AXIOM : Jurnal Pendidikan dan Matematika
(2026) Volume 15, No. 1, Page 42-56
https://doi.org/10.30821/axiom.v15i1.26491

Optimization of operational cost planning in integrated farming systems using a mixed-integer linear programming approach

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Received: 30 October 2025 / Accepted: 10 February 2026 / Published Online: 29 June 2026

Abstract
This study addresses the challenge of optimizing small-scale integrated farming systems (IFS) by minimizing operational costs while ensuring sustainable land use across agricultural, livestock, and aquaculture components. The main objective is to develop a Mixed-Integer Linear Programming (MILP) model that incorporates deterministic parameters such as land availability, labor allocation, and internal-external input flows. The model integrates multiple interrelated subsystems using production coefficients, resource constraints, and cost structures derived from actual smallholder scenarios. A two-period simulation was conducted to evaluate the model's effectiveness using fixed input values, reflecting rural farming conditions. The results demonstrate that the system achieved consistent outputs without requiring external purchases of manure, feed, or irrigation water. The total operational cost reached IDR 96,770,000, with optimized land and labor allocation across periods. This research contributes a novel MILP formulation tailored to integrated farming, providing practical insights for policymakers and practitioners. Its implications extend to the development of decision-support systems for rural agricultural planning. However, the model's deterministic assumption limits its adaptability to dynamic environments. Future work should explore stochastic variants and real-time input adjustments to improve model flexibility and realism.

Keywords: Integrated farming system, Mix integer linier programming, Operational cost minimization, Resource optimization, Sustainable agriculture planning

How to Cite: Sinaga, L.P., Habibi, R., & Panggabean, S. (2026). Optimization of operational cost planning in integrated farming systems using a mixed-integer linear programming approach. *AXIOM : Jurnal Pendidikan dan Matematika, 15*(1), 42-56. https://doi.org/10.30821/axiom.v15i1.26491

Introduction

The agricultural sector plays a vital role in ensuring food security and supporting national economic growth, particularly in developing countries such as Indonesia. However, this sector faces various structural and ecological challenges that hinder its productivity and long-term sustainability. Among the most pressing issues is the decline in soil fertility due to the excessive and prolonged use of chemical inputs. These practices not only degrade soil structure but also reduce land quality, directly affecting agricultural yields and farmers' income. Additionally, the continuous conversion of agricultural land for industrial, residential, and infrastructure development (particularly in urbanizing regions) has led to a substantial decrease in arable land (Badan Pusat Statistik, 2020; Santini et al., 2022).

Such conditions indicate that land degradation is a critical challenge in achieving sustainable agricultural development. This degradation results from changes in the physical, chemical, and biological properties of the soil, leading to a reduced capacity to support agricultural activities (AbdelRahman, 2023). Excessive reliance on chemical fertilizers is one of the main contributors to this degradation, as it pollutes soil and water and disrupts ecological balance. Over time, it also reduces microbial biodiversity, which is essential for maintaining soil fertility and productivity (Dincă et al., 2022).

In response to these multifaceted problems, there is an urgent need for more efficient, environmentally sound, and sustainable agricultural practices. The Integrated Farming System (IFS) has emerged as a promising solution by combining various agricultural components, such as crop cultivation, livestock, and aquaculture, into an interdependent and mutually reinforcing system. IFS is designed to enhance productivity with minimal reliance on external inputs, thereby reducing operational costs while fostering a resilient agroecosystem. The system integrates food crops, horticulture, livestock, and freshwater aquaculture, forming a circular flow of resources that minimizes waste and environmental damage.

IFS is conceptually structured around four core components: humans, crops, livestock, and aquaculture. Humans manage the system, crops provide both food and livestock feed, livestock produce consumable animal products and organic waste that serves as fertilizer, and aquaculture utilizes system by-products as nutrients while contributing protein-rich food sources (Arimbawa, 2015).The utilization of organic waste, such as manure, can reduce dependence on chemical fertilizers, lower production costs, and mitigate environmental pollution (Suwardike et al., 2024).The reciprocal interactions among these components form a self-sustaining and efficient agricultural ecosystem, as illustrated in Figure 1.

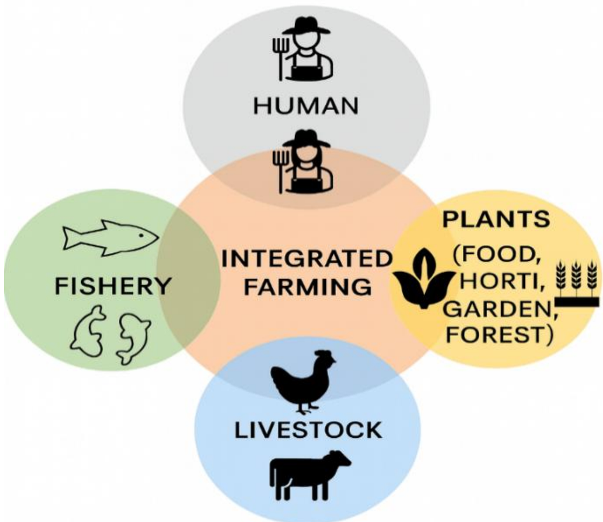

Figure 1. Illustration of the Integration of Core Components in an Integrated Farming System (Arimbawa, 2015).

Beyond ecological and structural design, agricultural operations are inherently tied to technical practices and cost efficiency. Operational aspects such as land management, input utilization, and production strategies significantly influence outcomes. The adoption of modern technologies and efficient management systems can enhance productivity while supporting sustainability. Practices such as water-saving irrigation, crop rotation, and sustainable intensification can increase yields without ecological harm. In parallel, effective supply chain management reduces post-harvest losses and contributes to economic gains for farmers (Benini et al., 2023; Ford, 2009)

Recent studies have emphasized the importance of managing operational costs to improve agricultural viability. In Indonesia, irrigation, fertilizers, and labor remain the largest cost components in rice farming, with efficiency levels reaching only 83% (Siagian & Soetjipto, 2020). Infrastructure programs such as the Farm Road Program have demonstrated potential to reduce operational costs for pepper farmers by up to 22% by improving market access (Mahendra, 2025). Modeling approaches such as Mixed Integer Nonlinear Programming have been successfully used to optimize the allocation of agricultural machinery and reduce per-unit costs (Sun et al., 2023). Likewise, the use of Mixed Integer Linear Programming (MILP) to optimize cropping systems and distribution logistics has proven effective in improving profitability for smallholder farmers in developing countries (López et al., 2022). In rice farming contexts, managing irrigation and labor costs remains critical to ensuring farm-level sustainability and profitability (Darma et al., 2025)

Mathematical modeling provides a valuable framework for understanding complex systems and optimizing resource allocation in agriculture. It enables data-driven decisions for predicting yields, scheduling planting, and managing inputs (Makowski, 2020; Mula et al., 2006; Rossing et al., 1997). Foundational studies have shown the relevance of mathematical programming in optimizing land use and farm planning (Heady, 1952; McCarl & Spreen, 1974; Ragsdale, 2004). Moreover, land-use transformation models that account for various agronomic and economic drivers can support crop rotation strategies and sustainable land management (Taratula et al., 2019).

Recent empirical applications further demonstrate the effectiveness of mathematical optimization models in enhancing farm income. A multi-objective linear programming model was used to identify optimal crop combinations in dryland regions with water scarcity, considering both resource constraints and market dynamics (Bhatia & Rana, 2020). Another study focused on optimizing integrated agricultural resources to increase farmer income in North Bolaang Mongondow Regency, Indonesia, using a linear programming approach (Wantasen et al., 2024). Their findings highlighted significant improvements in productivity and income through resource synergies. Similarly, a linear programming model applied to a coffee–goat IFS in North Sumatra revealed income improvements driven by simple shade systems, increased livestock units, and the strategic use of by-products such as organic fertilizer from goat manure and coffee pulp (Hida et al., 2023).

Recent advances in circular and integrated agriculture highlight how mathematical optimization supports resource-efficient systems. (Yue et al., 2022) developed an integrated crop–livestock–biogas–crop framework under uncertainty, showing that circular configurations can increase resource-use efficiency and reduce emissions. (Chen et al., 2023) applied a multi-objective, emergy-based model to examine trade-offs between economic returns, energy efficiency, and environmental performance in crop–livestock systems, while (Hang et al., 2021) designed an optimisation scheme that jointly manages greenhouse gas emissions and farm income. For intensive livestock–crop settings, (Taifouris & Martin, 2021) combined nutrition requirements, anaerobic digestion, nutrient recovery, and cropping within one model to compare integrated and separated systems in economic and environmental terms.

At broader scales, interval meta-goal programming has been used to coordinate agricultural water–land use under uncertainty (Najafabadi et al., 2023), and the MOCLAM model optimises crop–livestock allocation in semi-arid regions using multi-objective goal programming (Chand et al., 2024). Collectively, these studies confirm the promise of optimisation-based approaches for sustainable and circular agriculture, yet they mainly address environmental trade-offs or regional planning rather than farm-level, multi-period operational cost planning under strict land, labour, and input constraints.

Despite these advances, there remains a research gap in the application of integrated mathematical models that specifically optimize operational costs in multi-component farming systems under resource constraints, particularly within integrated farming practices in Indonesia. While previous studies focus on income maximization and productivity, limited attention has been paid to multi-period operational cost planning under practical constraints such as land availability, feed supply, and labor. This study aims to formulate and implement a MILP model to optimize the operational cost planning of an Integrated Farming System that incorporates crop production, livestock, and aquaculture. The proposed model integrates multi-period resource allocation constraints and production planning to achieve cost minimization objectives. The novelty of this research lies in its mathematical formalization of an integrated farming model that simultaneously considers land, labor, and input limitations across multiple time periods, an approach that is rarely addressed in existing agricultural optimization literature. The expected contribution of this study is twofold: first, it provides a replicable quantitative framework for planning and managing integrated agricultural operations; second, it offers practical insights for smallholder farmers and policymakers seeking to improve cost efficiency and system sustainability in rural farming communities.

Methods

This study employs a quantitative approach using a MILP framework to optimize operational costs in an IFS with closed-loop dynamics. The system comprises three interdependent subsystems: crop production, livestock, and aquaculture, that reinforce one another through circular utilization of resources such as crop residues, animal manure, and pond water to reduce dependence on external inputs.

System Definition and Parameterization

The model spans three consecutive production periods (T1-T3), representing seasonal cycles. The commodities modeled include maize, forage grass (as feed), cattle, poultry, and fish, with parameter values (e.g., input costs, labor, yields, feed conversion, water needs, and waste outputs) sourced from empirical studies and regional agricultural standards (Layek et al., 2023; Paramesh et al., 2020)

Land allocation is optimized across subsystems: crops are expected to supply feed for livestock; livestock generate manure for fertilizing crops; and ponds serve both as aquaculture units and rainwater collectors for irrigation, contributing to soil conservation and water efficiency. This integrated structure promotes land productivity, environmental sustainability, and rural development, embodying a circular farming model. The MILP model simulates these interactions to minimize operational costs while satisfying resource constraints, synergistic feedback loops, and sustainability objectives. A conceptual framework illustrating subsystem linkages is provided in Figure 2. Mathematical modeling in the optimization of operational cost planning in IFS can be illustrated by the scheme in Figure 3.

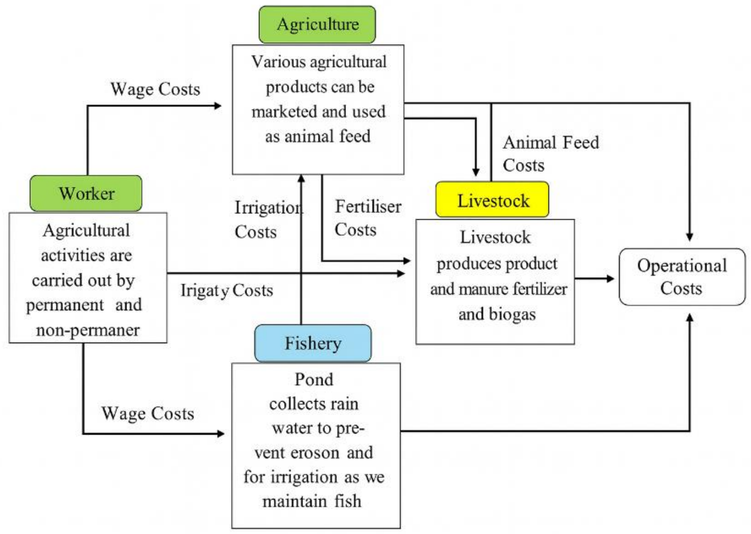

Figure 2. Operational cost relationship scheme in IFS

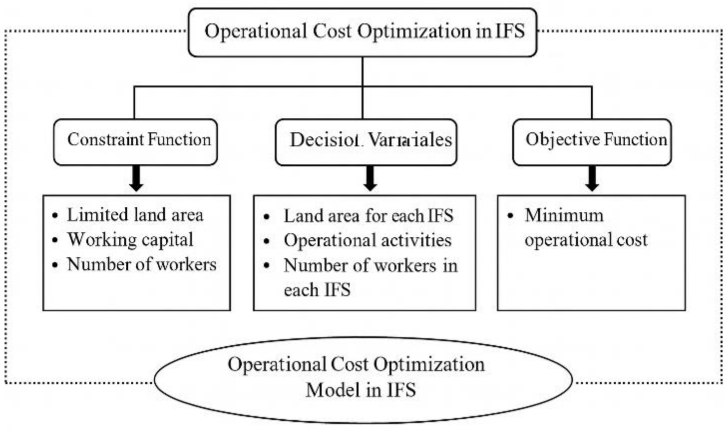

Figure 3. Optimization scheme of operational planning in IFS

Mathematical Model Formulation

Based on the conceptual framework, the mathematical modeling introduces indexed decision variables as presented in Table 1. The agricultural components, particularly land area and the number of workers (both permanent and temporary), are formulated as core decision variables in this IFS model. The number of crops, livestock, and fishponds influences several key factors, including the required land area, the cost of seed or livestock procurement, fertilization expenses for crops, feed costs for livestock and fish, and the allocation of labor, both permanent and non-permanent. The decision variables related to land allocation and labor assignment and to operational activities in the IFS are detailed in Table 2.

Table 1. Indexed decision variables.

Index	Description
i	Index representing agricultural components with the i -th type of crop
j	Index representing livestock components with the j-th type of animal
k	Index representing aquaculture components with the k-th type of fishpond
t	Index representing the t-th time period

Table 2. Decision variables.

Variabel	Description	Unit
LX_{it}	Land area allocated for the i-th type of crop in the t-th time period	ha
LY_{jt}	Land area allocated for the j-th type of livestock facility in the t-th time period	ha
LZ_{kt}	Land area allocated for the k-th type of fishpond in the t-th time period	ha
PX_{it}	Number of permanent workers employed in crop farming of plant i during time period t	person
PY_{jt}	Number of permanent workers employed in livestock farming of animal j in period t	person
PZ_{kt}	Number of permanent workers employed in aquaculture of pond k in period t	person
QX_{it}	Number of non-permanent workers employed in crop farming of plant i in period t	person
QY_{jt}	Number of non-permanent workers employed in livestock of animal j in period t	person
QZ_{kt}	Number of non-permanent workers employed in aquaculture of pond k in period t	person

Subsequently, several parameters relevant to the mathematical modeling are introduced and summarized in Table 3. These parameters cover both the production coefficients and the operational cost components of the IFS, including seed and feed requirements, labor demands, and associated input costs. The unified presentation facilitates a comprehensive understanding of the quantitative relationships embedded in the IFS model formulation.

Table 3. Parameters related to the production and operational cost of IFS.

Parameter	Description	Unit
a_{it}	Production coefficient of crop i in time period t	kg/ha
b_{jt}	Manure production coefficient of livestock j in time period t	kg/pen
c_{it}	Manure requirement coefficient for crop i in time period t	kg/ha
d_{jt}	Feed requirement coefficient for livestock j in time period t	kg/pen
e_{kt}	Feed requirement coefficient for fish in pond k in time period t	kg/pond
f_{it}	Water requirement coefficient for crop i in time period t	liters/ha
BX_{it}	Purchase price coefficient of crop seeds for crop i in period t	IDR/ha
BY_{jt}	Purchase price coefficient of livestock for animal j in period t	IDR/head
BZ_{kt}	Purchase price coefficient of fish seeds for pond k in period t	IDR/fish
CX_{it}	Purchase price coefficient of manure for crop i in period t	IDR/kg
CY_{jt}	Purchase price coefficient of animal feed for livestock j in period t	IDR/kg
CZ_{kt}	Purchase price coefficient of fish feed for pond k in period t	IDR/kg
DX_{it}	Purchase price coefficient of irrigation water for crop i in period t	IDR/m^3

The number of livestock or fish cultivated is directly influenced by the available land area, including livestock pens and fish ponds. Additionally, each agricultural activity requires labor, which consists of both permanent and non-permanent workers. Therefore, Table 4 presents the parameters related to land requirements and labor wages in the operation of the IFS.

Table 4. The parameters related to land requirements and labor wages in IFS.

Parameter	Description	Unit
g_{jt}	Coefficient of the number of livestock j raised per unit area of pen during period t	head/ha
h_{kt}	Coefficient of the number of fish in pond k raised per unit area of pond during period t	fish/ha
m_{kt}	Coefficient of water volume available in pond k per unit area during period t	m^3/ha
EX_{it}	Average monthly wage of permanent workers in crop farming of type i in period t	IDR
EY_{jt}	Average monthly wage of permanent workers employed in livestock j in period t	IDR

Parameter	Description	Unit
EZ_{kt}	Average monthly wage of permanent workers employed in aquaculture pond k in period *t*	IDR
FX_{it}	Average monthly wage of non-permanent workers employed in crop farming *i* in period *t*	IDR/ha-month
FY_{jt}	Average monthly wage of non-permanent workers employed in livestock *j* in period *t*	IDR/ha-month
FZ_{kt}	Average monthly wage of non-permanent workers employed in aquaculture *k* in period *t*	IDR/ha-month
TX_{it}	Duration of employment for non-permanent workers in crop farming of type i in period t	month
TY_{jt}	Duration of employment for non-permanent workers in livestock *j* in period t	month
TZ_{kt}	Duration of employment for non-permanent workers in aquaculture *k* in period t	month

The optimization model for the IFS is formulated using a MILP approach to simultaneously minimize operational costs and land utilization across agricultural, livestock, and aquaculture activities, considering constraints such as capital, land availability, and labor, with operational costs including expenditures on manure, animal and fish feed, as well as labor wages. Accordingly, the mathematical model for planning the IFS with the objective of minimizing operational costs is expressed as follows.

$$\text{Min Z} = Z_1 + Z_2 + Z_3 + Z_4 + Z_5 \tag{1}$$

s.t.

$$\sum_{i \in I} LX_{it} + \sum_{j \in J} LY_{jt} + \sum_{k \in K} LZ_{kt} \leq L_t, \qquad \forall t \in T \tag{2}$$

$$\sum_{i \in I} PX_{it} + \sum_{j \in J} PY_{jt} + \sum_{k \in K} PZ_{kt} = SP_t, \qquad \forall t \in T \tag{3}$$

$$\sum_{i \in I} (f_{it} \cdot LX_{it}) \leq A_t + \sum_{k \in K} (m_{kt} \cdot d_{kt} \cdot LY_{kt}), \qquad \forall t \in T \tag{4}$$

$$\sum_{j \in J} \left(g_{jt} \cdot d_{jt} \cdot LY_{jt} - B_{jt}\right) \leq \sum_{i \in I} (a_{it} \cdot LX_{it}), \qquad \forall t \in T \tag{5}$$

$$\begin{aligned} &\sum_{i \in I} (PX_{it} + QX_{it}) \cdot LX_{it} + \sum_{j \in J} \left(PY_{jt} + QY_{jt}\right) \cdot LY_{jt} \\ &\qquad + \sum_{k \in K} (PZ_{kt} + QZ_{kt}) \cdot LZ_{kt} \geq SP_t, \forall t \in T \end{aligned} \tag{6}$$

$$\begin{aligned} Z_1 = &\sum_{i \in I} (BX_{it} \cdot LX_{it}) + \sum_{j \in J} \left(BY_{jt} \cdot g_{jt} \cdot LY_{jt}\right) \\ &+ \sum_{k \in K} (BZ_{kt} \cdot h_{kt} \cdot LZ_{kt}), \forall t \in T \end{aligned} \tag{7}$$

$$Z_2 = CX_{it} \cdot \left(\alpha_t + \sum_{i \in I} (c_{it} \cdot LX_{it}) - \sum_{j \in J} \left(b_{jt} \cdot g_{jt} \cdot LY_{jt}\right)\right), \forall t \in T \tag{8}$$

$$\begin{aligned} Z_3 = &\sum_{j \in J} \left(CY_{jt} \cdot d_{jt} \cdot g_{jt} \cdot LY_{jt} + B_{jt}\right) + \sum_{k \in K} (CZ_{kt} \cdot e_{kt} \cdot h_{kt} \cdot LZ_{kt}) \\ &- \sum_{i \in I} (CY_{it} \cdot a_{it} \cdot LX_{it}), \forall t \in T \end{aligned} \tag{9}$$

$$Z_4 = DX_{it} \cdot \left(\varphi_t + \sum_{i \in I} (f_{it} \cdot LX_{it}) - \sum_{k \in K} (m_{kt} \cdot LZ_{kt}) \right), \forall t \in T \tag{10}$$

$$Z_5 = \sum_{i \in I} (FX_{it} \cdot PX_{it} + FX_{it} \cdot QX_{it}) + \sum_{j \in J} (FY_{jt} \cdot PY_{jt} + FY_{jt} \cdot QY_{jt}) + \sum_{k \in K} (FZ_{kt} \cdot PZ_{kt} + FZ_{kt} \cdot QZ_{kt}), \quad \forall t \in T \tag{11}$$

$$LJX_{it} \geq 0, LPX_{it} \geq 0, LX_{it} \geq 0, LY_{jt} \geq 0, LZ_{kt} \geq 0, \forall i \in I, \forall j \in J, \forall k \in K, \forall t \in T \tag{12}$$

$$PX_{it} \geq 0, PY_{jt} \geq 0, PZ_{kt} \geq 0, QX_{it} \geq 0, QY_{jt} \geq 0, QZ_{kt} \geq 0, \forall i \in I, \forall j \in J, \forall k \in K, \forall t \in T \tag{13}$$

$$LX_{it}, LY_{jt}, LZ_{kt} \in \mathbb{R}^+, PX_{it}, PY_{jt}, PZ_{kt} \in \mathbb{Z}^+ \tag{14}$$

Equation (1) establishes the objective of the model, which is to minimize the total operational cost of the integrated farming system. This total cost is decomposed into five components: seed procurement, organic fertilizer, animal and fish feed, irrigation water, and labor wages. Constraint (2) represents the land capacity constraint, ensuring that the total land allocated to crop, livestock, and fishery activities in each period does not exceed the available land area. Constraint (3) defines the production balance condition, which equates the total output generated from crop, livestock, and fish subsystems to the system production supply in each period. Constraint (4) captures labor availability, ensuring that the labor required for crop production activities is met by the available labor resources, consisting of the initial labor endowment and labor generated from fish farming activities. Constraint (5) enforces the feed and nutrient balance between livestock and crop subsystems, guaranteeing that livestock feed demand does not exceed the feed supplied by crop production. Constraint (6) ensures that total production from all subsystems satisfies system demand requirements in each period. Constraints (7)-(11) define the individual components of the total operational cost, including production costs for crops, livestock, and fish (Constraint 7), cost adjustments related to internal resource circulation (Constraint 8), livestock and fish-related operational costs (Constraint 9), labor-related operational costs (Constraint 10), and transportation and handling costs associated with production outputs (Constraint 11). Together, these constraints ensure the feasibility, resource consistency, and economic coherence of the integrated farming system across all planning periods. Equations (12)-(14) define the domain and feasibility conditions of all decision variables in the model. These constraints ensure non-negativity and appropriate domain specifications for land allocation and production variables, while enforcing integrality or continuity requirements consistent with the mixed-integer linear programming formulation.

Algorithmic Implementation, Solver Integration, and Simulation

The model is implemented in Python 3.11 using the Pyomo modeling framework and solved with the CBC (Coin-OR Branch and Cut) solver, an open-source solver suitable for MILP formulations. Computations are performed on a standard computing environment: Intel Core i7, 16 GB RAM, Windows OS. Average solution time is under 0.01 seconds, indicating a computationally efficient model for the specified scale. Simulation runs span the three planning periods with deterministic input data. Output includes optimal values of decision variables, production quantities, internal versus external input usage, and total system cost. These outputs are processed into tables and narrative form to support interpretive analysis.

The mathematical model is formulated for a multi-period planning horizon consisting of three periods (T1-T3). However, for the numerical simulation and result analysis presented in this study, only the first two periods (T1 and T2) are evaluated. This restriction is applied to illustrate the dynamic behavior of the integrated farming system while maintaining computational tractability and clarity of interpretation.

Results

The model integrates key parameters of production, cost, input requirements, and resource availability, including land, labor, feed, water, and fertilizer, from both internal and external sources. All parameters are assumed to be deterministic, using fixed numerical values that represent small- to medium-scale integrated farming operations in rural settings.

The simulation results indicate that the proposed model effectively minimizes total operational cost under integrated farming constraints, which implicitly improves the net economic performance of the system. Constraints account for resource conservation, input-output balance across subsystems, and capacity limitations.

The simulation results are presented for two consecutive time periods (T1 and T2), representing the initial stages of the multi-period planning horizon, with a fixed land availability $L_t = 2$ ha and a labor supply of 15 permanent workers per period. Crop seed costs BX_{it}, livestock costs BY_{jt} , and fish seed costs BZ_{kt} increase linearly with time, starting at IDR 2,000,000, IDR 1,500,000, and IDR 500,000 respectively. Land expansion wages are fixed per unit: EX_{it} = IDR2,500,000 for crops, EY_{jt} = IDR 2,800,000 for livestock, and EZ_{kt}= IDR 2,000,000 for aquaculture. Monthly wages for non-permanent workers are IDR 1,000,000 for crops FX_{it}, IDR 1,200,000 for livestock FY_{jt}, and IDR 900,000 for ponds FZ_{kt}. Crop yields a_{it} are 5 tons/ha for maize and 2 tons/ha for others; livestock and fish yields g_{jt} and h_{kt} are 1 unit per ha. Manure demand per ha c_{it} is 100 kg for maize and 40 kg for others, while manure production b_{jt} is 50 kg/head (cattle) and 0.8 kg/head (poultry). Feed requirements d_{jt} and e_{kt} are 400 kg/head (cattle), 65 kg/head (poultry), and 5 kg/pond (fish), respectively. Irrigation needs f_{it} are 5,000 L/ha for maize and 3,000 L/ha otherwise. Crop residues provide animal feed (800–1,000 kg/ha) and fish feed (30-80 kg/ha). Pond sludge contributes 25 kg/ha of manure, and water availability from ponds is m_{kt} = 500 m³/ha, of which h_{kt} = 70 m³/ha is usable. Input prices are set as follows: manure CX_{it} = IDR 2,500/kg, livestock feed CY_{jt} = IDR 3,000/kg, fish feed CZ_{kt} = IDR 4,000/kg, and irrigation water DX_{it} = IDR 200/m³. Labor intensity is assumed to be 2 persons/ha for crops, 1 for livestock, and 0.1 for ponds. Duration of employment for non-permanent labor is 1 month across all sectors.

The simulation of the integrated farming model over two planning periods resulted in a balanced utilization of land, labor, and internal resources across crop farming, livestock, and aquaculture components. In both periods (T_1 and T_2), the system achieved consistent production outputs, including 1.00 ton of maize, 3.00 tons of grass, 4 cattles, 100 chickens, and 1,000 fish per period. This yields a cumulative total of 2.00 tons of maize, 6.00 tons of grass, 8 head cattle, 200 chickens, and 2,000 fish over the entire simulation horizon. Table 5 presents the production results for each commodity in periods T_1 and T_2, as well as their aggregated values over the two-period planning horizon. In this study, the values reported in the "Total" column represent the cumulative production obtained by summing the results of T_1 and T_2 for each commodity. Consistently, Table 6 reports the total operational cost per commodity, which corresponds to the aggregated costs incurred over the same two periods ($T_1 + T_2$), as calculated based on the proposed MILP formulation.

Land allocation and labor employment remained stable across both periods: 1.000 ha of land for maize and 0.600 ha for grass, supported by 7 workers (5 permanents and 1 non-permanent for maize; 2 permanents and 1 non-permanent for grass). Similarly, 0.200 ha and 0.100 ha were allocated for cattle and poultry respectively, each staffed by 1 permanent and 1 non-permanent worker. Aquaculture utilized 0.100 ha per period, with the same labor configuration.

Table 5. Simulation Results - Production and Costs

Commodity	T1	T2	Total
Maize	1 ton	1 ton	2 ton
Grass	3 ton	3 ton	6 ton
Cattle	4 cattles	4 cattles	8 cattles
Chicken	100 chickens	100 chickens	200 chickens
Fish	1000 fish	1000 fish	2000 fish

Table 6. Total Cost Per Commodity

Commodity	Total Cost (IDR)
Maize	39,100,000
Grass	16,460,000
Cattle	17,150,000
Chicken	11,050,000
Fish	13,010,000

Notably, no additional purchases of manure, livestock feed, fish feed, or irrigation water were required in either period, indicating that internal resource flows fully met production needs. In terms of operational expenditure, the total production costs were IDR 39,100,000 for maize, IDR 16,460,000 for grass, IDR 17,150,000 for cattle, IDR 11,050,000 for poultry, and IDR 13,010,000 for aquaculture. The total operational cost over the two periods amounted to IDR 96,770,000 (Table 6), suggesting efficient integration and resource utilization within the system without reliance on external input procurement.

Discussion

These results resonate with and extend recent optimization studies on circular and integrated agriculture. (Yue et al., 2022) showed that a crop-livestock-biogas-crop system managed under uncertainty can enhance resource use efficiency and reduce emissions relative to conventional configurations. Similarly, (Hang et al., 2021) demonstrated that circular practices lower greenhouse gas emissions with only moderate income sacrifices. The present study corroborates the core insight that internal recycling loops are central to sustainable agriculture, but does so at the level of a small integrated farm that relies on low-tech flows of crop residues, manure, and pond sludge rather than biogas infrastructures or advanced waste-treatment facilities.

Compared with multi-objective models such as (Chen et al., 2023) who optimized integrated crop-livestock systems using emergy-based indicators, and (Chand et al., 2024), who developed the MOCLAM model for crop-livestock allocation in semi-arid India, the MILP model in this study adopts a single economic objective. While those studies explicitly quantify trade-offs between profit, energy efficiency, and environmental indicators, our model focuses on minimizing operational costs subject to strict resource-balance constraints. This design choice yields a transparent solution that is directly interpretable for farm-level planning; yet, the emergent pattern (high self-sufficiency and efficient use of internal residues) remains consistent with their broader sustainability conclusions.

With respect to resource-nexus optimisation, (Najafabadi et al., 2023) employed interval meta-goal programming to balance economic returns, water conservation, and food production at regional scale, whereas (Taifouris & Martin, 2021) integrated livestock, digesters, nutrient recovery, and cropping in a large-scale intensive system, quantifying substantial reductions in environmental impact. In contrast, the present model operates at the micro-scale of a representative integrated farm, and explicitly couples land, labor, feed, fertilizer, and water balances within a single MILP framework. By including aquaculture alongside crop and livestock components, the model also covers a broader set of subsystems than most existing optimisation studies, which typically restrict themselves to crop-livestock or livestock-biogas configurations. This combination fills an evident gap in the literature on optimisation for integrated farming systems in tropical rural contexts.

From a theoretical perspective, the results reinforce central claims of integrated farming system and circular economy frameworks. The emergence of a cost-efficient solution with no external purchases of manure, feed, or irrigation water provides a concrete optimisation-based illustration of how closed nutrient and biomass loops can sustain production. The conversion of livestock waste and crop residues into usable inputs for other subsystems mirrors the circular agriculture designs proposed by (Taifouris & Martin, 2021; Yue et al., 2022), but shows that similar principles can be operationalized through relatively simple on-farm practices rather than capital-intensive technologies.

The stability of land and labor allocations across periods also supports the systems-thinking view that well-balanced integrated systems can buffer seasonal variability without major structural adjustments. In the model, proportional distributions of workers and land reflect an implicit equilibrium between productivity targets and the availability of internal resources. This contrasts with the more dynamic trade-off frontiers reported in multi-objective settings (Chand et al., 2024; Chen et al., 2023), suggesting that, under certain conditions, a single economically driven optimum may already embody substantial environmental co-benefits when internal recycling is strongly enforced by constraints.

Practically, the model provides a decision-support tool for small to medium scale farmers and extension services interested in designing integrated crop-livestock-fish systems that minimize dependence on external inputs. The optimal solution translates directly into land-use plans, herd and flock sizes, and aquaculture pond areas that can be used as a baseline for farm budgeting and investment decisions. For policymakers, the findings illustrate how integrated farming programmes could prioritize configurations that simultaneously achieve cost reduction and local resource circularity, rather than subsidizing isolated sub-sectors.

Nevertheless, several limitations must be acknowledged. The current formulation is deterministic: labor availability, input qualities, climate, and prices are assumed to be fixed, whereas real-world conditions are inherently uncertain. In addition, the objective function focuses on operational cost without explicitly quantifying environmental outcomes such as greenhouse gas emissions, nutrient losses, or water pollution, which are central in recent circular agriculture studies (Hang et al., 2021; Yue et al., 2022). Future work should therefore extend the MILP to incorporate stochastic parameters and multi-objective or goal-programming structures, enabling formal analysis of trade-offs among profit, self-sufficiency, and environmental performance. Scaling the model to larger farm clusters or cooperatives, and integrating renewable energy options such as biogas from livestock waste, would further enhance its practical relevance and position it more directly within the broader family of circular agriculture optimisation models. While the present study reports numerical results for two planning periods, the proposed MILP framework is readily extensible to longer multi-period horizons, including the full three-period case.

Conclusion

This study responds to the scarcity of integrated mathematical models that explicitly optimise multi-period operational costs in small, multi-component farming systems operating under binding resource constraints, a topic that remains largely overlooked in the agricultural optimisation literature, particularly in the context of integrated farming in Indonesia. To address this gap, we developed and implemented a mixed-integer linear programming model for a crop-livestock-fish system and used it to identify a cost-efficient combination of land use, labour allocation, and internally sourced inputs. The optimal solution maintains stable production of maize, grass, cattle, poultry, and fish over two planning periods while still satisfying strict limits on land and labour. All needs for manure, feed, and irrigation water are met through the internal reuse of crop residues, livestock manure, and pond sludge, yielding a high level of input self-sufficiency and an operational cost profile that is feasible for small to medium rural enterprises.

The principal contribution of this research is a reproducible quantitative framework that connects operational cost minimisation with internal resource recycling in an integrated farming system, thereby offering concrete evidence of how circular economy concepts can be implemented at the farm level. In practical terms, the model can function as a decision-support tool for smallholder farmers, extension workers, and policymakers who aim to improve cost efficiency while lowering reliance on external inputs in rural development programmes. At the same time, the current formulation is deterministic and does not explicitly account for price fluctuations, climatic variability, or environmental outcomes such as greenhouse gas emissions and nutrient losses. Future work should extend the model to stochastic or dynamic settings and introduce multi-objective criteria that simultaneously consider profitability, self-sufficiency, and environmental performance, in order to strengthen its robustness and policy relevance.

Acknowledgement

The authors would like to express their sincere gratitude to the Institute for Research and Community Service (LPPM) and the Rector of Universitas Negeri Medan for their continuous support and facilitation throughout the implementation of this study.

Declarations

Author Contribution	:	LPS: Conceptualization, Formal Analysis, Funding Acquisition, Methodology, Project Administration, Resources, Supervision, Validation, Writing-Original Draft. RH: Conceptualization, Data Curation, Software, Validation, Writing-Review & Editing. SP: Investigation, Visualization, Validation, Writing-Review & Editing.
Funding Statement	:	This research was funded by the 2025 PNBP Research Grant of Universitas Negeri Medan under Rector's Decree No. 0194/UN33/KPT/2025 and Research Contract No. 0064/UN33.8/PPKM/PD/2025.
Conflict of Interest	:	The authors declare no conflict of interest.
Additional Information	:	Additional information is available for this paper.

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