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# Praxeological analysis of grade V elementary school mathematics textbooks on fractions

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**Abstract**
Textbooks are essential instruments in implementing the Merdeka Curriculum, serving as a bridge between curriculum policy and classroom practices. However, didactic challenges often arise when the presentation of material becomes confined to mechanical procedures rather than fostering deep conceptual understanding, particularly in fractions, which are highly abstract for elementary students. This study aims to analyze Grade V mathematics textbooks under the Merdeka Curriculum, focusing on fractions, through a praxeological lens. The analysis applies the four components of praxeology, task type (T), technique (τ), technology (θ), and theory (Θ), to evaluate the extent to which textbooks support effective learning. Using qualitative document analysis, data were drawn from the 2022 Grade V Mathematics Textbook published by the Ministry of Education, Culture, Research, and Technology, with attention to the relationship between the praxis block (tasks and techniques) and the logos block (technology and theory). Findings reveal that although the textbook integrates visual approaches and real-world contexts (such as local culture), procedural aspects remain dominant. A praxeological paradox was identified by visual illustrations often serve merely as technical justification (technology) without sufficient conceptual exploration (theory). Moreover, the limited variety of representations and rigid techniques in mixed operations risk reinforcing natural number bias and creating learning obstacles. The structure of fraction material in the textbook tends to prioritize procedural-mechanical aspects over independent conceptual exploration. More flexible and holistic instructional designs in textbooks are needed to reduce individual-instructional learning and to foster students' ability to generalize concepts to a broader mathematical domain.

**Keywords:** Conceptual understanding, Didactical transposition, Learning obstacles, Merdeka curriculum, Praxeology

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## Introduction

Textbooks play a vital role in learning as sources of information, references, and instructional aids (Muhamad & Hidayat, 2024). Rahayu et al. (2022) emphasize that a good textbook must present material clearly and systematically so students can understand and apply concepts. Textbooks also function as evaluation tools, since exercises allow students to measure their comprehension. Another key criterion of a good textbook is the inclusion of illustrations that support conceptual understanding, especially in fractions, which are often difficult for students (Lubis, 2024; Wibow, 2019). The main objective of a textbook is to support students' learning achievement based on the current curriculum. Thus, textbooks must meet standards of content validity, organization, readability, and alignment with learning objectives (Azzahra et al., 2022; Ramda, 2017; Rizqi et al., 2021).

Fractions are a fundamental topic in elementary mathematics, forming the basis for understanding ratios, proportions, rational numbers, decimals, and more complex operations (Putri et al., 2023; Rahayu et al., 2022). In the Merdeka Curriculum, number concepts, including fractions, are introduced progressively from early grades. At Phase C (Grade V), students are expected to understand, represent, compare, and operate with fractions in varied contexts (Fitrianawati, 2022). NCTM (2000) also identifies fractions as central to elementary mathematics, serving as a foundation for advanced concepts. Jahangiri (2022) similarly shows that strong fraction understanding supports readiness for higher-level mathematics. Therefore, fraction instruction must go beyond procedural mastery to include contextual, representational, and conceptual justification. Primasari et al. (2021) note that contextual approaches can enhance student interest and motivation in learning fractions.

Despite this, fractions remain one of the most challenging topics for elementary students. Difficulties extend beyond computation to understanding fraction meaning, numerator-denominator relationships, visual representation, and contextual applications (Fatimatuzzaroh, 2022; Rahma et al., 2024; Oktafiyani & Karlimah, 2021). Students often need concrete, visual, and contextual representations to grasp fractions meaningfully. Yet, when visuals are used merely to support procedures, they fail to build conceptual understanding. Thus, fraction presentation in textbooks requires critical examination to determine whether procedural tasks are balanced with conceptual explanations and mathematical justification.

A relevant theoretical framework is praxeology within *Anthropological Theory of Didactics* (ATD), developed by Chevallard (Sahara et al., 2025). In this perspective, mathematical activity is not only task-solving but also an organization of knowledge encompassing techniques and their mathematical justification (Chevallard, 2002; Chevallard, 2019; Chevallard et al., 2014; Chevallard et al., 2015). Praxeology enables analysis of how textbooks present material, construct techniques, and justify concepts (Putra et al., 2020; Utami et al., 2022).

In the context of this study, praxeology was employed to analyze the organization of fraction material in the Grade V mathematics textbook under the Merdeka Curriculum. This analysis is important because textbooks function not only as sources of content but also as instruments that shape how students understand concepts, through the types of tasks, solution procedures, representations, and mathematical explanations they provide (Bosch et al., 2006; Chevallard, 2019) . Several previous studies have applied the praxeological framework to analyze mathematics textbooks at the junior and senior high school levels, particularly in identifying the structure of content and potential learning challenges arising from textbook presentation (Chandra et al., 2025; Hendriyanto et al., 2023; Siagian et al., 2023; Yunianta et al., 2023). However, praxeological studies of fraction material in Grade V mathematics textbooks under the Merdeka Curriculum remain limited and require deeper investigation. Therefore, this research focuses on analyzing the praxeological structure of fraction material to examine the balance between tasks, solution techniques, conceptual explanations, and theoretical foundations presented in the textbook.

A number of previous studies have examined mathematics textbooks from various perspectives, including content feasibility, curriculum alignment, effectiveness of instructional strategies, and potential student learning obstacles (Maulana & Hidayat, 2024; Oktafiyani & Karlimah, 2021). In addition, several studies have documented the use of textbooks across different levels and topics, such as comparison, derivatives, and sets (Azzahra et al., 2022; Chandra et al., 2025; Hendriyanto et al., 2023). However, research specifically analyzing the praxeological structure of fraction material in Grade V mathematics textbooks under the Merdeka Curriculum remains limited. Most prior studies have tended to emphasize content alignment, media effectiveness, or instructional strategies, while analyses of the balance between tasks, techniques, conceptual justification, and theoretical foundations in textbooks have been relatively scarce.

This gap is important to examine because textbooks shape the way students understand fractions. When textbooks emphasize procedural techniques without sufficient conceptual explanation, students risk perceiving fractions merely as operational rules rather than as numerical concepts with multiple representational meanings. Such conditions may reinforce mechanistic thinking, limit representational flexibility, and create learning obstacles, including the natural number bias in fractions (Van Dooren et al., 2009; Van Hoof et al., 2015). Therefore, praxeological analysis is necessary to reveal the extent to which textbooks provide a balanced organization of knowledge between task-solving activities and the underlying mathematical explanations.

Based on this background, the study addresses the following research questions: How is the praxeological structure of fraction material organized in the Grade V Mathematics Textbook under the Merdeka Curriculum, particularly the concepts of fractions, addition, and subtraction? What learning obstacles are likely to arise as a result of the identified praxeological structure? These questions are significant because the Merdeka Curriculum requires students to think critically and generalize concepts and skills, which are integral to the *Profil Pelajar Pancasila*. If the textbooks used fail to adequately develop the logos block, a gap may emerge between the curriculum's expectations and actual classroom practice. Systematically identifying this gap is an essential first step toward addressing and improving it.

## Methods

This study employs a qualitative document analysis, as proposed by Bowen (2009), systematically applying Anthropological Theory of the Didactic (ATD) praxeology theory as the analytical framework. This approach was chosen because the primary data consist of written documents, specifically, mathematics textbooks, analyzed to understand how fraction material is structured, presented, and mathematically justified. Document analysis is appropriate as it enables a systematic, theory-based examination of the epistemic content of curriculum materials without requiring field data collection (Suryadi, 2013). This approach aligns what is described as an epistemological analysis of mathematical organization, a methodology recognized within the ATD tradition. The framework is used to examine the organization of mathematical knowledge in textbooks through four main components: type of task (T), technique ($\tau$), technology ($\theta$), and theory ($\Theta$).

The data source for this study is the Grade V Elementary School/ Islamic Elementary School Mathematics Textbook for the Merdeka Curriculum, Fitrianawati (2022b) published by the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia (ISBN 978-602-427-916-5). The analysis is limited to Chapter 3, Fractions, which includes fraction concepts such as mixed numbers and fraction comparisons, together with operations involving the addition and subtraction of fractions with both common and different denominators.

The unit of analysis encompasses all textual and visual elements within these sections: concept introductions, student exploratory activities (Eksplorasi), illustrative problems (Masalah), worked examples demonstrating multiple strategies (*Cara Lukas* [Lukas' method], *Cara Komang* [Komang's method], *Cara Nisa* [Nisa's method], *Cara Yohana* [Yohana's method], *Cara Asep* [Asep's method]), generalized summaries (*Ayo Menyimpulkan* [Let's conclude]), and practice exercises (*Ayo Berlatih* [Let's practice]). This inclusive scope ensures that no component of the praxeological organization is overlooked. Data analysis was conducted using the praxeological framework within the Anthropological Theory of the Didactic (ATD) developed by Chevallard.

This framework consists of two blocks: the praxis block and the logos block. The praxis block includes the type of task (T) and the technique ($\tau$), while the logos block encompasses technology ($\theta$) and theory ($\Theta$) (Lortie-Forgues et al., 2015). In this study, T refers to the mathematical activities or problems assigned to students; $\tau$ refers to the methods, procedures, strategies, or representations used to solve the tasks; $\theta$ refers to the mathematical explanations or justifications underlying the use of techniques; and $\Theta$ refers to the broader mathematical concepts or principles that serve as the foundation for those explanations. The analytical framework applied is Chellavard's four component praxeological model (Chevallard, 1999). Each component is operationalized and presented in Table 1.

**Table 1**. Operationalization of Praxeological Components

| Praxeological Component | Operational Definition | Identification Criteria in the Textbook |
|---|---|---|
| Type of Task (T) | Categories of mathematical problems or activities presented to students. | Any instruction, question, or activity requiring a specific mathematical action (recognizing, comparing, ordering, calculating, transforming, representing) |
| Technique ($\tau$) | Methods, procedures, algorithms, or representational strategies used to solve tasks | Any step-by-step procedure, visual model, algorithm, or strategy demonstrated or directed by the textbook for task completion |
| Technology ($\theta$) | Explicit mathematical discourse that justifies why a technique works | Any explanatory statement, rule, or justification provided regarding why a technique is valid or applicable |
| Theory ($\Theta$) | Broader mathematical concepts, definitions, or principles underlying the technology | Any concept, property, or definition referenced or implied as the foundation for the technological discourse |

The analysis was carried out in five systematic stages. First, all sections related to fractions were extracted and delimited. Second, each textual and visual element was coded into initial units. Third, the units were classified into the categories T, $\tau$, $\theta$, or $\Theta$ based on the criteria in Table 1. Fourth, the relationship between the praxis block and the logos block were examined for each identified task type, with coding of the presence, explicitness, and adequacy of each component. Fifth, cross-task patterns were synthesized to identify overall praxeological tendencies and their alignment with the international literature on fractions. Data reliability was maintained through several mechanisms. First, all textbook sections were read repeatedly, accompanied by systematic note-taking throughout the analysis process. Second, the categorization results were triangulated using established praxeological criteria proposed by Bosch & Gascón (2014) and benchmarks from international studies on fractions (Lamon, 2012; Siegler et al., 2010; Van de Walle et al., 2013). Third, the interpretations were verified through member checking by comparing them with findings from published praxeological studies of comparable textbooks (Hendriyanto et al., 2023; Yunianta et al., 2023).

## Results

The praxeological analysis identified fourteen distinct task types (T1–T14) across the entire chapter on fractions. These tasks were organized into four thematic clusters: (A) fraction concepts, comparison, and ordering; (B) addition of fractions with like denominators; (C) addition of fractions with unlike denominators; and (D) subtraction of fractions. The results are presented in Tables 2–5.

### Cluster A: Fraction Concepts, Comparison, and Ordering ($T_1$–$T_3$)

The first thematic cluster concerns fraction concepts, comparison, and ordering. Three task types (T1–T3) were identified within this cluster, reflecting the textbook's introduction to fundamental fraction ideas and representations. The corresponding praxeological elements are presented in Table 2.

**Table 2**. Praxeological Analysis: Fraction Concepts, Comparison, and Ordering

| Code | Type of Task (T) | Technique (τ) | Technology (θ) | Theory (Θ) |
|---|---|---|---|---|
| $T_1$ | Converting mixed numbers into improper fractions (e.g., 2¾, 3½ in the context of Yohana's jumps) | $\tau_1$: Algorithmic conversion (multiply the whole number by the denominator, then add the numerator)<br>$\tau_2$: Visual decomposition of the mixed number into full unit strips plus partial strips | A mixed number represents the combination of a whole number and a proper fraction; it can be expressed as an improper fraction | Fractions as numbers; improper fractions; mixed numbers as the sum of whole and fractional parts |
| $T_2$ | Comparing fractions (e.g., comparing portions of *wingko*; comparing visual representations) | $\tau_3$: Compare numerators (same denominator)<br>$\tau_4$: Compare whole number parts<br>$\tau_5$: Placement on the number line.<br>$\tau_6$: Visual part whole model | Fractions values can be compared by their numerators (when denominators are equal), their position on the number line, or their relative coverage of the whole | Fractions as parts of a whole; fractions as numbers on the number line; ordering rational numbers |
| $T_3$ | Ordering fractions using the number line (e.g., ordering 1¼, 7/2, 7/8) | $\tau_5$: Placement on the number line<br>$\tau_7$: Comparing fraction values | The order of fractions is determined by their position on the number line; larger fractions are located further to the right | Fractions as numbers; ordering rational numbers; density of the number line |

Table 2 shows that the initial cluster provides a reasonably solid foundation in fraction concepts. The use of the number line in $T_2$ and $T_3$ is noteworthy, as this representation aligns with evidence from Lortie-Forgues et al. (2015) that number line proficiency predicts later mathematical success. However, the logos block remains underdeveloped: the technology for $T_1$ does not explain why the conversion algorithm works (i.e., that whole numbers represent a certain number of unit fractions), and the theory for $T_3$ mentions density of the number line only implicitly without making it explicit. Critically, the five sub-constructs of fractions identified by Lamon, measure, quotient, ratio, operator, and part-whole, are entirely absent.

## Cluster B: Addition of Fractions with Like Denominators ($T_4$–$T_7$)

The second cluster addresses the addition of fractions with like denominators. Four task types (T4–T7) were identified, encompassing contextual problem solving, the interpretation of fractions as parts of a whole, the formulation of addition rules, and operations involving mixed numbers. Table 3 presents the corresponding praxeological analysis.

**Table 3**. Praxeological Analysis: Addition of Fractions with Like Denominators

| Code | Type of Task (T) | Technique (τ) | Technology (θ) | Theory (Θ) |
|---|---|---|---|---|
| $T_4$ | Calculating the total of fractions from a whole divided equally (e.g., 1/8 + 2/8 for portions of *martabak*) | $\tau_6$: Visual part whole model (circle/sector) $\tau_8$: Add numerators, keep denominator | Fractions with the same denominator can be added because their unit fractions are identical | Addition of fractions with like denominators; denominator as unit of measure |
| $T_5$ | Calculating the total of fractions from discrete objects expressed as fractions (e.g., 2/10 + 3/10 for erasers) | $\tau_6$: Visual part whole model (discrete objects) $\tau_9$: Express parts as fractions. $\tau_8$: Add numerators, keep denominator | The denominator represents the total number of equal-sized units; the numerator represents the count of those units | Meaning of numerator and denominator; fractions as measures of parts |
| $T_6$ | Formulating the rule for adding fractions with like denominators | $\tau_{10}$: Inductive generalization from worked examples | When fractions share the same denominator, addition is performed on the numerators while the denominator remains unchanged | Generalization of fraction addition; properties of fractions with like denominators |
| $T_7$ | Adding two mixed numbers with like denominators (e.g., 2⅕ + 3⅕) | $\tau_{11}$: Convert to improper fractions, add numerators, simplify. $\tau_{12}$: Simplify result. $\tau_{13}$: Separate whole-number part. $\tau_{14}$: Add whole numbers. $\tau_{15}$: Combine whole-number and fractional results | Mixed numbers can be added either by converting them into improper fractions or by separately operating on the whole-number and fractional parts | Mixed numbers; addition of rational numbers; equivalence of solution strategies |

Cluster B is pedagogically the richest section of the chapter: it provides concrete contexts, employs visual models, and even offers multiple solution strategies for $T_7$ (e.g., *Cara Lukas* [Lukas' method] and *Cara Komang* [Komang's method]). The technology for $T_4$–$T_5$ approaches adequacy by noting that fractions with like denominator functions as a unit of measure, analogous to adding quantities in the same physical unit (e.g., 1 meter + 2 meters = 3 meters, not 3/16 meters) is not made explicit. (Van de Walle et al., 2013) identify this denominator-as-unit understanding as foundational for meaningful fraction arithmetic. Its absence means that the generalization in $T_6$ (add numerators, keep denominator) risks being memorized as a rule rather than understood as a consequence of unit structure.

## Cluster C: Addition of Fractions with Unlike Denominators (T8–T9)

The third cluster concerns the addition of fractions with unlike denominators. Two task types (T8–T9) were identified, involving the addition of simple fractions and mixed numbers through the use of equivalent fractions and common denominators. Table 4 presents the corresponding praxeological structure, including the techniques, technologies, and theories associated with these tasks.

**Table 4**. Praxeological Analysis: Addition of Fractions with Unlike Denominators

| Code | Type of Task (T) | Technique (τ) | Technology (θ) | Theory (Θ) |
|---|---|---|---|---|
| $T_8$ | Adding two simple fractions with unlike denominators (e.g., 1/3 + 4/6 for parts of paper) | $\tau_6$: Visual part-whole model (rectangular strip).<br>$\tau_{16}$: Convert to equivalent fractions.<br>$\tau_{17}$: Find the least common multiple (LCM) of denominators.<br>$\tau_8$: Add numerators, keep common denominator | Fractions with unlike denominators must be converted into equivalent fractions with a common denominator before addition can be performed | Equivalent fractions; LCM; addition of fractions with unlike denominators |
| $T_9$ | Adding mixed numbers with unlike denominators (e.g., 1½ + 2¼ + 1/200 for distance) | $\tau_{11}$: Convert to improper fractions.<br>$\tau_{17}$: Find Least Common Multiple (LCM) number.<br>$\tau_8$: Add numerators.<br>$\tau_{12}$: Simplify.<br>$\tau_{13}$: Separate whole and fractional parts.<br>$\tau_{14}$: Add whole numbers.<br>$\tau_{17}$: Find LCM for fractional parts.<br>$\tau_{15}$: Combine results | Mixed numbers must first be expressed with equivalent fractional units (common denominator via LCM) before addition; whole and fractional components can be handled separately | Mixed numbers; equivalent fractions; LCM; arithmetic of rational numbers |

Cluster C introduces addition with unlike denominators through the LCM technique ($\tau_{17}$). The textbook provides two approaches for $T_9$ (Cara Yohana, which converts to improper fractions, and Cara Lukas, which separates whole and fractional parts, commendable from a pedagogical standpoint). However, the technology for $T_8$ explains only procedurally that denominators must be made the same, without clarifying why addition across different units is mathematically invalid. (Reys et al., 2014) note that this conceptual gap, understanding why a common denominator is required, rather than simply that it is mandated, a major source of learning difficulties in fractions.

## Cluster D: Fraction Subtraction (T10–T14)

The final cluster addresses fraction subtraction. It comprises five task types (T10–T14), including the subtraction of fractions and mixed numbers with both like and unlike denominators. These tasks involve visual models, equivalent fractions, and common-denominator procedures as key techniques for solving subtraction problems. Table 5 presents the praxeological structure identified in this cluster.

**Table 5**. Praxeological Analysis: Fraction Subtraction

| Code | Type of Task (T) | Technique (τ) | Technology (θ) | Theory (Θ) |
|---|---|---|---|---|
| $T_{10}$ | Subtracting fractions with like denominators (e.g., 2/3 – 1/3 for unfinished parts of a picture) | $\tau_6$: Visual part-whole model.<br>$\tau_{18}$: Subtract numerators, keep denominator | Subtraction of fractions with like denominators is performed on the numerators; the denominator, as the common unit, remains unchanged | Fractions as parts of a whole; subtraction of fractions with like denominators |
| $T_{11}$ | Determining the fractional remainder of a whole (e.g., Asep's leftover apples: 6/8 – 2/8) | $\tau_6$: Visual model of discrete objects.<br>$\tau_9$: Express parts as fractions.<br>$\tau_{18}$: Subtract numerators, keep denominator | The denominator represents the constant total of equal units; the numerator indicates the quantity subtracted or remaining | Meaning of numerator/denominator; fractions as measures; subtraction in part-whole contexts |
| $T_{12}$ | Subtracting mixed numbers with like denominators (e.g., 5⅔ – 2⅓ for distance) | $\tau_{11}$: Convert to improper fractions.<br>$\tau_{18}$: Subtract numerators.<br>$\tau_{12}$: Simplify into mixed numbers.<br>$\tau_{13}$: Separate whole/fractional parts.<br>$\tau_{18b}$: Subtract whole numbers.<br>$\tau_{19}$: Combine results. | Subtraction of mixed numbers follows the same logic as addition: either convert to improper fractions or operate separately on whole and fractional parts. | Mixed numbers; subtraction of rational numbers; inverse of addition |
| $T_{13}$ | Subtracting fractions with unlike denominators (e.g., 2/3 – 1/2 for unfinished parts of a painting) | $\tau_6$: Visual part-whole model (strip model).<br>$\tau_{20}$: Convert to equivalent fractions.<br>$\tau_{17}$: Find LCM.<br>$\tau_{18}$: Subtract numerators, keep common denominator | Fractions with unlike denominators cannot be subtracted directly; equivalent fractions with a common denominator must first be established | Equivalent fractions; LCM; subtraction with unlike denominators |
| $T_{14}$ | Subtracting mixed numbers with unlike denominators (e.g., 4½ – 2⅓ for rope length) | $\tau_{11}$: Convert to improper fractions.<br>$\tau_{17}$: Find LCM.<br>$\tau_{18}$: Subtract numerators.<br>$\tau_{12}$: Simplify.<br>$\tau_{13}$: Separate.<br>$\tau_{21}$: Subtract whole numbers.<br>$\tau_{17b}$: Find LCM for fractional parts.<br>$\tau_{18c}$: Subtract fractional parts.<br>$\tau_{19}$: Combine results | The principle of unit equivalence applies to subtraction of mixed numbers with unlike denominators; separating whole and fractional components is a valid strategy | Mixed numbers; equivalent fractions; LCM; arithmetic of rational numbers |

## Discussion

The analysis consistently shows that the textbook places greater emphasis on what should be done and how to do it (praxis block), but provides limited explanation of why a method is mathematically valid (logos block). These findings are consistent with praxeological studies of mathematics textbooks across different levels and contexts. The following discussion elaborates on these results from four perspectives: (1) the praxeological paradox in visual representations; (2) three potential learning obstacles; and (3) theoretical contributions along with proposed solutions.

### The Praxeological Paradox of Visual Representations

One of the most striking findings concerns the role of images and illustrations in the textbook. Across all 14 task types, a wide range of visual models are employed, including circle segments, rectangular strips, arrays of objects, and number lines. At first glance, this appears to signal a strong conceptual orientation. However, praxeological analysis reveals that nearly all of these visuals function within the praxis block as techniques (τ) for solving problems, rather than within the logos block as explanations of why a method is mathematically valid (θ). This is what we term the praxeological paradox: many images seem to explain concepts, but in reality, they merely support procedures.

This finding is consistent with those of (Abung, 2023; Rahayu et al., 2022), who emphasize that contextual settings and visual representations can indeed help students grasp the initial concepts of fractions. However, both studies also highlight that visualizations in textbooks more often serve as procedural aids rather than as tools for conceptual exploration, a pattern accurately reflected in the present study's results. Take, for example, the circle diagram used in T4 (Eksplorasi 3.2A). Students observe that combining 1/8 and 2/8 portions of martabak yields 3/8. The image does help students arrive at the correct answer. Yet, the textbook does not use the diagram to explain why the denominator remains 8, namely, because the number 8 designates the fractional unit (eighths), and when we add quantities expressed in the same unit, only the quantity increases, not the unit itself. This is precisely analogous to 1 meter + 2 meters = 3 meters, not 3/16 meters. Without such an explanation, the image merely confirms the answer rather than clarifying the underlying reason.

This finding has a strong theoretical foundation in the literature. (Van de Walle et al., 2013) distinguish between images used as models for mathematical relationships (logos function) and images used as models of problem situations (praxis function). According to them, the primary value of a representation in learning lies in its logos function, its ability to make mathematical structures visible and open to discussion. When images are used only in the praxis function, students may become adept at visualizing specific problems without truly understanding the underlying concepts. (Lamon, 2012) also cautions that part-whole images, such as shaded circles, while easy to grasp, can restrict students to a single meaning of fractions. Bruce et al. (2013)complement this view by asserting that comprehensive fraction understanding requires mastery of at least five meanings: part-whole, measure, quotient, operator, and ratio. When a textbook relies predominantly on only one meaning, it structurally limits the range of understanding students can achieve.

The number line that appears in T2 and T3 is a partial exception to this paradox. Its use aligns with the findings of Lortie-Forgues et al. (2015), which show that students' ability to accurately place fractions on the number line is closely related to their success in learning fraction arithmetic and algebra. The number line fosters the understanding that a fraction is a specific point within the continuum of numbers, rather than merely a pair of numbers. However, in the analyzed textbook, the number line is used only in the sections on comparing and ordering fractions, and is not employed again once arithmetic operations are introduced. This represents a missed opportunity. If the addition of 1/3 + 2/6 were also demonstrated on the number line, students would see for themselves why denominators must be made the same: because one cannot move along the line using different units simultaneously without first converting them. The absence of the number line in the arithmetic context is a clear gap in the textbook's logos block.

The rectangular strip model used in T8 (Cara Nisa, Eksplorasi 3.2D) is the example that comes closest to fulfilling a logos function. The illustration shows that 1/3 of the strip is equal in length to 2/6 of the strip. In this way, the image demonstrates the concept of equivalent fractions. However, the textbook does not make this conclusion explicit: the image is presented, but the principle itself is not stated. (Mack, 1995) documented that students may correctly apply the procedure for equivalent fractions yet still hold misconceptions about what it means for two fractions to be equivalent, precisely because instruction fails to connect the procedure with the underlying principle. This finding from three decades ago remains relevant to the textbook analyzed here.

### Three Potential Learning Obstacles

The imbalance between the praxis block and the logos block in this textbook has the potential to generate three types of learning obstacles for students. These obstacles are structural features of the way the textbook presents material which, without additional explanation from the teacher, tend to produce recurring misunderstandings. Lortie-Forgues et al. (2015) note that difficulties in learning fraction arithmetic fundamentally stem from weak understanding of why fraction operations work as they do, a condition precisely reflected in the underdeveloped logos block of this textbook.

The first obstacle is whole number bias. (Hunting & Davis, 1991) were among the first researchers to systematically document that children carry intuitions from whole numbers when learning fractions. They are accustomed to thinking that a larger number means more, that multiplication always produces a larger number, and that a single number cannot be represented by two numbers at once. These intuitions are valid for whole numbers but misleading for fractions. Van Hoof et al. (2015) showed that this bias can persist into high school if instruction does not explicitly highlight the differences between the properties of whole numbers and rational numbers. Van Dooren et al. (2009) also warned that students who remain fixated on whole number thinking tend to misapply proportional reasoning in inappropriate contexts, another manifestation of the same bias.

In the analyzed textbook, this bias is likely reinforced rather than reduced by the dominance of part-whole images without balancing explanations. When students view fractions as how many parts out of how many total parts, they tend to perceive the numerator and denominator as two separate whole numbers rather than as a single integrated value. The explanations in T4 and T5 begin to move in the right direction by mentioning the roles of numerator and denominator, but they stop short of making the crucial statement that 3/8 is one number, not two separate numbers. (Wu, 2011) strongly emphasized that the conceptual shift from fractions as a relation between two whole numbers to fractions as a single rational number with a position on the number line is the most important change in fraction learning at the elementary level. Such a shift can only be achieved if the number line is used consistently, something this textbook fails to do in most of its chapters.

The second obstacle is representational rigidity. (Lamon, 2012) showed that genuine understanding of fractions requires the ability to view fractions from multiple representational perspectives and to move flexibly among them. A student who only knows 3/4 as three parts out of four parts of a circle diagram will struggle when asked to place 3/4 on the number line, to interpret 3/4 as the result of dividing three objects equally among four people, or to use it as a multiplier that reduces a quantity. (Reys et al., 2014) emphasized that strong fraction understanding demands fluency across multiple representations, part-whole, measure, quotient, operator, and ratio, and that such fluency can only be developed through explicit practice in shifting between representations.

In the analyzed textbook, part-whole representations dominate (T4–T5, T8, T10–T11, T13), with limited use of the number line (T2–T3), and almost no representations of quotient or operator meanings. One clear example is the apple-sharing context in T11 (Asep divides 8 apples, 6 for Lukas and 2 for Yohana), which would have been highly suitable for introducing fractions as quotients. Yet the textbook continues to frame 6/8 as 6 out of 8 total apples, maintaining the part-whole perspective. Such representational limitations create a risk of rigidity. (Van de Walle et al., 2013) describe this kind of understanding as fragile, adequate for routine problems but prone to collapse when faced with different types of tasks. Oktafiyani & Karlimah (2021) empirically confirmed this: elementary students experienced significant difficulty when required to shift from concrete to symbolic representations of fractions, a difficulty rooted in the lack of practice in translating across representations during instruction.

The third obstacle is the underdeveloped explanation of equivalent fractions. Equivalent fractions are the key to performing addition and subtraction with unlike denominators (T8–T9, T13–T14). Yet in the textbook, they are introduced primarily as a technical step without clarifying why the procedure is valid. (Recorde, 1543), in one of the earliest formal explanations of arithmetic equivalence, established that two comparisons are said to be equivalent when both represent the same proportion. The underlying principle remains: equivalence is a profound mathematical property that requires justification, not merely a procedural step. Without such justification, students are prone to two types of errors: assuming that 1/3 and 2/6 are different numbers (failing to recognize equivalence), or applying the LCM procedure mechanically without understanding why the resulting fractions are equivalent to the originals. Maulana & Hidayat (2024), in their analysis of Grade IV mathematics textbooks, found a similar pattern: fraction material presentation does not fully build deep conceptual understanding, particularly in justifying procedures. The findings of this study extend and reinforce their results in the context of Grade V within the Merdeka Curriculum.

Like previous studies that employed the praxeological framework, this research shares similarities while also offering a more specific contribution. (Hendriyanto et al., 2023; Yunianta et al., 2023) demonstrated that praxeological analysis can reveal patterns in the organization of mathematical content in textbooks, particularly the relationship between tasks and techniques. This study strengthens their findings by showing that the same pattern, the dominance of the praxis block over the logos block, also occurs in elementary mathematics textbooks of the Merdeka Curriculum, specifically in the topic of fractions.

Chandra et al. (2025) as well as Siagian et al. (2023) highlight that textbook structures which place excessive emphasis on procedures can give rise to significant learning obstacles. This study extends their findings by identifying in greater detail three specific vectors of obstacles in the context of fractions: whole number bias, representational rigidity, and insufficient theorization of equivalence. Thus, this research not only confirms the existence of obstacles but also explains the structural mechanisms that produce them. Kusharyadi et al. (2024), in their praxeological analysis of sequence and series textbooks, likewise found that the gap between the praxis and logos blocks is a common characteristic of textbooks in Southeast Asia. The findings of this study confirm that such a pattern is not limited to a single grade level or subject area, but represents a systemic issue also present in elementary school textbooks.

From a broader perspective, Sahara et al. (2025) in their comparative ecological analysis of fraction arithmetic in Indonesia and Malaysia, found that textbooks in both countries still place stronger emphasis on procedural fluency than on conceptual exploration. This study is conceptually aligned with their findings while also deepening the analysis: whereas Sahara et al. examined ecological dimensions in a broad sense, the present research dissects in detail the praxeological components that underlie this imbalance.

Evaluating this textbook against international expert standards reveals both its strengths and its structural weaknesses. (Hunting & Davis, 1991) established that effective fraction learning must build explicit connections between the informal knowledge students already possess and the formal material being taught. In this respect, the textbook has a clear strength: the use of everyday contexts such as wingko, martabak, erasers, apples, ribbons, and distances makes the material feel familiar and accessible. Primasari et al. (2021) even found that such contextual approaches can increase students' interest and motivation in learning mathematics, including fractions. However, what is less well addressed is the conceptual bridge between informal contexts and formal procedures. The textbook presents concrete stories and then immediately jumps to procedural steps, without explicitly showing students why the taught method makes sense. Van Dooren et al. (2009) caution that overly strong attachment to specific contexts can actually limit students' ability to transfer concepts, contexts should be designed as bridges toward abstraction, not as ends in themselves.

(Lamon, 2012) provides the most comprehensive benchmark: fractions consist of five sub-constructs, and instruction that relies on only one sub-construct produces fragile understanding. Of these five, this textbook meaningfully develops only two. The quotient sub-construct is present contextually in T11 but is not didactically elaborated. The operator and ratio sub-constructs are entirely absent. Putra et al. (2020) found a related issue: the didactic knowledge of prospective elementary teachers about fractions, when analyzed through ATD, also showed limitations in understanding fraction meanings beyond the part-whole sub-construct. If the textbooks they studied during their own schooling likewise developed only that sub-construct, then a chain of conceptual limitations is formed, from textbook to student to prospective teacher back to textbook. This cycle needs to be broken through systematic improvement.

(Mack, 1995) found that students' prior knowledge of whole numbers can interfere with fraction learning if not explicitly addressed. In T4, students are directed to add 1/8 + 2/8 by adding the numerators. On the surface, this instruction resembles ordinary whole number addition and can encourage faulty generalization. Although the diagram shows the correct result (3/8), the textbook does not address the question of why the denominators are not added, a question that requires a logos-block explanation to resolve. Utami et al. (2022) in their study of praxeology-based didactic design, demonstrated that reflective why questions are precisely the critical points for building students' conceptual understanding, and such questions must be explicitly planned rather than left entirely to student initiative.

Siegler et al. (2010) demonstrated that understanding fraction magnitude is most effectively developed through the number line. The fact that this textbook uses the number line only in the sections on comparing and ordering, and not in the sections on arithmetic operations, represents a major missed opportunity. (Wu, 2011) argued that the root of many difficulties in fraction learning lies in the failure to teach fractions as numbers from the outset. The analyzed textbook does employ rich real-world contexts, but once the material progresses to arithmetic operations, the framing of fractions as numbers advocated by (Wu, 2011) is not consistently maintained. (Van de Walle et al., 2013) emphasized that effective fraction learning requires not only multiple representations but also explicit practice in moving among them. This textbook presents various representations side by side, but never explicitly invites students to connect them. As a result, the richness of visual representations has not translated into an equivalent richness of conceptual understanding.

(Reys et al., 2014) called for a balance among conceptual understanding, procedural fluency, and reasoning ability. This textbook succeeds reasonably well in building procedural fluency and providing concrete starting points. However, students' ability to reason about why a method is valid requires a strong logos block. The 'Ayo Menyimpulkan' (Let's Conclude) section of the textbook reflects a good intention to summarize concepts, but the outcome tends to be a summary of procedures rather than statements of mathematical principles. This represents an aspiration toward the logos block that has not been fully realized.

### Theoretical Contribution: The Praxeological Paradox and Its Solution

The main theoretical contribution of this study is the concept of the praxeological paradox in elementary mathematics textbooks. This paradox occurs when the extensive use of images and illustrations creates the impression that the textbook is rich in conceptual explanations (logos block), while in reality those visuals merely reinforce procedural solutions (praxis block). Previous studies such as (Chandra et al., 2025; Hendriyanto et al., 2023; Yunianta et al., 2023) did identify weaknesses in the logos block of textbooks, but they did not theorize how such weaknesses could be hidden behind visual abundance. The concept of the praxeological paradox fills this gap by providing a more precise explanatory mechanism.

This paradox arises from the process of didactic transposition, (Chevallard & Johsua, 1985) term for the transformation that mathematical knowledge undergoes when it is transferred from the academic domain into school textbooks. In academic mathematics, visuals such as number lines and area models are used as theoretical tools to articulate conceptual properties. Yet when these visuals enter Grade V textbooks, their theoretical function is stripped away, leaving only images as aids for solving problems. This is consistent with (Chevallard & Johsua, 1985) prediction that transposition inevitably transforms knowledge. However, this analysis shows that the transformation can go too far, so that the logos block exists visually, but is absent conceptually.

To address this paradox, we propose the principle of logos-explicit textbook design: every technique introduced must be accompanied by a clear statement explaining why the technique is valid, and every explanation must be firmly rooted in explicitly stated mathematical concepts. For fraction arithmetic, this means that the rule add the numerators, keep the denominator should be paired with the explanation because the denominator names the unit fraction, and we are adding how many of those units we have. This explanation must in turn be grounded in the theory that a fraction a/b is a single number that indicates how many parts of size 1/b are taken, a specific point on the number line (Wu, 2011). Utami et al. (2022) showed that praxeology-based instructional materials which explicitly include logos components have been proven to support students' conceptual understanding more effectively than materials oriented only toward techniques. Their findings provide empirical support for the logos-explicit design principle we propose.

It is important to note that logos-explicit design does not mean adding more pages or making the textbook thicker. What is needed is a change in how existing visuals are used. Instead of using a circle diagram merely to show that 1/8 + 2/8 = 3/8, the textbook could simply add the explanation: each slice is one eighth-unit; we count the slices: one eighth plus two eighths equals three eighths. Such a simple change is a didactic step, not a complex mathematical elaboration, yet it is precisely this articulation of the logos block that connects procedure with concept. Teachers also need to recognize that textbooks do not always provide complete logos explanations, so they must actively enrich instruction with reflective questions, discussions of the mathematical reasons behind procedures, and comparisons among solution strategies (Abung, 2023; Rahayu et al., 2022)

## Conclusion

This study analyzes the Grade V Mathematics Textbook of the Merdeka Curriculum using Chevallard's praxeological framework on the topics of fraction concepts, addition, and subtraction. The results show that the praxis block, tasks and solution techniques, is fully present, whereas the logos block, conceptual explanations and mathematical reasoning, remains inadequate. Images and illustrations are widely used, but they function more as aids for solving problems than for building conceptual understanding. This pattern is referred to as the praxeological paradox.

This textbook has three main weaknesses. First, it does not sufficiently help students understand fractions as rational numbers. Second, it relies too heavily on the part-whole meaning of fractions and gives little attention to other meanings such as measure, quotient, operator, and ratio. Third, it does not adequately explain the mathematical reasons behind key techniques, such as the use of common denominators, unit fractions, and equivalent fractions. These weaknesses risk creating learning obstacles: students may apply whole-number thinking to fractions, become fixated on a single representation, and view equivalent fractions merely as procedures. These findings indicate that textbooks need to be designed with the principle of logos-explicitness, meaning that every technique must be accompanied by an explanation of why it is mathematically valid. Teachers also need to supplement textbook procedures with conceptual explanations.

This study is limited to document analysis and has not examined classroom use or student responses. Future research should involve classroom observations, assessments of student understanding, and comparisons with textbooks from previous curricula or from other countries.

## Declarations


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|---|---|---|
| Author Contribution | : | DR: Conceptualization, Writing - Original Draft, Editing and Visualization.<br>DS: Writing - Review & Editing, Formal Analysis, Methodology and Supervision. |
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