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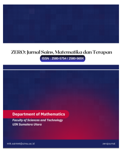

Exploration of Mathematical Concepts in Oceanography: A Literature Review

[1] **Dodi**
Sains and Marine Faculty, Universitas OSO, Pontianak, Indonesia

[2] **Zan Zibar**
Sains and Marine Faculty, Universitas OSO, Pontianak, Indonesia

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ABSTRACT

Mathematics is crucial in oceanography, enabling the modeling of complex ocean phenomena such as currents, waves, tides, and pollutant dispersion. This paper highlights the use of partial differential equations, numerical methods, and spatial statistics in simulating physical ocean processes. Key models include the Navier–Stokes equations for fluid flow, advection–diffusion models for pollutant transport, and wave models for ocean surface dynamics. These models are vital for practical applications like climate prediction, disaster mitigation, and marine ecosystem management. For example, modeling sea surface temperature aids in forecasting El Niño and La Niña events that impact rainfall and fisheries. In Indonesia, a maritime country highly exposed to ocean hazards, mathematical tools support marine research, policy planning, and sustainable development. This study presents an overview of mathematical models in oceanography, emphasizing their analytical strength and value in addressing environmental and resource challenges.

Corresponding Author:

Dodi,
Sains and Marine Faculty,
Universitas OSO, Pontianak, Indonesia
Email: dodi@oso.ac.id

1. INTRODUCTION

Oceanography, as a branch of science that studies the ocean and all its components, is an interdisciplinary field that integrates aspects of marine physics, chemistry, biology, and geology. However, one of the fundamental pillars in oceanographic data modeling and analysis is mathematics. Through mathematical approaches, complex oceanic phenomena such as ocean currents, waves, tides, and pollutant dispersion can be quantitatively described, predicted, and analyzed.

Mathematical concepts such as partial differential equations, numerical analysis, Fourier transformations, and spatial statistics are extensively utilized in modeling physical processes in the ocean. For instance, the Navier–Stokes equations are used to describe fluid movement in ocean currents, while advection-diffusion models form the basis for simulating the dispersion of pollutants or temperature in the ocean [1].

Moreover, the processing of ocean observation data from satellites and buoys heavily relies on statistical techniques and machine learning, enabling the analysis of long-term trends such as sea surface temperature changes due to global warming [2]. Thus, the exploration of mathematical concepts in oceanography is not only academically relevant but also strategic in supporting data-driven decision-making in marine resource management and disaster mitigation. In particular, mathematical models are critically needed to address pressing challenges such as coastal erosion, marine pollution, and the effects of global warming on ocean currents.

In Indonesia, as an archipelagic nation with the second-longest coastline in the world, a deep understanding of oceanic phenomena is crucial. Therefore, mathematical approaches serve as a highly potential tool to advance marine research and support maritime-based development policies. This paper aims to explore various mathematical concepts and their applications in oceanographic studies, both theoretically and practically.

2. RESEARCH METHOD

This study employs a qualitative descriptive method with a literature review (library research) approach. The aim of this approach is to identify, examine, and analyze various scholarly works that explore the interconnection between mathematical concepts and their applications in the field of oceanography [3][4].

The literature review was conducted through a systematic search of reliable sources, including scientific journals, reference books, conference proceedings, and research reports from official institutions. These sources were accessed through databases such as ScienceDirect, SpringerLink, and Google Scholar, as well as open-access references from institutions such as the Meteorology, Climatology, and Geophysical Agency (BMKG) and the National Oceanic and Atmospheric Administration (NOAA) [5][6]. Inclusion criteria for the reviewed studies were: publications from the last 15 years, peer-reviewed articles, and works that specifically discuss the application of mathematical models in oceanographic contexts. Exclusion criteria included non-scientific publications, articles lacking methodological clarity, and sources unrelated to marine phenomena or mathematical modeling.

The collected data are analyzed using a qualitative-descriptive analysis, by reviewing previous findings and synthesizing mathematical concepts within the context of marine phenomena [3][4].The focus of the study is directed toward: mathematical models used in oceanography, such as the Navier–Stokes equations, advection–diffusion equations, and wave models [1] the implementation of mathematics in modeling oceanographic phenomena such as ocean currents, tides, waves, and sea temperature dynamics [2][7] and advancements in computational methods and numerical simulations for mathematics-based oceanographic modeling [8][9].

3. RESULT AND ANALYSIS

The literature review shows that mathematics plays a crucial role in understanding the dynamics of the ocean. Various mathematical equations and numerical methods are used to model complex oceanic phenomena. Below is an analysis based on mathematical concepts in oceanography, accompanied by the corresponding mathematical equations and literature studies.

3.1 Ocean Current Models Using the Navier–Stokes Equations

One of the most important mathematical models in oceanography is the **Navier–Stokes equations**, which are used to describe the movement of fluids, including ocean currents. The equation is as follows:

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\vec{v}.\nabla\vec{v}\right)=-\nabla p+\mu\nabla^2\vec{v}+\vec{f}$$

Description:

ρ = fluid density; $\vec{v}$ = fluid velocity; t = time; p = pressure; μ = dynamic viscosity; $\vec{f}$ = wind force

As an example, to model ocean surface currents influenced by wind, this equation can be used to calculate the velocity of currents and the distribution of sea surface temperature along the coastline. This model is commonly used in ocean circulation simulations to predict changes in temperature and salinity within marine ecosystems.

Several studies have applied the Navier–Stokes equations. Fachrul [10] conducted a numerical study on the compressible two-dimensional steady flow through a cascade on a circular cylinder. The fluid flow is described by the Navier–Stokes equations, which are written in terms of flow function and vorticity, and solved using the Chebyshev collocation method in one direction and the high-order finite-difference method in the other direction. Tiwow [11] stated that the Navier–Stokes equations can be solved by applying boundary conditions related to fully developed laminar flow, thus obtaining:

(1) The average velocity of the fluid throughout a non-horizontal pipe:

$$V=\frac{D^2}{32\mu\ell}(\Delta p-\gamma\ell\,Sin\,\theta)$$

(2) The volumetric flow rate of fluid throughout a non-horizontal pipe:

$$Q=\frac{D^2}{128\mu\ell}(\Delta p-\gamma\ell\,Sin\,\theta)$$

(3) The mass flow rate of fluid throughout a non-horizontal pipe:

$$m=\rho\,\frac{D^2}{128\mu\ell}(\Delta p-\gamma\ell\,Sin\,\theta)$$

The Navier–Stokes equations have been widely used in Indonesia to understand and forecast key oceanographic phenomena. One prominent example is their application in modeling the seasonal current variations in the Indonesian Throughflow (ITF), a critical ocean current that transports warm water from the Pacific to the Indian Ocean through the Indonesian archipelago. This current significantly influences regional climate patterns and marine biodiversity.

A study by Susanto [12] applied Navier–Stokes-based Ocean circulation models to examine the variability of the ITF during monsoonal transitions. The results helped predict fluctuations in sea surface temperatures that are linked to coral bleaching events in eastern Indonesia, especially around the Banda Sea and the Lesser Sunda Islands.

Another practical application is seen in the work of BPPT (Badan Pengkajian dan Penerapan Teknologi) and BMKG, which used Navier–Stokes-based simulations in coastal hydrodynamic modeling to anticipate coastal flooding and abrasion in Northern Java, particularly in areas such as Semarang and Pekalongan, where land subsidence and tidal inundation are worsening. By coupling the Navier–Stokes equations with atmospheric forcing data, these models provided early warning indicators for extreme tide events (rob).

Moreover, researchers [13][14] have used variations of the Navier–Stokes equations in real-time wave and current modeling systems to support safer navigation and port management in Tanjung Perak Port.

These implementations demonstrate how the Navier–Stokes framework has facilitated the prediction of key environmental events in Indonesian waters, including: seasonal monsoon current shifts, temperature anomalies related to El Niño and La Niña, coastal erosion and flooding, and Wave-current interactions affecting marine transportation

Despite their usefulness, the accuracy of these models depends heavily on the resolution of input data, boundary conditions, and the numerical methods applied. Limitations may arise in nearshore environments where complex bathymetry and human activities require finer-scale modeling. Nonetheless, they remain an indispensable tool in oceanographic research and marine policy planning in Indonesia.

3.2. Ocean Wave Model with Nonlinear Wave Equations

Ocean waves, which are an important phenomenon in oceanography, are often modeled using nonlinear wave equations. One commonly used equation is the Korteweg-de Vries (KdV) equation, which describes waves in shallow water:

$$\frac{\partial \eta}{\partial t} + c_0 \frac{\partial \eta}{\partial x} + \alpha\eta \frac{\partial \eta}{\partial x} = \beta \frac{\partial^2 \eta}{\partial x^2}$$

Description:

$\eta\ (x, t)$ = wave height; c_0 = wave speed; $\alpha\ and\ \beta$ = constants depending on the wave characteristics

As an example, to observe the impact of tsunami waves after an earthquake, this equation is used in modeling soliton waves that can travel extremely long distances. This wave model is also used to predict the height and speed of waves in coastal areas. To date, the validity of a nonlinear wave equation is determined by the condition that the wave equation must exhibit linear characteristics, namely, in flat-bottomed waters and with very small amplitudes, the wavelength should match the wavelength predicted by linear wave theory [15]. This can be achieved by using the formulation process $\frac{\partial \eta}{\partial t}$ and $\frac{\partial \eta}{\partial x}$. For this purpose, the mathematical equation for the water surface from linear wave theory is used, which is $\eta = A \cos kx \cos \sigma t$ [15].

Indonesia, situated along the Pacific Ring of Fire, is highly vulnerable to tsunamis due to frequent seismic activity. The 2004 Indian Ocean tsunami and the 2018 Palu tsunami are two tragic events where understanding wave propagation was crucial for disaster response and mitigation.

Researchers from ITB and Kyoto University used nonlinear shallow water equations (including KdV-type soliton approximations) to simulate the run-up heights and arrival times of tsunami waves in Aceh. Their models revealed that the nonlinear interactions between wave crests led to unexpected amplification, particularly in Banda Aceh [16]. Nonlinear wave modeling was applied by researchers at ITS and BPPT to simulate soliton-like wave pulses in Palu Bay. These models helped explain the rapid arrival and intensification of tsunami waves following the earthquake [17]. In 2021, BPPT conducted a tsunami impact simulation using nonlinear wave equations for the south coast of Java, including Yogyakarta, Cilacap, and Pacitan, to support zoning regulations and evacuation planning [18].

To validate the use of nonlinear models like the KdV equation, they must align with linear wave theory under specific conditions: flat-bottomed waters, low amplitude waves, and long wavelengths. In such cases, wave behavior should match the expected pattern from the linear wave surface formula:

$$\eta = A \cos kx \cos \sigma t$$

Matching the terms ∂η/∂t and ∂η∂x is key to ensuring consistency between nonlinear and linear wave theory under ideal conditions [19].

Although nonlinear wave models require calibration and are sensitive to local bathymetry, they remain essential tools for Indonesia's oceanographic and disaster research community. The implementation of nonlinear wave models like the KdV equation in Indonesia has been instrumental in: reconstructing past tsunami events, predicting wave behavior in narrow or shallow coastal zones, supporting early warning systems, and informing disaster mitigation policy.

3.3. Ocean Tidal Model Using Harmonic Analysis

Harmonic analysis is a mathematical method used to decompose tidal patterns into a series of sinusoidal components (harmonic waves), each with specific frequency, amplitude, and phase. These components are associated with the gravitational forces of the Moon and the Sun, as well as the Earth's rotation. Ocean tidal models typically use a harmonic analysis approach, which states that tidal variations can be explained as the sum of several sinusoidal functions. The equation for the change in tidal height is:

$$\eta(t) = \sum_{n=1}^{N} a_n \; Cos\; (\omega_n t + \emptyset_n)$$

Description:

$\eta\ (t)$ = tidal height at time t; a_n = amplitude of the nth harmonic component

ω_n = angular frequency of the nth harmonic component; $\emptyset_n$ = initial phase of the nth harmonic component

Tidal movements refer to the periodic vertical movement of the sea caused by the interaction of celestial bodies (primarily the Moon and the Sun) with the Earth, as well as other forces such as gravitational and centripetal forces. According to research results [20], the highest average sea level elevation in 2016-2017 was recorded at Seblat (0.520 m) and Krui (1.293 m) in 2017, while the lowest elevations were at Seblat (0.8601 m) and Krui (0.7226 m) in 2016. The amplitude difference between the average tidal constants from altimetry data and tide station data at Seblat was 0.122 m, and at Krui, it was 0.115 m. Correlation testing between the sea level elevation from altimetry results and tide station data showed a strong relationship. The correlation coefficient at Seblat was 0.748, while at Krui, it was 0.618. These results were obtained from harmonic tidal analysis using tide station data and satellite Jason-2 Altimetry data.

Indonesia, with its extensive archipelagic coastline and exposure to tidal currents, has actively utilized harmonic analysis for various oceanographic and coastal applications, ranging from navigation to climate change monitoring. Research in Seblat and Krui during 2016–2017 combined Jason-2 satellite altimetry with tide gauge observations to analyze tidal patterns through harmonic decomposition. The study revealed: highest mean sea level in 2017 at Krui: 1.293 m; lowest in 2016 at Krui: 0.7226 m; amplitude difference between satellite and station data: 0.122 m (Seblat), 0.115 m (Krui); and correlation coefficients between datasets: 0.748 (Seblat), 0.618 (Krui) These values indicate a strong agreement between modeled and observed tidal data, validating the effectiveness of harmonic analysis in coastal monitoring [21].

The accuracy of tidal forecasts has significantly improved with the inclusion of historical harmonic constituents and real-time tide gauge data, reducing port delays and operational risks. The Indonesian Hydro-Oceanographic Center (Pushidrosal) employs harmonic analysis in tidal prediction software for major ports like Tanjung Priok, Belawan, and Surabaya. These models support: safe docking schedules, dredging planning, and marine traffic routing

These models help predict when and where extreme tides will combine with rainfall or land subsidence to cause severe flooding. In Northern Java, harmonic tidal models have been integrated with sea-level rise projections to map flood-prone areas during spring tide (pasang purnama). For example, Semarang and Pekalongan have seen recurring rob (tidal flooding). Harmonic forecasts are used in: urban planning, early warning systems, and zonation regulations for coastal development

Harmonic analysis not only aids in daily operational predictions, but also plays a strategic role in: long-term climate monitoring, detecting tidal anomalies and seasonal shifts, coastal hazard assessments, identifying resonance zones or tidal amplification regions, data assimilation in ocean models, providing boundary conditions for hydrodynamic simulations. Its strength lies in simplicity and robustness, especially when applied to regions with consistent tidal records. However, its effectiveness diminishes in highly dynamic estuaries or areas with significant nonlinear interactions.

3.4. Pollutant Dispersion Model with Advection-Diffusion Equation

The process of pollutant dispersion in the ocean can be explained using the advection-diffusion equation, which is a combination of the movement of water mass (advection) and the spreading of substances through diffusion. The equation used is:

$$\frac{\partial C}{\partial t} + \vec{v}\,.\nabla C = D\,\nabla^2 C$$

Description:

C = pollutant concentration; $\vec{v}$ = flow velocity; D = diffusion coefficient

As an example, in oil pollution in marine waters, this equation is used to predict the movement and dispersion of oil in the ocean, which is important for oil spill response and marine environmental protection policies.

In the research, it is mentioned that the solution to the advection-diffusion equation describing the dispersion process of concentration can be solved using the method of separation of variables. In this case, the solution for concentration values as a function of the x and y directions is solved using the similarity method, which can convert a partial differential equation into an ordinary differential equation. Meanwhile, dispersion in the z direction is handled using the Laplace transform [22]. The pollutant dispersion model in the atmosphere can be constructed based on the solution of the advection-diffusion equation with the boundary conditions of atmospheric stability in a region, and the calculations can be performed using MATLAB [22].

Several studies have used solutions to the Advection-Diffusion equation with specific methods related to mathematics. [20] According to the research results, the Adomian Laplace decomposition method can be used to obtain solutions for the advection-diffusion equation. The advection-diffusion equation is as follows:

$$\frac{\partial u}{\partial t} + C\frac{\partial u}{\partial} = D\frac{\partial^2 u}{\partial x^2}$$

These techniques are often implemented using computational tools like MATLAB, facilitating the simulation of pollutant spread in both air and water systems. Solutions to the advection-diffusion equation can be obtained through mathematical techniques, such as: separation of variables and similarity solutions for spatial (x, y) components, Laplace transform for the vertical (z) direction, adomian decomposition method and finite difference methods for numerical approximation [21],[23].

One of the most significant marine pollution events in recent years occurred in Balikpapan Bay, East Kalimantan, due to a ruptured underwater pipeline. The advection-diffusion equation was employed by researchers from BPPT and ITB to model the oil slick's spread, factoring in tidal currents and wind-induced surface drift. The simulation assisted in: identifying critical coastal areas for immediate cleanup, estimating arrival time of pollutants to aquaculture zones, supporting legal and environmental impact assessments [24]

Studies from LIPI and UNPAD applied the advection-diffusion equation to evaluate the spread of heavy metals and organic pollutants from the Citarum River into Karawang Bay. The simulation used tidal current data and dispersion coefficients specific to seasonal conditions. Findings supported: designing buffer zones, advising on seasonal fishing bans, and mapping vulnerable seagrass ecosystems [25].

In Benoa Bay, where eutrophication threatens coral health, researchers used advection-diffusion models to simulate nitrate and phosphate dispersion from land-based sources. The results helped local agencies in: planning wastewater treatment systems, adjusting land use regulations around tourism zones, and estimating the lag time between discharge and ecological impact [26]

3.5. Termohaline Circulation Models

The Thermohaline Circulation Model is an oceanographic model that describes the movement of seawater masses driven by differences in density resulting from variations in temperature (thermal) and salinity (haline). This model is one of the main drivers of the global ocean circulation system, commonly referred to as the Global Conveyor Belt or Thermohaline Circulation (THC). Thermohaline circulation represents the movement of seawater caused by temperature (thermo) and salinity (haline) differences, which influence water density. This process is essential for modeling the global circulation that drives the movement of seawater masses around the world, such as in the Global Conveyor Belt. The equations commonly used to model this circulation are convection-diffusion systems involving the distribution of temperature and salinity:

$$\frac{\partial T}{\partial t} + \vec{v}\,.\nabla T = \alpha_T\,\nabla^2 T$$
$$\frac{\partial S}{\partial t} + \vec{v}\,.\nabla T = \alpha_S\,\nabla^2 S$$

Description:
T = temperature; S = salinity; α_T and α_S = diffusion coefficients for temperature and salinity

This model is used to predict the movement of global ocean circulation, such as the El Niño phenomenon, which affects climate patterns worldwide. Thermohaline circulation is a primary mechanism for the transport of heat and nutrients in the ocean, playing a crucial role in the global climate system. Research shows that wind kinetic energy supplies approximately 778 GW of energy, with 80% converted into gravitational potential energy in the Antarctic Ocean. Energy loss due to convection reaches 202 GW, while the effect of cabbeling contributes 207 GW. In the Eastern Mediterranean, carbon (δ^{13}C) and oxygen (δ^{18}O) isotope analysis from sapropel sediments revealed deoxygenation during the Messinian period due to changes in salinity and temperature, which inhibited thermohaline circulation and affected primary productivity. In the Eastern Mediterranean Sea, salinity at the minimum zone (~1000 dbar) gradually increased by 0.002 per year, reaching a stable value of 38.76 in 2017. In addition, salinity and temperature anomalies between 600–1200 dbar showed a strengthening positive trend up to 2017, before stabilizing during 2018–2019, reflecting changes in water mass dynamics. Although the one-dimensional diffusion model approach in the Mediterranean Sea is considered practical, limited observation frequency and ecosystem complexity not fully represented remain significant challenges [27].

Indonesia's position between the Pacific and Indian Oceans makes it the central pathway for the Indonesian Throughflow (ITF), a key component of the global thermohaline circulation. Research conducted by Susanto and Gordon [28] using thermohaline models revealed that temperature and salinity gradients across the Makassar Strait drive the vertical and horizontal transport of water masses, directly influencing sea surface temperatures in the Indian Ocean. The model has helped: forecast heat transport anomalies during El Niño and La Niña years, understand deep water upwelling patterns in the Banda and Flores Seas, provide boundary conditions for coupled ocean-atmosphere models

The Agency for Meteorology, Climatology and Geophysics (BMKG) incorporates thermohaline circulation modeling in its seasonal forecasting systems. By analyzing sea temperature and salinity changes at depths of 100–1000 dbar in the Java Sea and Timor Sea, models are calibrated to predict: dry season anomalies (e.g., delayed rainfall onset), marine heatwaves impacting coral reef ecosystems, fish migration disruptions due to shifting thermocline depth [29]

These findings were critical in modeling ocean deoxygenation trends, which affect marine biodiversity and fisheries in Eastern Indonesia. A 2020 study by LAPAN and LIPI combined ARGO float data with thermohaline circulation models to detect long-term salinity increases in the Arafura and Banda Seas. These changes were linked to: intensified evaporation during prolonged dry seasons, reduced freshwater input from Papua and Maluku rivers, and disrupted vertical mixing, leading to hypoxic zones [28].

3.6. Tsunami and Ocean Surface Wave Models

Mathematical models used to predict tsunami waves following earthquakes or other underwater movements employ more complex equations, such as the Green–Naghdi equations for nonlinear surface waves:

$$\frac{\partial \eta}{\partial t} + \frac{\partial \eta^2}{\partial x} = g\ \frac{\partial^2 \eta}{\partial x^2}$$

Description:
η = sea surface elevation; g = gravitational acceleration; x = horizontal coordinate

The Green–Naghdi equations with general weight functions were derived in a simple way. A wave-absorbing beach was also considered in the general Green–Naghdi equations. The numerical solution for a level higher than 4 was not feasible in the past with the original Green–Naghdi equations. After the simplification of Green–Naghdi equations for shallow water waves, application of high level (higher than 4) equations was feasible. Numerical calculations have been performed by use of the simplified Level 5 and Level 7 Green–Naghdi equations, which will be presented in the next paper [30]. In each numerical test, the self-convergence of the Green–Naghdi equations should be performed first by use of Level 1, 3, 5, and 7 Green–Naghdi equations. Then, the converged solutions can be compared with known experimental values and/or other solutions. Numerical simulation of nonlinear water waves will be presented in the next paper [12].

Indonesia's geographical position along the Pacific Ring of Fire makes it one of the most tsunami-prone regions in the world. The Green–Naghdi equations, due to their accuracy in simulating nonlinear wave behavior in coastal topography, have been applied in several Indonesian research initiatives and disaster response efforts.

The GN model outperformed classical shallow water models by accurately representing the observed steep wave front and arrival time of the tsunami, assisting in post-disaster damage assessment. The 2018 Palu tsunami in Central Sulawesi provided a unique case of a localized tsunami with an unusual waveform caused by submarine landslides and fault rupture. Researchers from BPPT and Institut Teknologi Sepuluh Nopember (ITS) used Green–Naghdi-based numerical simulations to analyze: rapid wave formation in Palu Bay, wave resonance effects due to narrow bay geometry, amplification of wave height at the bay's end [29].

In 2021, BMKG and LAPAN collaborated on a multi-scenario tsunami hazard assessment for south coast regions of Yogyakarta, Cilacap, and Banyuwangi, utilizing Level 3 and Level 5 Green–Naghdi equations. These models helped in: simulating wave run-up heights under various earthquake magnitudes, identifying critical inundation zones, developing community-based evacuation maps [31]

Recent developments in Indonesia's InaTEWS (Indonesia Tsunami Early Warning System) have included experimental modules integrating GN-based solvers for fast wavefront simulation in narrow straits like Sunda Strait and Maluku. These efforts aim to reduce computational lag and improve warning accuracy for near-field tsunamis [32].

3.7. Coupled Ocean-Atmosphere Models

The interaction between the atmosphere and the ocean is often modeled using Coupled Ocean–Atmosphere Models, which integrate atmospheric fluid dynamics models with ocean circulation models. These models are essential for predicting weather and climate. The equations used in such models form a system that describes the interaction between sea surface temperature (SST) and other atmospheric variables, such as pressure and wind.

$$\frac{\partial T}{\partial t} = \alpha\,(\nabla^2 T - W)$$

$$\frac{\partial U}{\partial t} = \beta\,(\nabla^2 U - F)$$

Description:

T = sea surface temperature; U = wind velocity; $W\ and\ F$ = interaction factors

This model is used in the prediction of El Niño and La Niña phenomena, which influence global climate patterns. Rainfall simulation is one of the essential characteristics in the regional model over the MC that begs improvement. Local convective activities are the fundamental rainfall-producing mechanism in this region, which are still not well resolved in models. This is particularly true for climate modeling, as model grid reso lution is often coarse (tens of km) to accommodate decadal-scale simulations for the sake of affordable com putational cost. Unless the regional model resolution can be sufficiently high (e.g., 3 km or less), the numerical schemes for parametrizing convection are essentiall [17]. The results also indicate that the primary driver for the observed rainfall errors in the model is within the atmospheric component, and these errors are attributed to user choices of regional climate model configuration and, more fundamentally, the challenge in accurately resolving the convective rainfall process [17].

Studies by the BMKG Climate Modeling Center and Universitas Indonesia have used WRF-CROCO coupled models to simulate rainfall over Sumatra, Kalimantan, and Papua, especially during monsoonal transitions. The models showed that: high-resolution grids (≤3 km) significantly improve the prediction of convective rainfall, sea surface temperature anomalies in the Java Sea are closely linked to rainfall variability in Jakarta and Surabaya, the ocean-atmosphere feedback loops affect daily rainfall timing and intensity [33]

The Agency for Meteorology, Climatology, and Geophysics (BMKG) uses CESM (Community Earth System Model) and NEMO-IFS coupled models to analyze ENSO events and their impacts on eastern Indonesian fisheries, particularly in the Arafura Sea and Banda Sea. These models helped: predict drought periods affecting agriculture in NTT and Maluku, forecast fish migration patterns in response to SST shifts, support seasonal planning for aquaculture [34]

As part of LAPAN's climate risk initiatives, coupled models simulated wind–SST interactions to assess flood risks in urban areas such as Semarang and Pekalongan. The results showed that: warm SST anomalies in the Java Sea contributed to intense localized rainfall, and coupled models enabled the identification of high-risk periods for coastal inundation (banjir rob) [35]

4. CONCLUSION

Mathematics is a fundamental pillar in the field of oceanography, enabling scientists and policymakers to understand, simulate, and predict complex marine phenomena. Through mathematical modelling, including the use of partial differential equations, numerical methods, and statistical analysis, oceanographers can analyze the behavior of ocean currents, waves, tides, pollutant dispersion, thermohaline circulation, and tsunami propagation. Each model, from the Navier–Stokes equations to the Green–Naghdi and coupled ocean-atmosphere systems, serves a unique role in translating theoretical principles into practical applications.

In the Indonesian context, where the nation faces frequent marine-related hazards and heavily relies on ocean resources, these mathematical models have proved essential. They have supported early warning systems, guided marine spatial planning, informed disaster mitigation strategies, and enhanced the sustainable management of fisheries and coastal development.

Despite challenges such as data resolution limitations and the complexity of ocean dynamics, the integration of advanced mathematical techniques with computational tools continues to improve the accuracy and usefulness of these models. This highlights the strategic importance of mathematical oceanography in addressing climate change, protecting marine ecosystems, and ensuring resilient maritime futures—especially for archipelagic nations like Indonesia.

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