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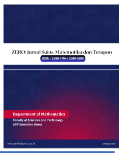

The Trinomial Tree Method in Pricing European Gold Option with Volatility Forecasting Using the GARCH (1,1) Model

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ABSTRACT

This study enhances the pricing accuracy of European gold options by integrating GARCH (1,1)-based volatility forecast into the trinomial tree method. GARCH (1,1) captures key characteristics of financial return series, such as heteroscedasticity and volatility clustering, while the trinomial tree offers greater flexibility than traditional models by allowing three price movements at each node. This integration provides a more realistic and robust framework for option pricing under dynamic market conditions. Using gold price data from October 2017 to October 2024, the model forecast annualized volatilities of 16.59%, 17.33%, and 17.66% for one, two, and three months. For call options, prices increase with longer maturities, ranging from Rp194,048 to Rp207,385. Conversely, put options become more valuable when the strike price exceeds the market prices, reaching up to Rp107,778. The proposed model offers practical value for more accurate pricing and investment strategies.

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1. INTRODUCTION

In investment decision-making, two primary factors to consider are return and risk. The relationship between these factors is typically positive and linear, where higher risk is associated with higher expected returns, and vice versa [1]. However, investors inherently seek to minimize risk, which has led to the development of derivative product designed for risk mitigation, such as options. Options are financial instruments that allow investors to speculate on the price movements of underlying assets, including company stocks, currencies, agricultural commodities, and more. An option provides the holder with the right, but not the obligation, to buy or sell an asset at a predetermined price within a specific period [2]. Based on the timing of execution, options are categorized into European-style which can only be exercised at expiration, and American-style, which can be exercised any time before or at expiration.

Among various underlying assets, gold is often regarded as a safe-haven asset during periods of economic uncertainty or inflation [3],[4]. In Southeast Asian countries like Indonesia, gold serves not only as a financial asset but also plays a vital role in cultural and social practices, including dowries and inheritance. Despite its prominence, gold-linked financial instruments such as options have not yet been fully developed or widely adopted in these markets, partly due to the lack of pricing models that account for local market behavior. Financial markets in

Southeast Asia also exhibit structural and behavioral characteristics that differ from those in developed economies, such as different levels of investor participation, regulatory environments, and price dynamics. These distinctions underscore the need for pricing models that reflect local market realities rather than relying solely on assumptions suited to more established financial systems. Addressing this gap is essential for supporting financial innovation, market development, and more effective risk management in the region.

Accurately determining the price of an option is crucial to assess its suitability for investment. Various models have been developed to price options, including the Black-Scholes model [5], the Binomial method [6], and Monte Carlo simulation [2]. Each method has its advantages and assumptions. Previous studies, such as those by Ding [7] and Meng [8], have shown that tree-based methods, particularly the Binomial tree, can outperform other models in terms of pricing accuracy, especially when volatility is estimated using more adaptive models.

However, many of these studies have primarily focused on stock options in developed markets, often under assumptions that may not align with the dynamic nature of alternative assets like gold or with the specific characteristics of emerging economies. This highlights the need for option pricing approaches that incorporate more flexible volatility modeling and are tailored to the behavior of assets and markets in regions such as Southeast Asia.

Volatility represents the degree of uncertainty in the future movement of asset prices. It describes both the frequency and magnitude of fluctuations in financial asset prices, serving as a key indicator of market risk and uncertainty in investment decision-making [9]. In practice, financial time series data such as daily gold prices exhibit time-varying volatility, typically characterized by phenomena such as volatility clustering. This characteristic makes the assumption of constant variance unrealistic for modeling financial returns [10].

To address this issue, Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity (ARCH) model, which allows for time-dependent conditional variance based on past disturbances [11]. However, ARCH models often require many lag terms and can result in inefficient estimation. To overcome these limitations, Bollerslev (1986) developed the Generalized ARCH (GARCH) model, which incorporates both past residuals and past variances to better capture the volatility dynamics with fewer parameters [12]. The GARCH model is especially effective in modeling financial returns due to its ability to capture volatility clustering and provide more realistic volatility estimates.

Given these advantages, the GARCH model is widely applied in financial modeling, particularly for assets like gold that experience significant price fluctuations. Among various GARCH specifications, the GARCH(1,1) model is one of the most frequently used due to its simplicity, robustness, and strong empirical performance in capturing volatility clustering in financial return series [13],[14],[15]. Recent studies have shown that GARCH(1,1) not only effectively models time-varying volatility but also accommodates characteristics such as thick-tailed returns. Its use of a single lag for both the conditional variance and past squared residuals also makes it computationally efficient and suitable for a wide range of financial time series data [16],[17].

This study advances prior methodologies by integrating GARCH (1,1)-based volatility forecast into the Trinomial tree option pricing framework. While previous research often assumes constant volatility, this combined approach allows for a more realistic representation of market behavior. Estimating volatility through GARCH (1,1) enhances the accuracy of the input parameters used in the pricing model, ultimately resulting in more representative and reliable option prices.

The Trinomial tree method itself extends the Binomial model by allowing three possible movements such as upward, downward, or unchanged at each time step [18],[19]. This added flexibility makes the Trinomial tree particularly suitable for capturing the dynamic behavior of gold prices [20], and it has been found to outperform the Binomial approach in terms of accuracy and convergence [21]. By integrating GARCH-based volatility into the Trinomial framework, this research aim to produce more accurate and realistic pricing of European-style gold option contracts, while also examining the impact of strike price and time to maturity on option values.

In the Indonesian context, where gold holds deep cultural and economic significance, this research offers both academic and practical contributions. By providing a more precise framework for pricing gold options, the model can assist investors, analysts, and financial institutions in developing financial products and strategies that are better aligned with actual market behavior.

2. RESEARCH METHOD

To position this study within the context of prior research, Table 1 summarizes selected studies based on the modeling approach used for volatility forecasting and option pricing, with a particular focus on gold and analogous commodities. While many studies apply traditional models under the assumption of constant volatility, recent literature has increasingly explored GARCH-family models for more accurate volatility estimation, especially in commodity markets. However, few studies integrate such volatility models into option pricing frameworks.

Table 1. Summary of Methods in Previous Studies and This Study's Positioning

Study	Volatility Model	Option Pricing Method	Asset	Notes

Muşetescu et al. (2022) [13]	GARCH	-	Crude oil	Forecasting volatility
Ghani & Rahim (2019) [16]	ARMA-GARCH	-	Natural rubber	Volatility modeling
Nissa et al. (2020) [20]	Constant	Trinomial	Stock option	Tree-based pricing with constant volatility
Yurttagüler (2024) [22]	ARCH, GARCH, EGARCH, TGARCH	-	Gold	Volatility modeling
This Study (2025)	GARCH (1,1)	Trinomial	European gold option	Combines GARCH-based volatility with trinomial pricing

To strengthen conventional option pricing approaches, this study integrates two well-established frameworks: GARCH (1,1) for volatility forecasting and the Trinomial tree for option pricing. Traditional pricing models, such as Black-Scholes or Trinomial approaches with constant volatility, often fail to capture time-varying nature of gold price movements, particularly in emerging markets where price dynamics tend to be more volatile. By forecasting volatility with GARCH (1,1), the model captures volatility clustering, resulting in more realistic input parameters. The Trinomial tree, in turn, offers greater flexibility than the Binomial tree by allowing three possible movements at each node, which improves numerical stability and convergence. This combination enhances pricing accuracy by aligning the model structure more closely with real market behavior.

2.1 Volatility Forecasting using GARCH (1,1)

To model the volatility of gold return data, this study employs the Generalized Autoregressive Conditional Heteroskedasticity model of order (1,1), or GARCH (1,1), as introduced in [23]. This model was chosen for its balance between simplicity and strong empirical performance. While more complex GARCH-family models exist, GARCH (1,1) remains widely used in practice and has demonstrated robust capability in capturing volatility clustering across various financial assets. Supporting this choice, Yurttagüler [22] has evaluated several ARCH-family models such as ARCH, GARCH, EGARCH, and TGARCH using gold price data from Türkiye (2005-2023) and concluded that GARCH (1,1) was the most appropriate for modeling gold price volatility. Given its effectiveness and parsimony, GARCH (1,1) is considered well-suited for integration with the Trinomial Tree framework in this study.

The GARCH (1,1) specification captures time-varying conditional variance σ_t^2 as a function of both past squared innovations (ε_{t-1}^2) and past variances (σ_{t-1}^2). The model is specified as:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (1)$$

where $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$. Here, $\varepsilon_t = r_t - \mu$ represents the return innovation, with μ as the conditional mean of the return series.

Volatility, which is the standard deviation of returns, is derived directly from the conditional variance by taking its square root. In other words, $\sigma_t = \sqrt{\sigma_t^2}$ where σ_t is interpreted as the forecasted volatility at time t.

The return series r_t, which reflects the relative change in gold prices over time, is computed using the following formula:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \quad (2)$$

where S_t denotes the gold price at time t [24]. The data used in this study are daily gold price obtained from www.exchange-rates.org, covering the period from October 2017 to October 2024.

To ensure the appropriateness of the model, several diagnostic checks were conducted [25]. The Augmented Dickey-Fuller (ADF) test was first applied to verify the stationarity of the return series. Next, the presence of ARCH effects was confirmed using the Lagrange Multiplier (LM) test, which justifies the application of GARCH modeling. In addition, ACF and PACF plots were analyzed for preliminary model identification in the context of ARIMA-based residual diagnostics. All statistical procedures and estimations are performed using EViews 10 due to its robust built-in econometric tools and ease of use for time series modeling.

The long-run variance V_L and the weight parameter γ are defined such that $\omega = \gamma V_L$ and $\gamma + \alpha + \beta = 1$. After estimating ω, α, and β, the values of γ and V_L can be derived as $\gamma = 1 - \alpha - \beta$ and $V_L = \frac{\omega}{\gamma}$.

Theorem 1. Assume $\omega > 0$ and $\alpha, \beta \geq 0$, then the GARCH (1,1) equation has a stationary solution if and only if $\mathbb{E}[\log(\alpha e_t^2 + \beta)] < 0$. In this case, the solution is uniquely given by:

$$\sigma_t^2 = \omega(1 + \sum_{j=1}^{\infty}\prod_{i=1}^{j}(\alpha e_{t-i}^2 + \beta)$$

[26].

Parameters in the GARCH (1,1) model are estimated using the Maximum Likelihood Estimation (MLE) method under the assumption of conditional normality:

$$L = \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{r_t^2}{2\sigma_t^2}\right)$$

with the log-likelihood function given by:

$$\ln L = \sum_{t=1}^{n} [-\ln(\sigma_t^2) - \frac{r_t^2}{\sigma_t^2} \quad (3)$$

After constructing the log-likelihood function, the optimal parameters are obtained by maximizing its value. Once estimated, the multi-step-ahead forecast of conditional variance at time $n + t$, given information up to time n, is:

$$\mathbb{E}[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L) \quad (4)$$

where $V_L = \frac{\omega}{(1-\alpha-\beta)}$ denotes the long-run variance. The corresponding volatility forecast σ_{n+t} is derived by taking the square root of the forecasted variance. Finally, because the data consists of daily gold prices, the daily volatility forecast is annualized by multiplying it by $\sqrt{365}$.

2.2 Option Pricing using Trinomial Tree Method

The trinomial tree method is an extension of the binomial tree model which assumes three possible outcomes for the underlying asset price (up, down or remain unchanged). First introduced by Boyle in 1986, the trinomial method is based on matching the mean and variance of continuous and discrete distributions. It discretizes the continuous time interval $[0, T]$ into n equal subintervals with time partition points $0 = t_0 < t_1 < \cdots < t_n = T$, where $t_j = j\Delta t$, $\Delta t = \frac{T}{n}$, and $j = 0,1,2, \ldots, n$ [27].

Compared to the binomial model, the trinomial tree offers better numerical stability and faster convergence, especially with fewer time steps, as supported by Josheski [21]. The presence of a middle state (no price change) enables a more accurate approximation of the lognormal distribution of asset prices. Although Monte Carlo simulation is widely used to model random price paths and estimate option values by averaging discounted payoffs [2], it is computationally intensive and more commonly applied in American option pricing [8]. Therefore, this study adopts the trinomial tree method for its balance between computational efficiency and accuracy in pricing European-style options under time-varying volatility.

In each time step Δt, the asset price S_n may move up by a factor u, down by a factor d, or remain unchanged by a factor m with corresponding probabilities p_u, p_d, and p_m [18]. The trinomial method is built upon several assumptions: constant volatility σ, asset prices following a geometric Brownian motion (GBM), a constant risk-free interest rate r, no arbitrage opportunities, absence of transaction costs or taxes, permission for short selling, divisibility of assets, and no dividends paid during the contract life.

Six key parameters govern the model: u, m, d, p_u, p_m, and p_d. The parameter estimation is typically guided by the following assumptions [1]:

1. The discrete model's expected price matches that of the continuous model
2. The discrete model's price variance matches that of the continuous model
3. $ud = 1$
4. $p_u + p_m + p_d = 1$
5. $p_m = \frac{2}{3}$

Once the parameters are established, the possible movements of the asset price at each node can be calculated as:

$$S_{ij} = \begin{cases} u^i S_0, i \geq 1 \\ S_0, i = 0 \\ d^{|i|} S_0, i \leq -1 \end{cases} \quad (5)$$

For European-style options, the payoff at maturity is given by:

$$C_{iN} = \max(S_{ij} - K, 0) \quad (6)$$

$$P_{iN} = \max(K - S_{ij}, 0) \quad (7)$$

Backward induction is then applied to obtain the present value of the option:

$$C_{i,j} = e^{-r\Delta t}(p_u C_{i+1,j+1} + p_m C_{i,j+1} + p_d C_{i-1,j+1}) \quad (8)$$

$$P_{i,j} = e^{-r\Delta t}(p_u P_{i+1,j+1} + p_m P_{i,j+1} + p_d P_{i-1,j+1}) \quad (9)$$

[28].

3. RESULT AND ANALYSIS

3.1 Volatility Forecasting

Before estimating volatility with the GARCH (1,1) model, a sequence of analyses is conducted, including stationarity testing, ARIMA model identification, and testing for ARCH effects.

1. Stationarity Test

Stationarity is tested using the Augmented Dickey-Fuller (ADF) test. The null hypothesis is that the return series contains a unit root (non-stationary). Table 2 presents the ADF test result.

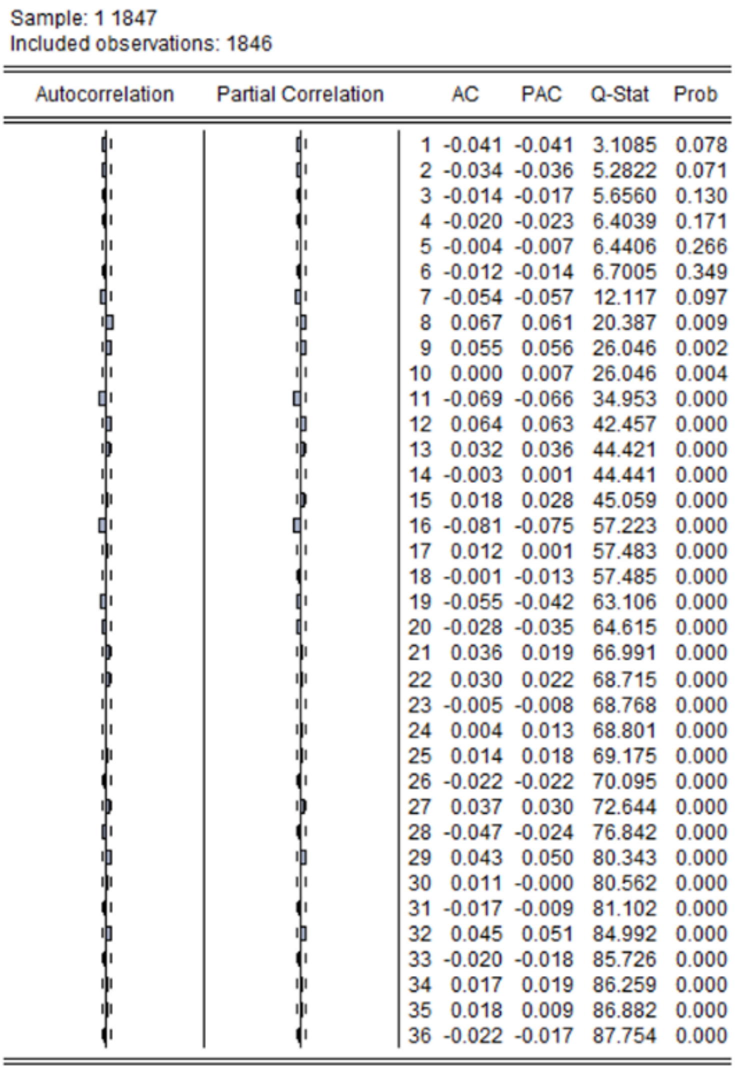
Table 2. ADF Test Result

ADF Statistic	**Critical Value (5%)**	**P-value**
-44.70591	-2.862903	0.0001

Since the test statistic is smaller than the critical value and the p-value is below 0.05, the null hypothesis is rejected, indicating that the gold return data is stationary.

2. ARIMA Model Identification

The identification of ARIMA model orders is conducted using ACF and PACF plots. Based on the ACF and PACF in Figure 1, significant spikes appear at lags 1, 8, 9, 11, 12, and 16. As a starting point and in accordance with the parsimony principle, AR (1), MA (1), and ARMA (1,1) are evaluated.



Sample: 1 1847
Included observations: 1846

Lag	AC	PAC	Q-Stat	Prob
1	-0.041	-0.041	3.1085	0.078
2	-0.034	-0.036	5.2822	0.071
3	-0.014	-0.017	5.6560	0.130
4	-0.020	-0.023	6.4039	0.171
5	-0.004	-0.007	6.4406	0.266
6	-0.012	-0.014	6.7005	0.349
7	-0.054	-0.057	12.117	0.097
8	0.067	0.061	20.387	0.009
9	0.055	0.056	26.046	0.002
10	0.000	0.007	26.046	0.004
11	-0.069	-0.066	34.953	0.000
12	0.064	0.063	42.457	0.000
13	0.032	0.036	44.421	0.000
14	-0.003	0.001	44.441	0.000
15	0.018	0.028	45.059	0.000
16	-0.081	-0.075	57.223	0.000
17	0.012	0.001	57.483	0.000
18	-0.001	-0.013	57.485	0.000
19	-0.055	-0.042	63.106	0.000
20	-0.028	-0.035	64.615	0.000
21	0.036	0.019	66.991	0.000
22	0.030	0.022	68.715	0.000
23	-0.005	-0.008	68.768	0.000
24	0.004	0.013	68.801	0.000
25	0.014	0.018	69.175	0.000
26	-0.022	-0.022	70.095	0.000
27	0.037	0.030	72.644	0.000
28	-0.047	-0.024	76.842	0.000
29	0.043	0.050	80.343	0.000
30	0.011	-0.000	80.562	0.000
31	-0.017	-0.009	81.102	0.000
32	0.045	0.051	84.992	0.000
33	-0.020	-0.018	85.726	0.000
34	0.017	0.019	86.259	0.000
35	0.018	0.009	86.882	0.000
36	-0.022	-0.017	87.754	0.000

Figure 1. ACF and PACF plots

Model selection is based on the lowest AIC value. The AIC values are:

1. AR (1): -6.496438
2. MA (1): -6.496568
3. ARMA (1,1): -6.497409

Thus, the ARMA (1,1) model is selected as the best-fitting model and is used in subsequent analysis.

3. ARCH Effect Test

Heteroscedasticity and ARCH effect testing are conducted on the residuals of the ARMA(1,1) model. Using EViews 10, the p-value from the ARCH-LM test is 0.0127 (Figure 2), which is below 0.05, indicating the presence of ARCH effects.

Heteroskedasticity Test: ARCH

F-statistic	4.378876	Prob. F(2,1841)	0.0127
Obs*R-squared	8.730492	Prob. Chi-Square(2)	0.0127

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 12/27/24 Time: 09:51
Sample (adjusted): 4 1847
Included observations: 1844 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.99E-05	9.06E-06	8.817002	0.0000
RESID^2(-1)	0.031742	0.023264	1.364391	0.1726
RESID^2(-2)	0.059985	0.023264	2.578408	0.0100

R-squared	0.004735	Mean dependent var	8.80E-05
Adjusted R-squared	0.003653	S.D. dependent var	0.000370
S.E. of regression	0.000370	Akaike info criterion	-12.96710
Sum squared resid	0.000251	Schwarz criterion	-12.95812
Log likelihood	11958.66	Hannan-Quinn criter.	-12.96379
F-statistic	4.378876	Durbin-Watson stat	2.003132
Prob(F-statistic)	0.012670		

Figure 2. ARCH-LM test result

Additionally, the LM statistics for lags 1 and 2 are 8.736075 and 8.73134, respectively, both exceeding the chi-square critical values (3.84 and 5.9915). These results confirm the presence of ARCH effects, validating the application of the GARCH model.

4. Parameter Estimation

The GARCH (1,1) model is estimated by maximizing the log-likelihood function in (3), assuming that $r_t \sim \mathcal{N}(0, \sigma_t^2)$. Since the return series is defined as $r_t = \mu_t + \varepsilon_t$, and the conditional mean μ_t is assumed to be zero, it follows that $r_t = \varepsilon_t$. Consequently, the conditional variance in (1) can be expressed as:

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (10)$$

where r_t is the gold return calculated using (2). The model parameters are estimated using Microsoft Excel Solver. The optimal values obtained are:

$$\omega = 0.000002189, \qquad \alpha = 0.072801, \qquad \beta = 0.902428$$

These estimates satisfy the standard GARCH conditions: $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta < 1$, ensuring a valid and stable variance process. Substituting the estimated parameters into (10) yields the fitted GARCH (1,1) model:

$$\sigma_t^2 = 0.000002189 + 0.072801 r_{t-1}^2 + 0.902428 \sigma_{t-1}^2 \quad (11)$$

The stationarity of the model is verified using the condition $\mathbb{E}[\log(\alpha e_t^2 + \beta)] < 0$ as stated in Theorem 1. A simulation was conducted using Python to estimate this expectation by generating a large number of random values from the standard normal distribution. With 100,000 iterations, the simulation yielded a value of -0.030027, confirming that the model satisfies the stationarity requirement.

The estimated parameters imply $\alpha + \beta = 0.975229$ and a weight parameter $\gamma = 0.024771$. These values indicate strong persistence in volatility with a slow but consistent toward the long-run variance V_L. The presence of a positive γ confirms that a portion of the conditional variance is always tied to the long-term variance, providing stability and a mean-reverting structure to the model.

These parameters characteristics highlight the theoretical advantages of the GARCH (1,1) model over simpler volatility forecasting methods such as moving average (MA) and exponentially weighted moving averages (EWMA). While MA and EWMA techniques rely on fixed-length windows and apply uniform or exponentially decaying weights to past observations, they do not explicitly model volatility as a dynamic process nor include a mechanism for long-term reversion. As emphasized by Hull [1], the inclusion of a mean–reverting component through the γ-weighted long-run variance makes GARCH (1,1) a more robust and theoretically appealing model, particularly for assets like gold that exhibit volatility clustering and eventual return to equilibrium.

5. Volatility Forecasting

The volatility for the next 1, 2, and 3 months is forecasted using (4) and the values are shown in Table 3.

Table 3. Forecasted Volatility using GARCH (1,1)

Time Horizon	Variance	Volatility
1 month	0.0000754	0.0086809
2 months	0.0000823	0.0090686
3 months	0.0000855	0.0092456

To improve forecasting accuracy, a machine learning-based approach is also used. The data is split into 80% training and 20% testing. Using Python, new parameters are estimated as:

$$\omega = 0.000001769, \qquad \alpha = 0.05, \qquad \beta = 0.9299$$

These values satisfy the GARCH model conditions and yield the following model:

$$\sigma_t^2 = 0.000001769 + 0.05r_{t-1}^2 + 0.9299\sigma_{t-1}^2 \tag{11}$$

Forecasted volatility using this machine learning approach is shown in Table 4.

Table 4. Forecasted Volatility with Machine Learning Approach

Time Horizon	Variance	Volatility
1 month	0.0000774	0.008798
2 months	0.0000824	0.009079
3 months	0.0000852	0.009228

3.2 Option Valuation

In valuing European-style gold options using the trinomial method, the initial asset price (S_0) is taken as the most recent historical price, which is 1,388,060 on October 31, 2024. The analysis is conducted by considering three cases based on the relationship between the initial market asset price (S_0) and the strike price (K), namely $S_0 > K, S_0 = K$, and $S_0 < K$, with the respective strike prices assumed to be 1,200,000; 1,388,060; and 1,500,000. The annual risk-free interest rate used in this study is set at 6\% [29].

Option prices are computed for maturities of 1–3 months using $n = 12$ time steps. Volatility is forecasted via the GARCH(1,1) model and annualized by multiplying daily estimates with $\sqrt{365}$, resulting in 0.16585, 0.17326, and 0.17664, respectively. The initial assumption $p_m = \frac{2}{3}$ is replaced with the empirical value $p_m = \frac{1}{100}$, obtained by classifying returns based on a threshold of 0.0001. Under the constraint $ud = 1$, the up and down factors are defined as:

$$u = e^{\sigma\sqrt{\frac{200}{198}\Delta t}}, d = \frac{1}{u} \tag{13}$$

The corresponding transition probabilities are:

$$p_u = \left(r - \frac{99}{198}\sigma^2\right)\sqrt{\frac{\Delta t}{\frac{800}{198}\sigma^2}} + \frac{99}{200}, p_d = \frac{99}{100} - p_u \tag{14}$$

Using these formulas, the parameters are computed for each time to maturity $T = 1,2,3$ months with $n = 12$ time steps. Table 5 presents the computed values.

Table 5. Estimated Parameters of the Trinomial Tree Method

Time	u	d	p_u	p_d
1 month	1.01399	0.98621	0.50656	0.48344
2 months	1.02073	0.97969	0.51023	0.47978
3 months	1.02596	0.97470	0.51305	0.47695

After obtaining the model parameters, the next step is constructing the asset price tree using (3). Each node is represented as a pair (i, j), where i indicates the number of up (if $i > 0$), down (if $i < 0$), or neutral (if $i = 0$) movements, and j denotes the step number ($j = 0,1,2, \dots, n$). Table 6 shows selected asset price outcomes for each maturity.

Table 6. Possible Gold Price Movements at Each Step

(i, j)	1 month	2 months	3 months
(0,0)	1,388,060	1,388,060	1,388,060
(1,1)	1,407,475	1,416,840	1,424,088
(0,1)	1,388,060	1,388,060	1,388,060
(−1,1)	1,368,913	1,359,865	1,352,944
⋮	⋮	⋮	⋮
(12,12)	1,639,830	1,775,653	1,887,773

At maturity, the option value at each terminal node is calculated using the payoff function (Eq. (4) for calls and Eq. (5) for puts). The option value at earlier nodes is determined using backward induction with discounting, as per Eq. (6) or Eq. (7), depending on the option type. The discount factor $e^{-r\Delta t}$ reflects the time value of money. Figure 3 illustrates the trinomial tree structure used to compute the call option value with $K = 1{,}200{,}000$ and $T = 1$ month.

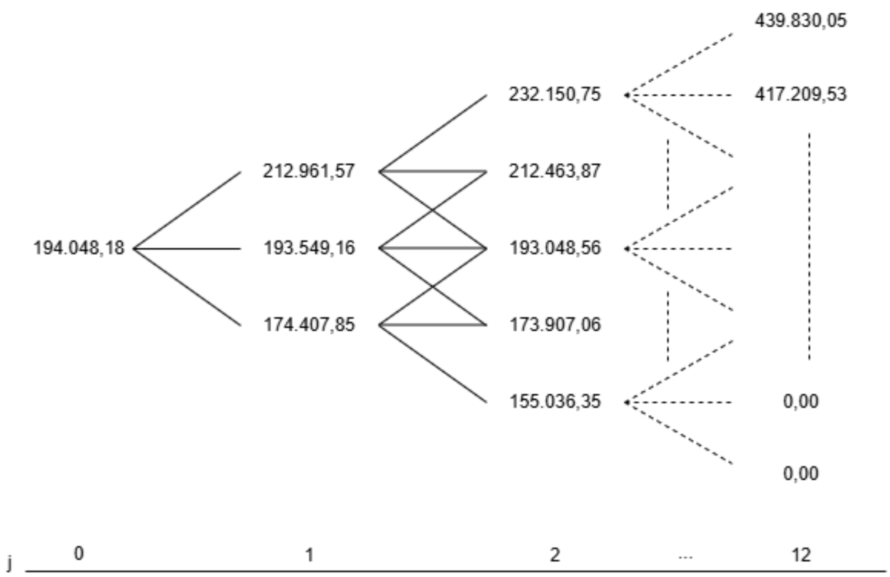


Figure 3. Trinomial Tree for Option Valuation ($K = 1{,}200{,}000$; $T = 1$ months)

The backward discounting process is applied recursively from the terminal nodes to the root node $(0,0)$, which represents the option price at time zero. As illustrated in Figure 3, the option value at the root node is 194,048.18. This procedure is repeated for each maturity and strike price scenario. The resulting option values are summarized in Table 7-9.

Table 7. Option Values for $T = 1$ month

(i,j)	$S_0 > K$		$S_0 = K$		$S_0 < K$	
	Call	Put	Call	Put	Call	Put
$(0,0)$	194,048	4.41	29,615	22,694	1,862	106,322
$(1,1)$	212,962	0,00	39,989	14,227	3,034	88,701
$(0,1)$	193,549	2.89	29,078	22,732	1,451	106,533
$(-1,1)$	174,408	9.07	18,784	31,585	644.40	124,874
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$(12,12)$	439,830	0,00	251,770	0,00	139,830	0,00
$(0,12)$	188,060	0,00	0,00	0,00	0,00	111,940
$(-12,12)$	0,00	25,055	0,00	213,115	0,00	325,055

Table 8. Option Values for $T = 2$ months

(i,j)	$S_0 > K$		$S_0 = K$		$S_0 < K$	
	Call	Put	Call	Put	Call	Put
$(0,0)$	200,410	414.96	45,616	31,810	8,914	105,934
$(1,1)$	227,889	104.17	61,362	19,921	13,614	83,092
$(0,1)$	199,221	215.60	44,655	31,994	8,048	106,305
$(-1,1)$	171,561	750.35	28,970	44,504	3,950	130,403
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$(12,12)$	575,653	0,00	387,593	0,00	275,653	0,00
$(0,12)$	188,060	0,00	0,00	0,00	0,00	111,940
$(-12,12)$	0,00	114,928	0,00	302,988	0,00	414,928

Table 9. Option Values for $T = 3$ months

(i,j)	$S_0 > K$		$S_0 = K$		$S_0 < K$	
	Call	**Put**	**Call**	**Put**	**Call**	**Put**
(0,0)	207,385	1,470	58,748	38,093	18,159	107,778
(1,1)	240,949	484.54	78,811	23,838	26,759	82,198
(0,1)	205,634	1,197	57,387	38,442	14,809	106,275
(−1,1)	171,860	2,539	37,349	53,520	9,026	135,609
⋮	⋮	⋮	⋮	⋮	⋮	⋮
(12,12)	687,773	0,00	499,713	0,00	387,773	0,00
(0,12)	188,060	0,00	0,00	0,00	0,00	111,940
(−12,12)	0,00	179,374	0,00	367,434	0,00	479,374

The option price at node (0,0) of each tree represents the fair price of the option at time zero. For the call option under the condition $S_0 > K$, the price increases with maturity: Rp194,048 for 1 month, Rp200,410 for 2 months, and Rp207,385 for 3 months. This upward trend reflects the increasing probability that the option will end in-the-money over a longer time horizon.

When $S_0 = K$, the call values are lower: Rp29,615, Rp45,616, and Rp58,748, indicating a reduced probability of ending in-the-money compared to the $S_0 > K$ case. In contrast, for $S_0 < K$, the call option values are significantly lower: Rp1,862, Rp8,914, and Rp18,159, due to the option being out-of-the-money across all maturities.

Regarding put options, the values are negligible for $S_0 > K$: Rp4.41, Rp414.96, and Rp1,470. This is expected as the option holder has no advantage in selling at a strike price below the market price. When $S_0 = K$, put values are modest: Rp22,694, Rp31,810, and Rp38,093. However, for $S_0 < K$, the put option becomes highly valuable: Rp106,322, Rp105,934, and Rp107,778, since the right to sell above the market price provides clear payoff advantages.

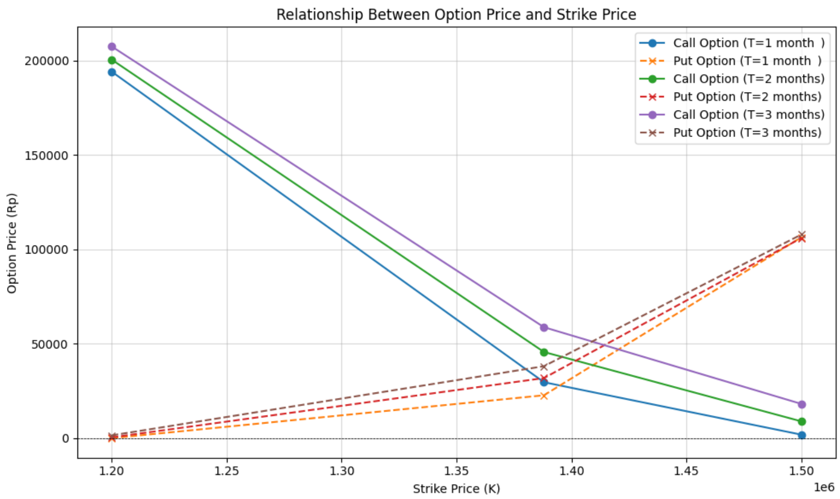


Figure 4. Effect of Strike Price on Option Value

Figure 4 illustrates that call option values decline with increasing strike prices, while put option values increase. This pattern aligns with standard financial theory: call options become less valuable as they move further out-of-the-money, and put options become more valuable as they move further in-the-money.

Figure 5 shows that both call and put option values generally increase with time to maturity. A longer time horizon provides more opportunities for the underlying asset to experience price movements favorable to the option holder, thereby increasing the option's time value.

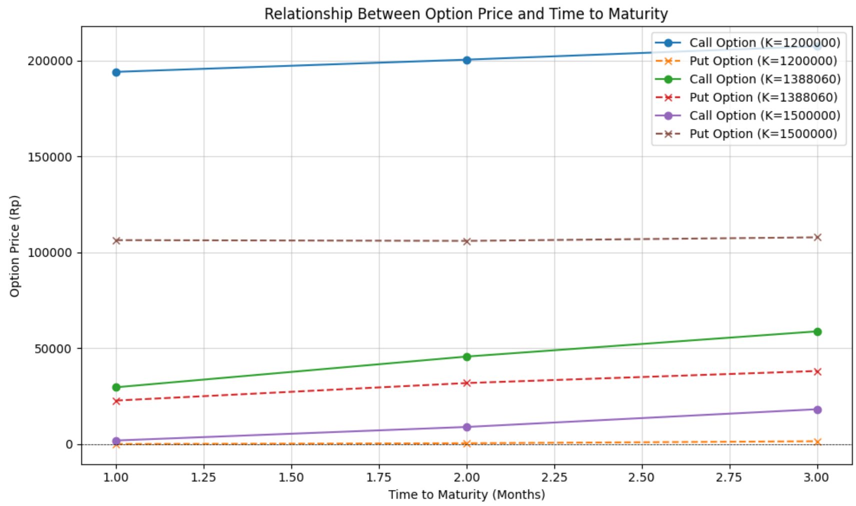


Figure 5. Effect of Time to Maturity on Option Value

4. CONCLUSION

This study applied the GARCH(1,1) model to forecast the volatility of gold return data and used the trinomial method to determine the price of European-style gold options. The GARCH(1,1) model was specified as

$$\sigma_t^2 = 0.000002189 + 0.072801 r_{t-1}^2 + 0.902428 \sigma_{t-1}^2$$

with volatility calculated as the square root of the conditional variance. The forecasting results produced by this model were found to be comparable to those obtained using machine learning-based approaches.

In the option pricing stage, a fixed probability of $\frac{1}{100}$ was employed in the trinomial model, with transition probabilities estimated from historical data. The analysis revealed that call option prices decrease as the strike price increases, while put option prices exhibit the opposite trend, increasing with higher strike prices. Furthermore, both call and put option prices increase with longer time to maturity, reflecting the greater potential for favorable movements in the underlying asset price over extended periods. Overall, the findings support the effectiveness of the GARCH(1,1)-based volatility forecast combined with the trinomial method for evaluating European gold options under different market conditions.

From a practical perspective, this approach can guide traders and investors in strategy selection. For instance, if investors expect rising gold prices, long-term call options (e.g., T = 3 months) may yield better returns. Conversely, long-term put options are more beneficial when anticipating a price decline. When the strike price equals the current gold price, short-term options (T = 1 month) may be preferred due to lower costs. Future research may extend this framework by exploring other GARCH specifications such as EGARCH or GJR-GARCH to better capture volatility asymmetry, extending the model to American options or other commodities (e.g., crude oil, silver), and integrating machine learning to enhance predictive performance.

5. REFERENCES

[1] J. C. Hull, *Risk Management and Financial Institutions,* New Jersey: John Wiley & Sons, 2023.

[2] S. Ren and X. Zhang, "Chooser option pricing based on Black-Scholes Model and Monte-Carlo Simuations," *BCP Business & Management,* vol. 32, pp. 63-71, 2022, doi: https://doi.org/10.54691/bcpbm.v32i.2870.

[3] R. Robiyanto, B.A. Nugroho, A.D. Huruta, B. Frensidy, and Suyanto, "Identifying the Role of Gold on Sustainable Investment in Indonesia: The DCC-GARCH Approach," *Economies,* vol. 9, no. 119, 2021, doi: https://doi.org/10.3390/economies9030119.

[4] S. Kumari, V. S. Nerlekar, and N. Raut, "Gold as a Safe Haven: Analyzing the Investment Patterns of Indian Women," *Journal of Informatics Education and Research,* vol. 4, no. 3, pp. 535-540, 2024.

[5] C. A. Ugomma, "An Empirical Study of Stock Price Returns Using Black-Scholes Option Pricing Model," *International Journal for Research in Applied Science & Engineering Technology (IJRASET),* vol. 11, pp. 2058-2066, 2023, doi: https://doi.org/10.22214/ijraset.2023.55479.

[6] Y. Miao, "A Binomial Tree-Based Empirical Study on the Biases of American Call Option," *Highlights in Business, Economic and Management,* vol. 40, pp. 470-476, 2024, doi: https://doi.org/10.54097/z09bzy93.

[7] H. Ding, "Research on option pricing of Tesla based on Black-Scholes-Merton, Binomial Tree, and Cox-Ingersoll-Ross Models," *Highlights in Business, Economics and Management,* vol. 22, pp. 35-40, 2023, doi: https://doi.org/10.54097/mwd4s763.

[8] Y. Meng, "Comparison of Least Square Monte Carlo Algorithm and Binomial Tree Model for Pricing American Options," in *Proc. 2022 4th Int. Conf. on Economic Management and Cultural Industry (ICEMCI 2022),* pp. 2019-2028, doi: https://doi.org/10.2991/978-94-6463-098-5_227.

[9] H. Yıldırım and F. V. Bekun, "Predicting volatility of bitcoin returns with ARCH, GARCH, and EGARCH models," *Future Business Journal,* vol. 9, no. 1, p. 75, 2023, doi: https://doi.org/10.1186/s43093-023-00255-8.

[10] S. Sulastri, L. Novieyanti, and Sukono, "Penentuan Harga Opsi Barrier Menggunakan Metode Trinomial Kamrad Ritchken Dengan Volatilitas Model GARCH," *Jurnal Ilmu Manajemen dan Bisnis,* vol. 10, no. 1, pp. 83-91, 2019, doi: https://doi.org/10.17509/jimb.v10i1.16163.

[11] A. Khera, A. Goyal, and M. P. Yadav, "Capturing the stock market volatility: a study of sectoral indices in India using symmetric GARCH models," *International Journal of Management Practice,* vol. 15, no. 6, pp. 820-833, 2022, doi: https://doi.org/10.1504/IJMP.2022.126536.

[12] F. Sekmen and G. A. Ravanoglu, "The modelling of exchange rate volatility using Arch-Garch models: the case of Turkey," *MANAS Soosyal Araştırmalar Dergisi,* vol. 9, no. 2, pp. 834-843, 2020, doi: https://doi.org/10.33206/mjss.541309.

[13] R. -C. Muşetescu, G.-E. Grigore, and S. Nicolae, "The use of GARCH autoregressive models in estimating and forecasting the crude oil volatility," *European Journal of Interdisciplinary Studies,* vol. 14, no. 1, pp. 13-38, 2022, doi: https://doi.org/10.24818/ejis.2022.02.

[14] S. Sharma, V. Aggarwal, and M. P. Yadav, "Comparison of linear and non-linear GARCH models for forecasting volatility of select emerging countries," *Journal of Advances in Management Research,* vol. 18, no. 4, pp. 526-547, 2021, doi: https://doi.org/10.1108/JAMR-07-2020-0152.

[15] C. Spulbar, F. R. Birau, J. Trivedi, I. T. Hawaldar, and E. L. Minea, "Testing volatility spillovers using GARCH models in the Japanese stock market during COVID-19," *Investment Management and Financial Innovations,* vol. 19, no. 1, pp. 262–273, 2022, doi: https://doi.org/10.21511/imfi.19(1).2022.20.

[16] I. M. Md Ghani and H. A. Rahim, "Modeling and Forecasting of Volatility using ARMA-GARCH: Case Study on Malaysia Natural Rubber Prices," in *Proc. International Rubber Conference, IOP Conf. Series: Materials Science and Engineering,* 2019, doi: https://doi.org/10.1088/1757-899X/548/1/012023.

[17] D. L. Olson and D. Wu, *Predictive Data Mining Models,* Singapore: Springer Science+Business Media, 2017.

[18] K. K. Langat, J. I. Mwaniki, and G. K. Kiprop, "Pricing Options Using Trinomial Lattice Method," *Journal of Finance and Economics,* vol. 7, no. 3, pp. 81–87, 2019, doi: https://doi.org/10.12691/jfe-7-3-1.

[19] A. K. Sinha, "How to Implement a Trinomial Option Pricing Model in MS-Excel?" *Journal of Accounting and Finance,* vol. 21, no. 5, 2021, doi: https://doi.org/10.33423/jaf.v21i5.4731.

[20] Q. Nissa, N. Satyahadewi, and H. Perdana, "Penentuan Harga Opsi Beli Tipe Eropa Menggunakan Metode Trinomial," *Bimaster: Buletin Ilmiah Matematika, Statistika dan Terapannya,* vol. 9, no. 3, 2020, doi: https://doi.org/10.26418/bbimst.v9i4.42742.

[21] D. Josheski and M. Apostolov, "A review of the binomial and trinomial models for option pricing and their convergence to the Black-Scholes model determined option prices," *Econometrics Ekonometria Advances in Applied Data Analysis,* vol. 24, no. 2, pp. 53–85, 2020, doi: https://doi.org/10.15611/eada.2020.2.05.

[22] İ. Yurttagüler, "Analysis of BIST Gold Index Volatility With Autoregressive Conditional Heteroscedasticity Models," İnsan ve Toplum Bilimleri Araştırmaları Dergisi, vol. 13, no. 3, pp. 1256-1276, 2024, doi: https://doi.org/10.15869/itobiad.1390321.

[23] T. Bollerslev. "A conditionally heteroskedastic time series model for speculative prices and rates of return," *The Review of Economics and Statistics,* pp. 542-547, 1987.

[24] R. S. Tsay, *Analysis of Financial Time Series*, 3rd ed., Hoboken, NJ: John Wiley & Sons, 2010.

[25] Z. C. Stiglingh and M. B. Seitshiro, "Quantification of GARCH(1,1) Model Misspecification with Three Known Assumed Error Term Distributions," Journal of Financial Risk Management, vol. 11, no. 3, pp. 549–578, 2022, doi: https://doi.org/10.4236/jfrm.2022.113026.

[26] B. Williams, *GARCH(1,1) Models*, Bachelor's thesis, Ruprecht-Karls-Universität Heidelberg, Faculty of Mathematics and Computer Science, July 2011.

[27] D. J. Higham, *An Introduction to Financial Option Valuation*, Cambridge: Cambridge University Press, 2004.

[28] D. Lilyana, B. Subartini, Riaman, and A. K. Supriatna, "Calculation of call option using trinomial tree method and black-scholes method: Case study of Microsoft Corporation," *Journal of Physics: Conference Series*, vol. 1722, no. 1, 2021, doi: https://doi.org/10.1088/1742-6596/1722/1/012064.

[29] Badan Pusat Statistik Indonesia, "BI Rate, 2024." [Online]. Available: https://www.bps.go.id/id/statistics-table/2/Mzc5IzI%3D/bi-rate.html. [Accessed: Dec. 2, 2024].