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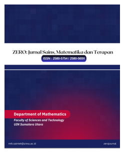

Designing an Optimal Route Network for the Synchronized Trans Gadjah Mada Electric Bus (TGMEB) using Max-Plus Algebra

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ABSTRACT

We develop an optimal route network and synchronized timetable for the Trans Gadjah Mada Electric Bus (TGMEB), designed to cover the entire campus with 4 buses and 28 stops. Using literature review, field observation, and online mapping, we create three network candidates—one-, two-, and three-terminal—and model each as a max-plus linear discrete-event system. Service period and periodic departures are derived from eigenvalue–eigenvector analysis and implemented in Scilab 5.5.2 with a max-plus toolbox. The two-terminal layout performs best: its average inter-stop travel time is 32% faster than the other alternatives while keeping departures synchronized and coverage intact. The results confirm that jointly selecting the network architecture and its timetable yields superior campus operations. This is the first campus-scale study that co-designs a multi-terminal electric-bus route network and synchronized timetable via max-plus algebra, optimizing departure throughput and average inter-stop travel time. Unfortunately, this design has not been tested on the field.

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1. INTRODUCTION

The Trans Gadjah Mada Electric Bus (TGMEB) is an electric bus designed to provide transportation services for the entire intra-campus community at Gadjah Mada University. In period 1: April 1, 2022 to August 13, 2024, BLTG operates serving an area of approximately 50% of the northern UGM area, with 23 stops with two buses and two route networks, where each bus serves 1 route network. These two route networks pass through the same roads and stops, only in opposite directions, with the same departure terminal and departure time. Passengers can choose the route network depending on the shortest distance to be traveled. In period 2: August 14, 2024 to March

23, 2025, still in the same service area, even reduced to around 40% due to road repairs, two buses operate on two different route networks, namely the route network for the eastern and western campus areas. These two route networks are not yet connected. Each route network has its own terminal with a considerable distance. This is quite inconvenient for passengers who have to change route networks, as conveyed in the Instagram account @aset.ugm. In period 3: March 24, 2025 to present, 1 route network was added to the west area of the south side of the campus. So there are a total of 3 route networks with a service area of approximately 65%, served by 3 buses, with 1 terminal. The departure schedule from the terminal for each route network is the same, so these three route networks can be said to be synchronized at the terminal, meaning passengers can change routes at the terminal and buses are available according to the same schedule. The duration between departure schedules is 20 minutes, with a 10-minute break every two departures. In general, passengers who want to change routes must go to the terminal first. The image of the route networks in the three periods above can be seen in the following image.

Period 1 (50% service area) Period 2 (40% service area) Period 3 (65% service area)

Figure 1. TGMEB route network from April 2022 to March 2025

From the situation of the development of TGMEB above, it can be obtained that not all areas of the UGM Campus can be served by TGMEB with an optimally synchronized route network. Based on information from the Facilities and Infrastructure Management Staff of the Asset Directorate of Gadjah Mada University in the period after period 2, up to 3 more buses will be added, so that a total of 5 buses will be operated. For this reason, in this study, the optimal route network that serves the entire campus area will be designed and analyzed by operating 4 buses in 4 route networks with varying numbers of terminals as places to connect between route networks and synchronize bus departure schedules. The consideration of using 4 buses is so that 1 bus can be used as a backup, while 4 route networks because it is estimated that each route network has a travel time from and back to the terminal of around 20 minutes. There are three alternative route networks that will be designed and analyzed, namely alternatives 1, 2 and 3, where the route network with 1, 2 and 3 terminals, respectively. Handling of bus synchronization problems with multiple terminals has been discussed in [1], [2].

Max-Plus Algebra, which is the set of all real numbers **R** equipped with maximum (max) and plus (+) operations, has been widely used to model network problems involving synchronization problems [3], [4], [5]. Especially in the problem of network modeling and synchronized scheduling for railway transportation, Max-Plus Algebra has been discussed in [6], [7]. Especially in the problem of rail scheduling by using Max-Plus algebra can be seen in [8], [9], [10] . While the discussion of the problem of delays in train travel time using Max-Plus algebra can be seen in [11]. Network modeling and synchronized scheduling for bus transportation using Max-Plus Algebra has been discussed in [12], [13], [14]. These studies only discussed the synchronization of departure schedules. Furthermore, the routes in these studies were already determined. The problem of synchronized scheduling optimization for transportation problems involving modeling using Max-Plus Algebra has also been discussed in the following article [15] The optimization carried out in these articles focuses on a network model modeled with Max-Plus Algebra into a Linear Max-Plus System, which is then combined with other optimization methods. The discussion of the optimal solution problem in general, both locally and globally has been discussed in [16]. The Closed Shop Scheduling Optimization Method using Max-Plus Automata as discussed in [17] is a combination of an optimization method with Max-Plus Algebra, especially for Automata. The use of Max-Plus Automaton in train scheduling analysis and its optimization has been discussed in [18]. Other optimization problems discuss the optimization problems of predictive control models [19], [20] and optimal control in a Max-Plus Linear System, as discussed in [21], [22], [23]. The optimization problems involving uncertainty in Max-Plus Systems are discussed in [24], [25].

The thing that is the novelty of this research, which has not been done much is to review the problem of multi-terminal electric bus transportation that serves in a limited area in the campus area. In this study, a bus route will be designed to cover the entire campus area. The renewal of this research also adds aspects of optimization, in particular are the average travel time between the minimum bus stops and the maximum amount of departure.

This study aims to design an optimal route network and scheduling for TGMEB that is synchronized and covers the entire campus area, where there are 4 buses operating. In this study, the optimization problem discussed is by comparing three bus transportation route network designs that serve all area in UGM. All three design networks have variety the number of terminal namely 1 terminal, 2 terminals and 3 terminals. This study will start to designing and modelling these network in max-plus algebra. Meanwhile, the optimal route network criteria will be determined from the maximum number of departures and the average time for passengers to reach between stops that is minimal. Optimization problems like this as far as the author knows have not been discussed, and this is a novelty in this study. So, this research aims to design an optimal route network and scheduling for the Trans Gadjah Mada Electric Bus (TGMEB) which is synchronized and covers the entire campus area, where there are 4 buses operating with 28 stops. The implementation of TGMEB network synchronization can be a starting point for developing an automated transportation system at the university. The network and electric buses used are the basis for developing a smart and green campus.

2. RESEARCH METHOD

This research is a design and analysis study to optimize the design results by comparing the performance of the design results according to the research objectives. The research stages are as follows.

2.1 Literature Study

To discuss the problems in this research there are several basic concepts in Max-Plus Algebra that must be understood and can be studied in [3], [4], [5], [6]. The concepts understood from the literature include Max-Plus Algebra, Matrix over Max-Plus Algebra, Graph and Max-Plus Algebra, Eigenvalues and Eigenvectors of Max-Plus. For computational calculations it is necessary to understand the use of the Scilab 5.5.2 Computer Program [40]with Toolboxes MAXPLUSV04032016 [41], to run commands related to Max-Plus Algebra.

2.2 Observational Study

In this section, data is collected through participatory observation studies and non-participatory observations. Information collected includes campus maps, schedules, route networks, many stops and travel time between stops. At the non-partial observation stage, *Google Earth* is used (https://earth.google.com/web/?hl=id) as a tool to visualize the overall campus area map and design of the new route network to be made.

2.3 Route Network Design

This section will design a new route network for the entire campus area based on data from Instagram accounts, the website, and field observations. Field observations will ensure road access and estimate the placement of new bus stops. In addition to observations, *Google Maps* (https://www.google.com/maps) will also be used to estimate travel times between bus stops. Three alternative route networks will be created: a route network with one terminal, two terminals, and three terminals.

2.4 Modeling and Scheduling Using Max-Plus Algebra

Based on several alternative route networks that have been created, a max plus algebraic model is then formed that accommodates synchronization related to scheduling, which ensures passengers can change route networks at terminals where buses and schedules are definitely available. The model will be a Max-Plus Equation System in the matrix equations over Max-Plus Algebra. In determining the scheduling periodicity, the concept of matrix eigenvalues in the equations above will be used, while eigenvectors will be used to determine departure times that guarantee the schedule to be periodic. Computations in scheduling will be carried out with the help of the Scilab 5.5.2 Computer Program. Scilab is software developed for numerical computation. Scilab provides many packeges of computational algorithms, particularly those in max-plus algebra. The results of Scilab's computations are highly readable for scheduling and eigen value problems. It will be shown how many maximum departure periods in the scheduling for each alternative route network.

2.5 Average Travel Time Between the Furthest Bus Stops

From the schedules obtained for each route network alternative, the average travel time between the furthest stops is calculated. The furthest stop is chosen to represent the stops within each route network alternative. This is because calculating all stops would be lengthy and complex. This analysis aims to determine how efficient the route network is when passengers must transfer buses to reach their destination.

2.6 Optimal route network conclusion

The optimal alternative route network will be considered through the indicators obtained in stages 2.4 and 2.5 above, namely the number of departure schedules and the minimum average travel time between the furthest stops.

3. RESULT AND ANALYSIS

3.1 Alternative Route Network Design

This design assumes each bus will serve one route, so each route network will consist of four routes. This assumption is given for two main reasons, namely 1) the campus plans to have 5 buses, 4 for use ready and 1 as a backup and 2) there is no overlap between buses if the route is served by more than one bus. All four routes are connected by terminals, either directly or indirectly, allowing passengers to reach any two existing stops. The design took into account the entire campus area, allowing for road access and the construction of bus stops. A list of the new bus stops in the design can be found in the following table.

Table 1. Design of the New BLTG Bus Stop

Bus Stop Label	Bus Stop Name
A A	Graha Sabha Pramana (GSP)
AB	Central Library
air conditioning	Faculty of Forestry
AD	Faculty of Agriculture
AE	Faculty of Animal Husbandry
A.F.	Wisdom Park
AG	Campus Mosque
AH	Faculty of Culture
AI	Faculty of Cultural Sciences (FIB)
A.J.	University Club (UC)
AK	Kosudgama Pharmacy
AL	Directorate of Community Service
AM	PKGM FK-KMK UGM
AN	Diploma III Forest Management Campus
AO	Secretariat of BEM KM UGM
AP	Ratnaningsih Sendowo Residence UGM
AQ	Sardjito Hospital
AR	Faculty of Engineering
US	Faculty of Biology
AT	Faculty of Mathematics and Natural Sciences
AU	Faculty of Social and Political Sciences Flat A
AV	Faculty of Dentistry
AW	Mandiri ATM Yogyakarta UGM
AX	UGM Silviculture Laboratory
AY	Master of Management (MM)
AZ	Ratnaningsih Kinanti UGM Dormitory
BA	PAU Postgraduate & Masters
BB	UGM Valley

There are three alternative route networks selected in the design, namely one route network with 1, 2 and 3 terminals, with a total travel distance of around 20 minutes for each route.

Route Network with 1 terminal (Alternative 1)

Network takes the terminal at AA: Graha Sabha Pramana and four routes serving the area as shown in the left figure below, while the right figure shows the weighted graph of alternative route network 1. The weights in the graph indicate the total distance traveled for each route. The travel time includes passenger transfer time of approximately 30 seconds.

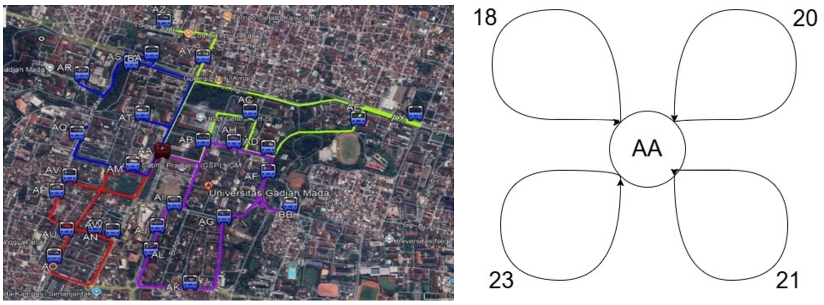

Figure 2. Map of Alternative Route and Network Graph Route with 1 Terminal at the Center and 4 Bus Loop Routes

Route Network with 2 terminals (Alternative 2)

Network takes the terminals at AT: Faculty of Mathematics and Natural Sciences and AA: Graha Sabha Pramana. Four routes serve the area as shown in the left image below, while the right image shows the weight graph of alternative route network 2.

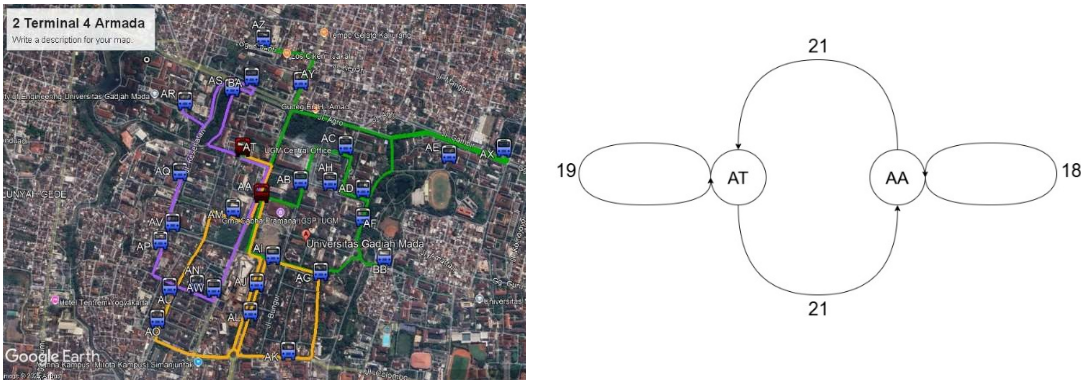

Figure 3. Map of Alternative Route and Network Graph Route with 2 Terminals, 2 Bus Loop Routes and 2 Bus Arc Routes

Route Network with 3 terminals (Alternative 3)

This route network takes terminals at BA: PAU Pascasarjana & Magister , AM: PKGM FK-KMK UGM and AD: Faculty of Agriculture . Four routes serve the area as shown in the left image below, while the right image shows the weight graph of alternative route network 3.

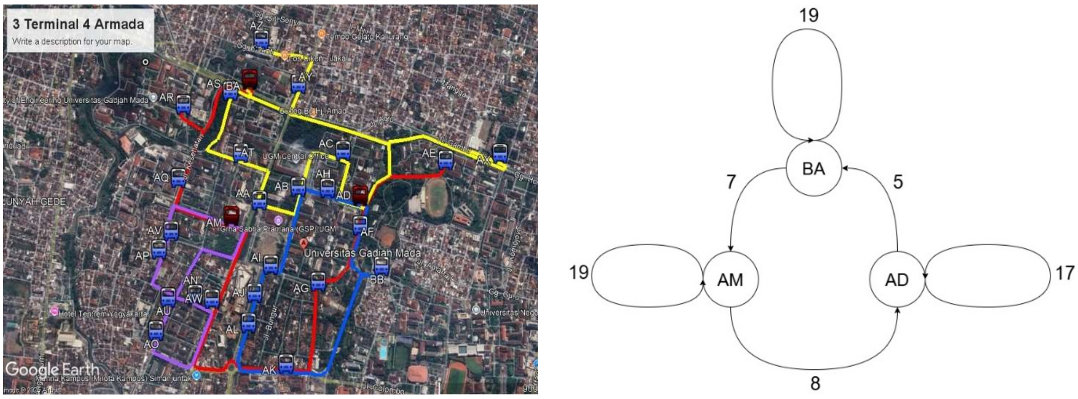

Figure 4. Map of Alternative Route 3 and Network Graph With 3 Terminals, 3 Bus Loop Routes and 1 Route around the bus between terminals

3.2 Modeling and Scheduling Each Alternative Route Network

Alternative Modeling and Scheduling 1

Let $x_1(k+1)$ is the departure time of the bus ($k+1$)th from terminal 1. To ensure synchronization, the departure time for $k+1$ each bus must wait for all buses to return to the terminal after completing their respective routes. This results in

$$\begin{aligned} x_1(k+1) &= x_1(k) \otimes 23 \oplus x_1(k) \otimes 18 \oplus x_1(k) \otimes 22 \oplus x_1(k) \otimes 20x_1(k) \\ &= x_1(k) \otimes 23 \end{aligned} \quad (1)$$

Thus, the scheduling will be periodic with a period of 23 minutes. Continuous scheduling (without breaks) from 6:30 a.m. to 4:16 p.m. will yield 25 departures.

Alternative Modeling and Scheduling 2

Suppose $x_i(k+1)$that is the departure time of the i-th bus $k+1$from the *i -th terminal,* with *i* = 1, 2. Suppose also that AT is the 1st terminal, and AA is the 2nd terminal, then the *k* -th +1 departure time from the 1st terminal is the maximum time between the *k-th departure time* from the 1st terminal plus the travel time from the AT-to-AT route or the *k-th departure time* from the 2nd terminal plus the travel time from the AA to AT route. Likewise, in the same way, the *k-th +1* departure time from the 2nd terminal can be determined, so that the following max-plus equation system is obtained

$$x_1(k+1) = \max(x_1(k)+19, x_2(k)+21)$$
$$x_2(k+1) = \max(x_1(k)+21, x_2(k)+18)\ . \quad (2)$$

If the max operation is denoted by ⊕, the addition operation is denoted by ⊗the equation which can be written as follows

$$x_1(k+1) = x_1(k) \otimes 19 \oplus x_2(k) \otimes 21$$
$$x_2(k+1) = x_1(k) \otimes 21 \oplus x_2(k) \otimes 18. \quad (3)$$

If written with a matrix equation, it is obtained.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 19 & 21 \\ 21 & 18 \end{bmatrix} \otimes \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (4)$$

For example, $A = \begin{bmatrix} 19 & 21 \\ 21 & 18 \end{bmatrix}$, with the help of Scilab 5.5.2, the eigenvalue of matrix *A can be obtained* as 21 and the eigenvector (0, 0) T. Continuous scheduling from 06.30 to 16.16 will result in 28 departures.

Alternative Modeling and Scheduling 3

Suppose $x_i(k+1)$is the departure time of the i-th bus $k+1$from the *i -th terminal,* with *i* = 1, 2, 3. Also suppose AM is the 1st terminal, AD is the 2nd terminal and BA is the 3rd terminal. There are 4 routes in this alternative, namely AM-AD-BA, AM-AM, AD-AD and BA-BA. From the graph it appears that the total travel time of the AM-AD-BA route is 20 minutes, so it appears larger than the other routes. This largest total travel time will determine the scheduling period.

Synchronized scheduling is defined as follows. The earliest departure time starts from Terminal 1, followed by scheduling at Terminal 2, 8 minutes later, then 5 minutes later the departure time at Terminal 3, 7 minutes later the second departure for Terminal 1, and so on. Thus, the departure times are as follows:

$$x_1(0) = 0, x_1(k+1) = x_3(k)+7, x_2(k+1) = x_1(k)+8, x_3(k+1) = x_2(k)+5 \quad (5)$$

Thus, the interval between scheduled departures is 20 minutes. Continuous scheduling from 6:30 a.m. to 4:16 p.m. would yield 29 departures.

3.3 Travel Time Analysis between Bus Stops

For this analysis, four bus stops were selected, located far enough apart to represent the entire service area. These four stops are AO, AK, AX, and AZ. The travel time between these stops was then calculated for each alternative route network. The calculation results for each alternative route network can be seen in the following table.

Table 2. Travel Time Between the Furthest Stops

Alternative 1					Alternative 2					Alternative 3				
Travel Time (minutes)	Objective				Travel Time (minutes)	Objective				Travel Time (minutes)	Objective			
Origin	AO	AK	AX	AZ	Origin	AO	AK	AX	AZ	Origin	AO	AK	AX	AZ
AO	0	15	19	27	AO	0	3	20	26	AO	0	30	35	42
AK	26	0	24	32	AK	16	0	19	25	AK	28	0	22	29
AX	25	19	0	13	AX	12	15	0	6	AX	22	36	0	7
AZ	17	15	11	0	AZ	10	13	11	0	AZ	18	29	12	0
Total: 243 , Average: 20.25					Total: 176 , Average: 14.67					Total: 310 , Average: 25.83				

From the results in Table 2 above, it appears that Alternative Route Network 2 is the most efficient in reaching between stops, because on average it has the minimum travel time.

From the results given in sub-chapters 3.2 and 3.3 above, it appears that during the TGMEB operating hours of 06.30–16.16, alternative route networks 1, 2, and 3 can be scheduled in 25, 28, and 29 departures, respectively.

Meanwhile, on average, the travel time between stops for alternative route networks 1; 2, and 3 is 20.25; 14.67, and 25.83, respectively. In terms of the number of departures, alternative 2 is fewer than alternative 3, but only a difference of 1 round. However, in terms of efficiency, alternative 2 is much more efficient than alternative 3. Thus, it can be concluded that the most optimal choice is alternative route network 2.

3.4 Discussion

The results of this study indicate that a route network with two terminals (Alternative 2) is the most optimal solution. This is evident from the comparison of the number of departures, scheduling periods, and travel time efficiency between the furthest stops.

Theoretically, the use of Max-Plus Algebra to model transportation scheduling systems has been widely discussed in the literature. In [3]and [4] it is explained that Max-Plus Algebra is very effective for modeling discrete event systems such as transportation schedules because it can capture the dynamics of synchronization between system elements. This study applies this principle in the context of a campus electric bus network, similar to the rail transportation modeling discussed by [7]and [8], as well as the bus transportation modeling in [19] and [21]. Where in [19] and [21] the preparation of the bus departure schedule periodically is also carried out through the calculation of eigen and eigen vectors, only for a lot of departure in a certain hour range is not discussed.

This study shows that a two-terminal configuration provides the optimal balance between synchronization complexity and operational efficiency. This finding is also in line with research findings [20] that suggest that network structures that are too simple or too complex can lead to inefficiencies in bus network performance. From an optimization perspective, the study findings reinforce the idea presented in [25]and [27] that the optimal solution in a Max-Plus Linear system does not always lie at the extremes (e.g., just one terminal or as many terminals as possible), but rather in a configuration that considers the trade-off between the number of departures and user travel efficiency.

In fact, there's another possibility for the two-terminal alternative not discussed in this study, which readers can explore for themselves. This possibility involves two terminals, where the loop route isn't located at each terminal, but rather is combined within one. However, this possibility appears to be less optimal than Alternative 2 above, as the route structure tends to accumulate at one terminal.

In addition, this study enriches the literature on the application of Max-Plus in bus transportation in Indonesia, complementing previous studies that have focused mostly on rural [23]or urban transportation [24]. By considering the campus context, this study adds a new dimension, namely the need for synchronization that is not only on technical aspects, but also on the comfort of users who are mostly students and academic staff with dense and specific travel patterns.

Thus, the results of this study not only have practical contributions in campus transportation management, but also theoretical contributions in modeling and optimization of the Max-Plus system, especially for networks with multi-terminals on a medium scale.

4. CONCLUSION

This study succeeded in designing the optimal route and scheduling network for TGMEB using Max-Plus algebra. Of the three alternative route networks that have been designed and analyzed, the results obtained that the design of the route with two terminals where there are two loop -shaped bus routes and two connected bow -shaped bus routes, are the most optimal choice. This alternative is able to provide quite a large amount of departure in the range of operating hours, with the average travel time between the most far farthest bus stops compared to other alternatives. The structure of two terminals allows good schedule synchronization, time efficiency, and flexibility of passenger movements. This finding is expected to be a recommendation for the TGMEB manager in improving environmentally friendly transportation services within the University of Gadjah Mada. TGMEB scheduling synchronization is performed assuming normal traffic along the route. Events that cause traffic congestion will naturally impact the schedule provided by the model of Max-Plus Algebra. Further research into adding a delay tolerance limit to the system is warranted.

5. REFERENCES

[1] I. Sugiarto, F. J. Permana, and A. Yosep, “Bus schedule modelling in Bandung, Indonesia, using max-plus algebra,” *IOP Conf Ser Mater Sci Eng*, vol. 830, no. 2, pp. 1–6, 2020, doi: 10.1088/1757-899x/830/2/022004.

[2] T. Šfiligoj and A. Peperko, “A max-plus algebraic model of bus bunching and control strategies,” in *Transportation Research Procedia*, Elsevier B.V., 2025, pp. 88–94. doi: 10.1016/j.trpro.2025.02.013.

[3] F. Baccelli, G. Cohen, G. J. Olsder, and J.-P. Quadrat, *Synchronization and linearity : an algebra for discrete event systems*. Paris: Wiley, 2001. Accessed: Feb. 02, 2024. [Online]. Available: https://www.rocq.inria.fr/metalau/cohen/documents/BCOQ-book.pdf

[4] B. Heidergott, G. J. Olsder, and J. W. van der Woude, *Max Plus at Work : Modeling and Analysis of Synchronized Systems : a Course on Max-Plus Algebra and Its Applications*. 2006.

[5] M. A. Rudhito, *Aljabar Max-Plus dan Penerapannya*. Yogyakarta: Sanata Dharma University Press, 2016.

[6] Z. Liu, “A Novel Model for Train Operation Adjustment in High-Speed Railway based on Max-plus Algebra,” *International Journal of Circuits, Systems and Signal Processing*, vol. 14, pp. 881–887, 2020, doi: 10.46300/9106.2020.14.114.

[7] R. Farrando, N. Farhi, Z. Christoforou, and A. Urban, “A Max-Plus Model Deriving the Effect of a Skip-Stop Policy on the Train Frequency of a Mass Transit Line,” *SSRN Electronic Journal*, pp. 2509109_1-2509109_10, 2022, doi: 10.2139/ssrn.4089614.

[8] E. Carnia, R. Wilopo, H. Napitupulu, N. Anggriani, and A. K. Supriatna, “Modified Kleene Star Algorithm Using Max-Plus Algebra and Its Application in the Railroad Scheduling Graphical User Interface,” *Computation*, vol. 11, no. 1, pp. 1–13, 2023, doi: 10.3390/computation11010011.

[9] J. Uyttendaele, “Timetable compression using max-plus automata applied to large railway networks,” *TOP*, vol. 31, no. 2, pp. 414–439, 2023, doi: 10.1007/s11750-022-00641-5.

[10] K. Sagawa, “A Railway Timetable Scheduling Model based on a Max-Plus-Linear System,” *2020 59th Annual Conference of the Society of Instrument and Control Engineers of Japan, SICE 2020*, pp. 1575–1580, 2020.

[11] M. Hoekstra, “Control of Delay Propagation in Railway Networks Using Max-Plus Algebra,” Delft University of Technology, Delft, 2020. Accessed: Feb. 02, 2024. [Online]. Available: http://resolver.tudelft.nl/uuid:b5816386-0ed9-4760-a6de-f0db0c3a5226

[12] W. Winarni, “Penjadwalan Jalur Bus dalam Kota dengan Model Petrinet dan Aljabar Max-Plus (Studi Kasus Busway TransJakarta),” *CAUCHY: Jurnal Matematika Murni dan Aplikasi*, vol. 1, no. 4, pp. 192–206, 2011, doi: 10.18860/ca.v1i4.1796.

[13] A. Nait-Sidi-Moh, M. A. Manier, and A. El Moudni, “Spectral analysis for performance evaluation in a bus network,” *Eur J Oper Res*, vol. 193, no. 1, pp. 289–302, Feb. 2009, doi: 10.1016/j.ejor.2007.08.047.

[14] H. Newcomb, *Modeling Bus Bunching with Petri Nets and Max-Plus Algebra*. Portland State University Library, 2014. doi: 10.15760/honors.66.

[15] X. Qiu, L. Tao, P. Li, and J. Yang, “Synchronization Optimization Model of BRT System Based on Max-Plus Algebra Petri Net,” *ICTE 2013*, pp. 911–917, 2013, doi: 10.1061/9780784413159.134.

[16] C. Wang and Y. Tao, “Locally and globally optimal solutions of global optimisation for max-plus linear systems,” *IET Control Theory & Applications*, vol. 16, no. 2, pp. 219–228, 2021, doi: 10.1049/cth2.12219.

[17] D. Merdekawati, “Closed Shop Scheduling Optimisation using Max-Plus Automata,” *J Phys Conf Ser*, vol. 1341, no. 4, 2019, doi: 10.1088/1742-6596/1341/4/042015.

[18] A. Edthofer, “Train schedule analysis and optimization with the max-plus automaton,” *32nd European Modeling and Simulation Symposium, EMSS 2020*, pp. 161–164, 2020, doi: 10.46354/i3m.2020.emss.022.

[19] J. Xu, T. van den Boom, and B. De Schutter, “Optimistic optimization for model predictive control of max-plus linear systems,” *Automatica*, vol. 74, pp. 16–22, 2016, doi: 10.1016/j.automatica.2016.07.002.

[20] J. Xu, B. De Schutter, and T. J. J. van den Boom, “Model predictive control for max-plus-linear systems via optimistic optimization,” *IFAC Proceedings Volumes*, vol. 47, no. 2, pp. 111–116, 2014, doi: 10.3182/20140514-3-fr-4046.00063.

[21] P. M. Dower, “An Adaptive Max-Plus Eigenvector Method for Continuous Time Optimal Control Problems,” *Springer INdAM Series*, pp. 211–240, 2018, doi: 10.1007/978-3-030-01959-4_10.

[22] H. Zhang, Y. Tao, and Z. Zhang, “Strong solvability of interval max-plus systems and applications to optimal control,” *Systems & Control Letters*, vol. 96, pp. 88–94, 2016, doi: 10.1016/j.sysconle.2016.07.005.

[23] M. Akian, “An Adaptive Multi-Level Max-Plus Method for Deterministic Optimal Control Problems,” *IFAC-PapersOnLine*, vol. 56, no. 2, pp. 7448–7455, 2023, doi: 10.1016/j.ifacol.2023.10.628.

[24] C. Wang, Y. Tao, and H. Yan, “Optimal input design for uncertain max-plus linear systems,” *International Journal of Robust and Nonlinear Control*, vol. 28, no. 16, pp. 4816–4830, 2018, doi: 10.1002/rnc.4285.

[25] B. G. Fitzpatrick, K. Holmbeck-Cannell, M. Laffin, and Y. Wang, “Optimal perimeter patrol via max-plus probability,” *2014 American Control Conference*, pp. 5248–5253, 2014, doi: 10.1109/acc.2014.6859306.

[26] B. Heidergott, G. J. Olsder, and J. van der Woude, *Max Plus at Work*. Princeton: Princeton University Press, 2006. [Online]. Available: http://about.jstor.org/terms

[27] Subiono, “Aljabar Min-Max Plus dan Terapannya,” 2015.
[28] D. Systemes, “Scilab,” 2015, *Scilab Enterprises, France*: 5.5.2. Accessed: Jan. 20, 2025. [Online]. Available: https://www.scilab.org/news/scilab-552-release
[29] S. Subiono, D. Adzkiya, and K. Fahim, “Toolbox Aljabar Max-Plus dan PetriNet,” 2016, *Departemen Matematika, Institut Teknologi Sepuluh Nopember, Surabaya*: MAXPLUSV04032016. Accessed: Oct. 23, 2024. [Online]. Available: http://koncomatematika.blogspot.com/2016/05/cara-menginstall-toolbox-max-plus-scilab.html