Tanzanian Public Firms," *Journal of Policy and Development Studies*, vol. 17, no. 1, pp. 168–180, Dec. 2024, doi: 10.4314/jpds.v17i1.14.

[20] L. M. De Castro, G. Dhillon, T. Cardão-Pito, M. Crathorne, T. Garcia, and D. Borrego, "MARKOWITZ EFFICIENT FRONTIER AND CAPITAL MARKET LINE-EVIDENCE FROM THE PORTUGUESE STOCK MARKET," 2017. [Online]. Available: www.european-jms.com

[21] R. Zhang, "Optimizing Investment in the Automotive Industry: A Modern Portfolio Theory Approach with Tesla, GM, and Hyundai," 2025, pp. 303–316. doi: 10.2991/978-94-6463-652-9_31.

[22] M. Brière, K. Oosterlinck, and A. Szafarz, "Virtual currency, tangible return: Portfolio diversification with bitcoin," Nov. 01, 2015, *Palgrave Macmillan*. doi: 10.1057/jam.2015.5.

[23] S. Corbet, B. Lucey, A. Urquhart, and L. Yarovaya, "Cryptocurrencies as a financial asset: A systematic analysis," *International Review of Financial Analysis*, vol. 62, pp. 182–199, Mar. 2019, doi: 10.1016/J.IRFA.2018.09.003.

[24] S. Sahu, J. H. Ochoa Vázquez, A. F. Ramírez, and J. M. Kim, "Analyzing Portfolio Optimization in Cryptocurrency Markets: A Comparative Study of Short-Term Investment Strategies Using Hourly Data Approach," *Journal of Risk and Financial Management*, vol. 17, no. 3, Mar. 2024, doi: 10.3390/jrfm17030125.
[25] N. Borri, Y. Liu, A. Tsyvinski, and X. Wu, "Cryptocurrency as an Investable Asset Class: Coming of Age," Oct. 2025, [Online]. Available: http://arxiv.org/abs/2510.14435
[26] D. G. Baur and T. Dimpfl, "Asymmetric volatility in cryptocurrencies," *Econ Lett*, vol. 173, pp. 148–151, Dec. 2018, doi: 10.1016/J.ECONLET.2018.10.008.
[27] K. Guesmi, S. Saadi, I. Abid, and Z. Ftiti, "Portfolio diversification with virtual currency: Evidence from bitcoin," *International Review of Financial Analysis*, vol. 63, pp. 431–437, May 2019, doi: 10.1016/j.irfa.2018.03.004.
[28] J. Almeida and T. C. Gonçalves, "A systematic literature review of investor behavior in the cryptocurrency markets," Mar. 01, 2023, *Elsevier B.V.* doi: 10.1016/j.jbef.2022.100785.
[29] S. Hussain and J. H. Chen, "The return and volatility spillovers among decentralized finance (DeFi) assets," *Res Int Bus Finance*, vol. 79, p. 103071, Jul. 2025, doi: 10.1016/J.RIBAF.2025.103071.
[30] J. Yae and G. Z. Tian, "Volatile safe-haven asset: Evidence from Bitcoin," *Journal of Financial Stability*, vol. 73, p. 101285, Aug. 2024, doi: 10.1016/J.JFS.2024.101285.
[31] L. Danilo and J. Thelissaint, "Risk and Reward of Crypto Portfolios: A Comparative Assessment of Selection Approaches," 2025.