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The analysis of students' ability to solve HOTS problems on the topic of sequences and series according to the Krulik and Rudnick stages

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Abstract
This study aimed to analyze the problem-solving abilities of low-performing students in addressing Higher Order Thinking Skills (HOTS) questions on sequences and series based on the stages proposed by Krulik and Rudnick. A descriptive qualitative approach was employed in this research. Thirty senior high school students completed a problem-solving test, from which two students who obtained the lowest scores were purposively selected for in-depth analysis and semi-structured interviews. Data were collected through written tests and interviews and analyzed using Miles' stages of qualitative data analysis. The results indicated that at the read and think stage, students were able to identify relevant information but demonstrated a limited understanding of the relationships among the given data. At the explore and plan stage, students attempted to organize information but encountered difficulties, particularly when dealing with sequence-related problems. At the select a strategy stage, students identified the appropriate formula; however, they showed hesitation in applying it correctly. At the find an answer stage, students performed computational procedures but often produced inaccurate results. Finally, at the reflect and extend stage, students tended to draw conclusions without verifying the correctness of their solutions. Overall, the main difficulties were evident in conceptual understanding, consistency in strategy application, and reflective evaluation.

Keywords: High Order Thinking Skills (HOTS), Krulik and Rudnick problem-solving stage, Problem-solving skills, Sequences and series

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Introduction

Problem-solving skills are a very important aspect in education, and every student needs to have these skills in order to face various challenges, especially in mathematics learning (Hobri, 2020). Mathematical problem solving is a process in which students use intellectual activities to find solutions to a problem, where teachers are required to provide a place and opportunity for students to innovate (Xu & Qi, 2022). Problem-solving indicators include understanding the problem, designing a problem-solving strategy, and solving the problem (Adelia et al., 2020). One theory used to analyze problem-solving abilities is Krulik and

Rudnick's theory (1995), which consists of five stages of problem solving, namely read and think, explore and plan, select a strategy, find an answer, and reflect and extend. These stages by Krulik and Rudnick are not independent and sequential, but each stage has a different goal that can be achieved through the use of relevant sub-skills. To support students with low problem-solving skills, it is necessary to identify the obstacles faced by students in solving story problems involving HOTS (Higher Order Thinking Skills) (Suseelan et al., 2023).

Problem-solving skills refer to students' ability to solve complex and unusual problems (Rambe and Afri, 2020). Higher Order Thinking Skills (HOTS) are complex and gradual thinking processes to find solutions in problem solving (Rahayu et al., 2020). In addition to HOTS questions, non-routine questions can also help students develop problem-solving skills, where solving them requires unusual methods or strategies (Keleş & Yazgan, 2025). Non-routine questions are questions that require deeper thinking because the procedures are not as clear or the same as the procedures learned in class (Aisyah, 2008, dalam Suardi et al., 2023). PISA-standard questions (such as HOTS and non-routine) are able to measure students' problem-solving skills, not just their ability to solve ordinary problems (Bidasari, 2017). However, in reality, the mathematical problem-solving skills of students in Indonesia are still low. The 2022 PISA (Programme for International Student Assessment) results show that Indonesia's mathematics literacy score was 366, lower than in PISA 2018 (OECD, 2023).

Previous research related to problem-solving ability analysis, namely the study by Sesa et al., (2022) showed that students with high problem-solving abilities based on the Krulik and Rudnick criteria met five indicators, those in the medium category met three indicators, and those in the low category met one indicator, with the categorization of students' problem-solving abilities using score intervals. Unlike Sesa et al., (2022), this study aims to analyze the problem-solving abilities of students in the low category only but in depth to solve HOTS problems with categorization using standard deviation. This is based on the 2022 PISA results, where Indonesian students' scores in mathematics are still relatively low. This research is important to provide an overview of how students with low problem-solving abilities solve HOTS problems.

Methods

The type of research used is descriptive research with a qualitative approach. Data collection was conducted in June-July 2024 at SHS 2 Bondowoso with 30 students from class X1 A1 as the prospective research subjects. Two students with the lowest problem-solving abilities were selected as research subjects. The reason for selecting students with low abilities was that the researcher wanted to analyze their weaknesses so that teachers could later help design good learning strategies to improve their students' mathematical problem-solving abilities.

The research began with preliminary observations, developing a research design, and coordinating with school teachers to determine the time and prospective research subjects. Next, instruments were developed, including test sheets and interview guidelines, which were then validated by two expert validators with instrument results of 3.67 and 3.8 (valid), respectively. Data collection was conducted through HOTS-type problem-solving tests on sequences and series with three descriptive questions, as shown in Table 1. The reason for

choosing sequences and series was that this material could be related to contextual problems and could be solved using various methods, requiring students to have good problem-solving skills and higher-order thinking skills (HOTS) to obtain the correct answers (Maryati & Fadhilah, 2021).

Table 1. HOTS Problem Solving Test Questions

No.	Questions	HOTS Question Types
1	Pada saat jam pelajaran olahraga, peserta didik diminta untuk berbaris memanjang ke belakang. Julia merupakan nama salah satu anak yang ikut berbaris. Peserta didik diminta menyebutkan bilangan ganjil dari barisan depan ke belakang dimulai dari 5, 7, 9, … dan Julia menyebut bilangan 45. Peserta didik diminta menyebutkan bilangan genap secara berurutan dari belakang ke depan dimulai dari 2, 4, 6, … dan Julia menyebut bilangan 46. Berapa banyak anak yang ada dalam barisan tersebut? Translated version: During a physical education class, students were asked to line up in a single row from front to back. Julia was one of the students in the line. The students were instructed to mention odd numbers sequentially from the front to the back, starting with 5, 7, 9, …, and Julia mentioned the number 45. The students were then instructed to mention even numbers sequentially from the back to the front, starting with 2, 4, 6, …, and Julia mentioned the number 46. How many students are there in the line?	C4
2	Eric melamar kerja sebagai barista di dua *café* berbeda yang menawarkan gaji berbeda pula. Pada *café* pertama, gaji yang ditawarkan sebesar $Rp1.500.000{,}00/bulan$ dan akan bertambah $Rp110.000{,}00$ setiap bulannya namun dengan syarat bahwa Eric harus bekerja selama 2 tahun terlebih dahulu. Pada *café* kedua, gaji yang ditawarkan tetap yaitu $Rp2.250.000{,}00/bulan$. Berdasarkan kedua pilihan tersebut, manakah pilihan terbaik yang harus dipilih Eric agar ia mendapatkan gaji yang maksimal jika Eric bekerja selama 4 tahun lamanya? Translated version: Eric applied for a barista position at two different cafés, each offering a different salary scheme. At the first café, the initial monthly salary offered is IDR 1,500,000 and increases by IDR 110,000 each month, provided that Eric works there for at least two years. At the second café, the salary offered is a fixed amount of IDR 2,250,000 per month. Based on these two options, which choice should Eric select in order to obtain the maximum total salary if he works for four years?	C5
3	Giselle dan Karina akan menonton drama musikal di auditorium kampus. Bangunan auditorium tersebut umumnya dibuat melengkung seperti Gambar 1. Jumlah maksimal barisan kursi pada auditorium tersebut sebanyak 10 barisan ke belakang dan banyaknya kursi setiap baris tidak sama. Jumlah kapasitas maksimal penonton sebanyak 710 penonton. Jika kursi setiap baris mengalami penambahan secara tetap, tentukan banyak kursi pada setiap baris yang dapat disusun dalam auditorium jika minimal kursi pada barisan pertama sebanyak 20 kursi!  Translated version: Giselle and Karina are going to watch a musical drama at the campus auditorium. The auditorium is generally designed in a curved shape, as shown in Figure 1. The maximum number of seating rows in the auditorium is 10 rows extending toward the back, and the number of seats in each row is not the same. The maximum seating capacity of the auditorium is 710 spectators. If the number of seats in each row increases by a constant amount, determine the number of seats in each row that can be arranged in the auditorium, given that the minimum number of seats in the first row is 20!	C6

The tests given to students were scored using an assessment rubric based on the Krulik and Rudnick stages, with a total of 15 items. Based on their scores, students were grouped into three categories of problem-solving ability (Table 2), and the two students with the lowest

scores were selected as research subjects. The test results of the 2 subjects were then analyzed according to the indicators and descriptions of problem solving by Krulik and Rudnick (Table 3), followed by structured interviews to explore the problem-solving process that was not obtained through the test. The validity of the test and interview data was tested using the member check technique, where students were given back the test and interview results for verification. After validity testing, the data was then presented as the basis for drawing conclusions.

Table 2. Problem Solving Skills Category

Category	Achievement of Problem-Solving Skills
High	$x \geq (\bar{x} + SD)$
Medium	$(\bar{x} - SD) \leq x < (\bar{x} + SD)$
Low	$x < (\bar{x} - SD)$

Source: Azwar (2009)

Description:

x : Students' mathematical problem-solving skills

$\bar{x}$: Average students score

SD : Standard Deviation

Table 3. Problem Solving Indicators and Descriptions Krulik and Rudnick

Problem-solving stages	Problem Solving Indicators	Research Problem Solving Descriptors	Code
Read and Think	Rewrite the information they know and ask questions based on their own understanding and words.	• Identify important information in the form of what is known in the question	1a
		• Identify the questions in the question	1b
		• Representation of the problem	1c
Explore and Plan	Seeking the information needed to develop an initial plan for solving the problem	• Organizing information	2a
		• Finding sufficient information (needed or unnecessary information)	2b
Select A Strategy	Write down and present the steps to solve the problem, which may involve selecting a formula.	• Selecting a formula or problem-solving strategy	3a
		• Writing down the steps to solve the problem	3b
		Sub-descriptors: • Pattern recognition • Working backwards • Guessing and testing • Simulation or experimentation • Reduction/expansion • Organized list/exhaustive list • Logical deduction • Divide and conquer	
Find An Answer	Performing troubleshooting procedures and obtaining results or solutions	• Estimating	4a
		• Using computational skills	4b
		• Using algebraic skills	4c
Reflect and Extend	Review each calculation process and draw a final conclusion.	• Checking answers	5a
		• Finding alternative solutions	5b

Modified from (Ruliani et al., 2018)

Results

The test data analysis results were then scored and categorized. The categorization of problem-solving ability test results was divided into high, medium, and low categories. The results of the problem-solving ability categorization are shown in Table 4.

Table 4. Problem-Solving Ability Categorization Results

No.	Ability Category Problem Solving	Number of Students	Achievement of Problem-Solving Skills
1	High	5	$x \geq 64$
2	Medium	20	$49 \leq x < 64$
3	Low	5	$x < 49$

The results of the problem-solving ability categorization showed that 66.67% of students had problem-solving abilities in the moderate and high categories, while 16.67% of students were in the low category. Of the five students with low problem-solving abilities, the two students with the lowest scores were selected for analysis. The research subject codes can be seen in Table 5.

Table 5. Research Subject Code

No.	Code	Problem-Solving Ability Score	Problem-Solving Ability Category
1	R1	46	Low
2	R2	44	Low

Both subjects have characteristics that tend to be different. The characteristics of subject R1 include being communicative but long-winded during interviews, structured test results, lacking self-confidence, and being indecisive. The characteristics of subject R2 include being less communicative but more to the point, less structured, lacking confidence in test and interview results, and being indecisive.

Results of data analysis of students' problem-solving abilities in the read and think stages

The first stage is read and think with indicators of rewriting known and asked information according to their understanding and in their own words. Descriptors at this stage include identifying important information in the form of what is known in the question (code: 1a), identifying questions in the question (code: 1b), and representing problems (code: 1c).

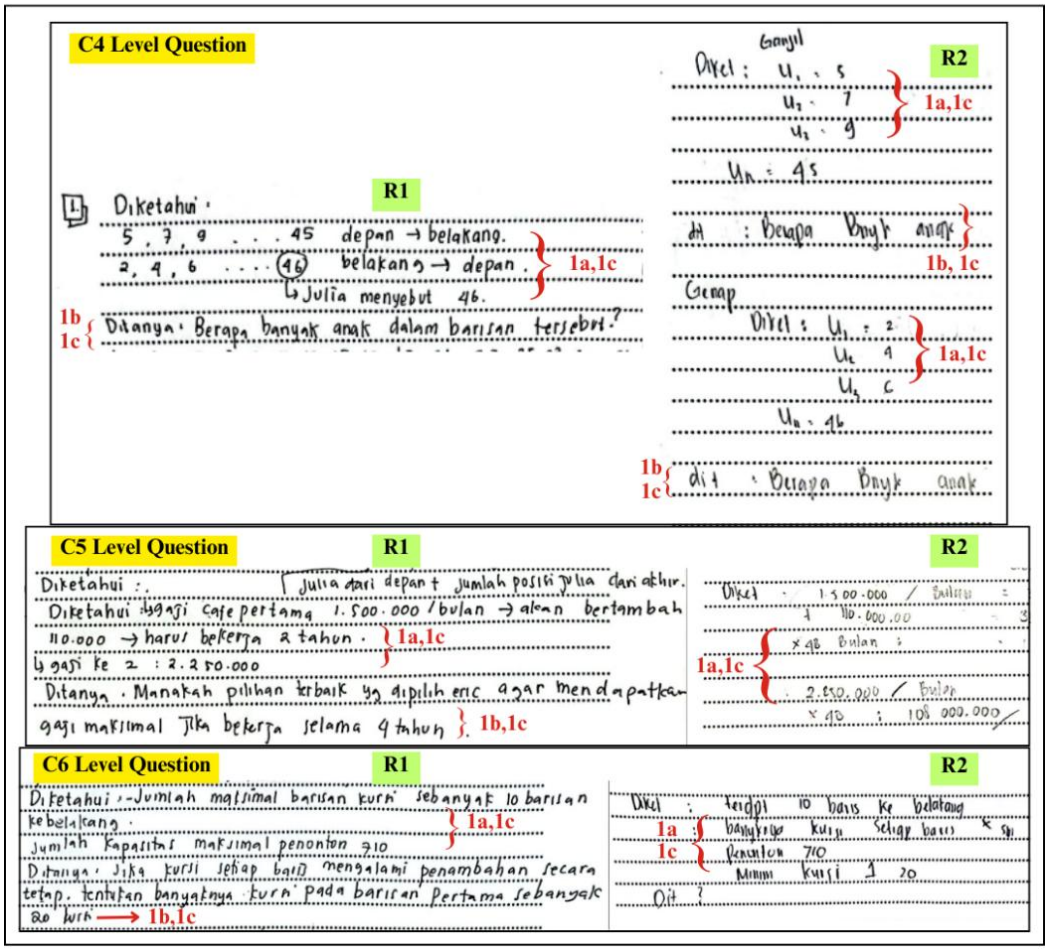

Figure 1. R1 & R2 Test Results in the Reading and Thinking Stages

Translation of Figure 1:
In Figure 1, subjects R1 and R2 wrote down what they knew (*diketahui*) with code 1a and what they were asked in the questions (*ditanya*) with code 1b. In both sections, the subjects represented their information based on their own understanding (code 1c).

Based on Figure 1, subjects R1 (left) and R2 (right) identified important information and questions in the question by representing them verbally and symbolically. In the analysis question (C4), both subjects represented important information and questions verbally according to their respective understanding, for example, by finding the number of children in the row.

The following is an excerpt from the interview with R2:

P : From question number 1, how did you identify what was known and what was asked in the question?

R2 : I read it first, then wrote down what was known from the question, such as the odd and even numbers in the sequence. After that, I wrote down what was asked in the question, which was how many children were in the sequence.

In evaluating (C5) and creating (C6) questions, both subjects represent important information and questions verbally, for example by summarizing important information known in the question and restating the question. From this, it can be concluded that R1 and R2 can

identify important information, questions, and represent problems in questions at the analyzing (C4), evaluating (C5), and creating (C6) levels.

Results of data analysis of students' problem-solving abilities in the explore and plan stages

The second stage is the explore and plan stage with indicators of seek the information needed to develop an initial plan for solving the problem. The descriptors in this stage include organizing information marked with code 2a and seeking sufficient information marked with code 2b. Based on Figure 2 below, R1 organizes information by sorting the numbers in the front-back and back-front sequences marked with code 2a in the analyze level question (C4). R2 can organize information by restating the values of $U_1, U_2,$ *and* U_3 as well as the value of U_n in each front-back (odd) and back-front (even) sequence marked with code 2a. R1 and R2 seek sufficient information by finding the values of a and b marked with code 2b.

In the evaluation level questions (C5), both subjects were unable to organize information and find sufficient information, whereas in the creation level questions (C6), both subjects were able to organize information and find sufficient information.

The following is an excerpt from the R1 interview.

P : How did you find information and develop an initial plan for solving problem number 3?

R1 : Here, I initially looked for the value of a first, but it turned out to be more difficult, so I tried to find the value of b first. Initially, I tried a value of b equal to 12, but the result of a was less than 20. Therefore, I tried with a smaller number, and it turned out that if the value of b was smaller, the result of a became larger and met the requirement of a minimum of 20 seats.

From this, it can be concluded that R1 and R2 can organize information and find sufficient information for problems at the analyzing (C4) and creating (C6) levels, but cannot organize information and find sufficient information for problems at the evaluating (C5) level.

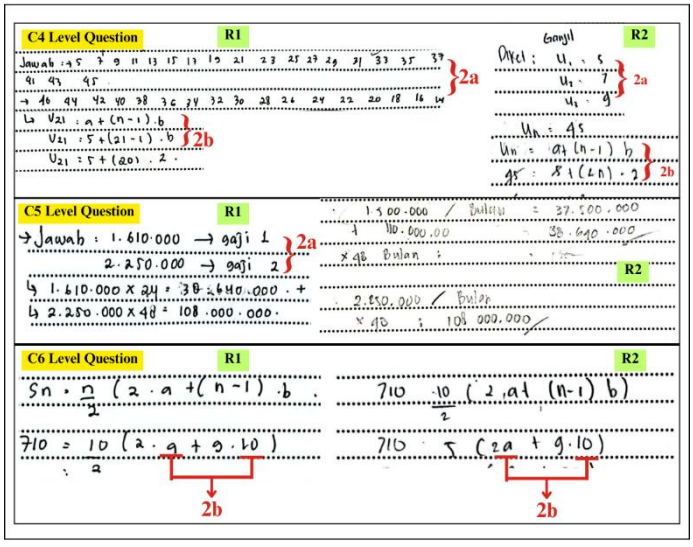

Figure 2. R1&R2 Test Results in the Explore and Plan Stages

Translation of Figure 2:
In Figure 2, subjects R1 and R2 organized information (marked with code 2a) and searched for sufficient information (marked with code 2b).

Results of data analysis of students' problem-solving abilities in the select a strategy stages

The third stage is select a strategy with indicators of describe and present steps to solve the problem, which may involve selecting a formula. The descriptors at this stage select a formula or problem-solving strategy marked with code 3a or describe the steps to solve the problem with code 3b. In Figure 3, regarding analyzing (C4), both subjects were able to choose the correct formula, but there was uncertainty in both subjects, where both asked their friends to confirm whether the formula they chose was appropriate for the question or not.

The following is an excerpt from the R2 interview.

P : How did you choose the formula or strategy you would use to solve question number 2?

R2 : I did not use a formula for question number 2, so I calculated it manually.

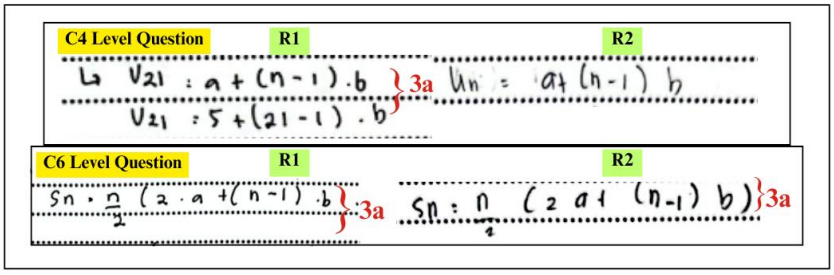

Figure 3. R1&R2 Test Results in the Select A Strategy Stages

Translation of Figure 3:
In Figure 3, subjects R1 and R2 chose the relevant formula marked with code 3a. The figure only shows the answers for levels C4 and C6 because at level C5, both subjects were unable to choose a formula (they did not use any formula).

In evaluating question C5, neither R1 nor R2 wrote down the formula or strategy steps on their answer sheets because they forgot which formula to use, resulting in the wrong strategy being chosen. Both subjects worked on the question manually. In solving question C5, the subjects should have used the S_n formula to calculate the salary at café 1. In the creation question (C6), R1 was able to choose the formula correctly on his own, while R2 was unable to choose the correct formula. In the interview, R2 mentioned that he did not know the correct formula to use, so he had to ask his friend. From this, it can be concluded that both subjects in the analysis (C4), evaluation (C5), and creation (C6) questions were not yet fully able to choose the correct and appropriate formula because they had doubts in choosing the formula. However, even though they were hesitant in choosing the formula, they were still able to apply the formula to get the answer.

Results of data analysis of students' problem-solving abilities in the find an aswer stages

The fourth stage is the stage of finding an answer. Indicators at this stage perform problem-solving procedures and obtain results or solutions. Descriptors at this stage include estimating/calculating, marked with code 4a, using computational skills with code 4b, and using algebraic skills with code 4c. Based on Figure 4 of the analysis question (C4), R1 estimates Julia's position in the odd and even rows, marked with code 4a. R1 and R2 can perform computational skills by calculating the answer, marked with code 4b. R1 and R2 can also perform algebraic skills with the ability to operate symbols, marked with code 4c. Both subjects find the value of the sequence for the front-back and back-front sequences, but only R1 adds up the final result, which is 43 children.

In the evaluation question (C5), R1 and R2 estimate the salary at café 1 for 4 years with an additional bonus of Rp110,000.00, on the condition that they must work for 2 years first. R1 and R2 can use computational skills, where R1 calculates the difference in salary between cafés 1 and 2, marked with code 4b. R1 and R2 cannot use algebraic skills because neither of them uses arithmetic formulas, so there are no variables to operate on. Both subjects calculated the total salary for Café 1 manually without using a formula, whereas students should have used the S_n formula to calculate the total salary. From this, it can be concluded that R1 and R2 can estimate and use computational skills, but cannot use algebraic skills because there are no variables to operate on in question C5.

In the creation question (C6), R1 and R2 did not estimate. R1 and R2 can perform computational skills by calculating the final result marked with code 4b. R1 and R2 can also use algebraic skills by operating symbols marked with code 4c. R1 and R2 find the value of the initial sequence using the arithmetic sequence formula and assume the difference in the sequence to be any number, resulting in a = 26 with b = 10. However, after finding the values of a and b, the subjects should continue to find the number of seats in each sequence from 1 to 10, so the answer to question C6 is not entirely correct.

Here is an excerpt from the interview with R1:

P : Explain the steps you used to solve question number 3!

R1 : At first, I used the sequence formula, but it didn't work. Finally, I tried using the series formula. Then, I obtained the value Sn = 710 and n = 10. Because a was being sought, I first assumed the value of b to be b = 10. From there, I obtained the value of a to be 26. After finding the value of a, I entered it again to find the value of Sn. By entering n = 10, a = 26, and b = 10, I obtained S10 to be 710.

From this, it can be concluded that R1 and R2 can find an answer, but the answer obtained is incomplete and not fully resolved. Additionally, both subjects can perform computational skills well on questions at levels C4, C5, and C6.

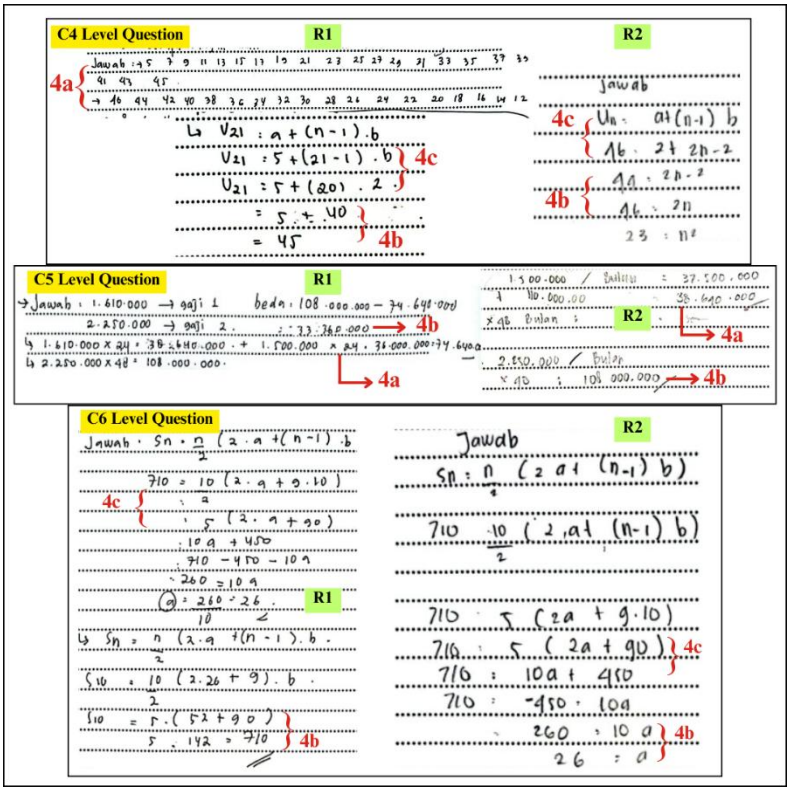

Figure 4. R1&R2 Test Results in the Find An Answer Stages

Translation of Figure 4:
In Figure 4, subjects R1 and R2 in questions at levels C4, C5, and C6 estimated answers marked with code 4a, performed computational skills marked with code 4b, and performed algebraic skills marked with code 4c.

Results of data analysis of students' problem-solving abilities in the reflect and extend stages

The fifth stage is the stage of describing and communicating. The indicators at this stage are reviewing each calculation process and giving a final conclusion. The descriptors at this stage include checking answers marked with code 5a and finding alternative solutions with code 5b. Based on Figure 5 in the analysis question (C4), R1 and R2 rechecked the answers marked with code 5a. R1 was able to draw a conclusion by adding up the number of children in each row, resulting in a final total of 43 children for the total number of children in that row. R2 only concludes that there are 21 children in the even rows and 23 children in the odd rows without adding the two to get the correct final result. In the interview, R1 explains that there are other alternative solutions that can be used, such as the arithmetic sequence formula, while R2 does not have any other alternative solutions.

On the evaluation question (C5), R1 and R2 were able to draw correct and accurate conclusions and check the answers. In addition, only R1 was able to find another alternative solution to question C5.

The following is an excerpt from R1's interview:

P : Are there any other alternative solutions/strategies for solving the problem? If so, please explain!

R1 : It should be possible to use arithmetic, but I still don't know how.

In the creation question (C6), R1 and R2 were able to draw correct but incomplete conclusions. In addition, R1 and R2 checked the answers, but only R1 was able to find alternative solutions. From this, it can be concluded that only R1 checked the answers and found alternative solutions.

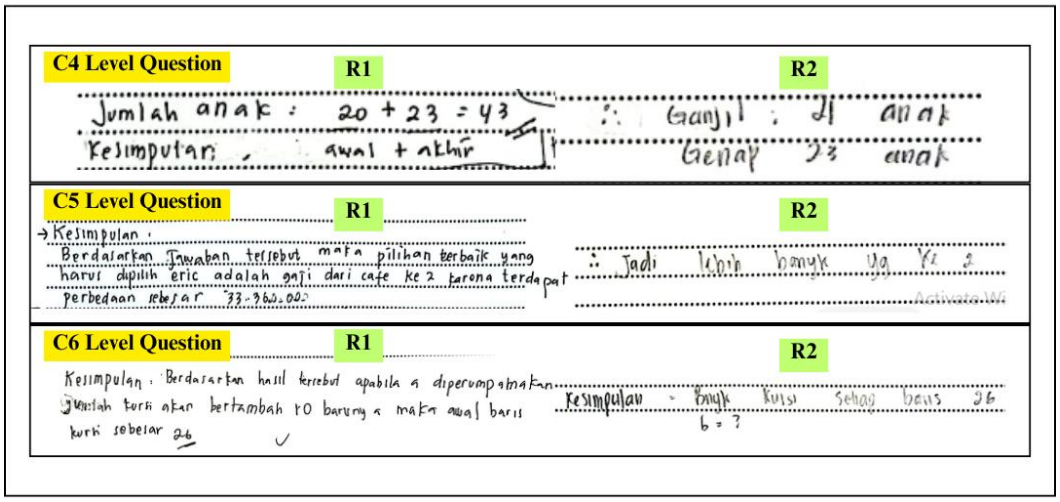

Figure 5. R1&R2 Test Results in the Reflect and Extend Stages

Translation of Figure 5:
In Figure 5, subjects R1 and R2 made conclusions (kesimpulan) at all three levels of questions. Codes 5a and 5b can only be done during the interview stage.

Discussion

The results of categorizing problem-solving abilities in this study show that 5 students are in the high category, 20 students are in the medium category, and 5 students are in the low category. This distribution shows that the majority of students are in the medium category, in line with the findings of Putri et al., (2018), Amaliah et al., (2021), dan Salvia et al., (2022) who reported a similar distribution, namely that most students have medium problem-solving skills. However, this study focuses on students with low problem-solving abilities to enable a more in-depth analysis, particularly in identifying the obstacles they face in solving problems in order to provide input for improving the learning design.

The results of the data analysis of students with low problem-solving abilities at the five stages can be seen in Table 6.

Table 6. Similarities and Differences Between the Two Subjects in Problem Solving

Krulik and Rudnick's Problem-Solving Stages	Similarities and Differences	Level C4	Level C5	Level C6	Conclusion
Read and Think	Similarities	Can identify important information, can identify questions, and can represent problems	Can identify important information, can identify questions, and can represent problems	Can identify important information, can identify questions, and can represent problems	- Can identify important information - Can identify questions - Can represent problems
	Differences	R1 represents important information verbally, while R2 represents it symbolically.	-	-	Both subjects represent important information in different ways.
Explore and Plan	Similarities	Can organize information and find sufficient information	Unable to find sufficient information	Can organize information and find sufficient information	Can organize information and find sufficient information on questions at levels C4 and C6.
	Differences	R2 still has doubts about organizing and searching for information because students ask their friends.	R1 controls the information, while R2 does not control the information and leaves it random.	-	R2 is not yet fully capable of organizing information and searching for sufficient information.
Select A Strategy	Similarities	Can choose a formula correctly and appropriately	Unable to select a formula	Can choose a formula correctly and appropriately	Can correctly and accurately select a formula for questions at levels C4 and C6
	Differences	-	-	Can choose formulas but with different capabilities because R2 cannot choose its own formula	R2 can choose a formula correctly but with low ability
Find An Answer	Similarities	Estimating, able to use algebraic skills, able to use computational skills, able to find the correct answer	Estimating, can use computational skills, cannot use algebraic skills, can find the answer but it is not quite right.	Does not estimate, can use algebraic skills, can use computational skills, can find answers but they are incomplete.	- Can use computational skills - Can find an answer
	Differences	- The final results differ in that the R2 result is less accurate because the subjects did not complete the task - R1 can be estimated, while R2 cannot	-	-	- Both subjects can find an answer but with different abilities - R2 does not estimate question C4
Reflect and Extend	Similarities	Can draw a conclusion but with different results	Can draw correct and accurate conclusions	Can draw conclusions but they are incomplete	Can draw a conclusion, but it is not entirely accurate.
	Differences	R1 draws the correct conclusion, checks the answer, and finds alternative solutions, while R2's conclusion is inaccurate, does not check the answer, and does not find alternative solutions.	R1 can find other alternative solutions and check the answers, while R2 cannot.	R1 can check the answers and find other alternative solutions, while R2 cannot.	- R2 is inaccurate in drawing conclusions - R2 does not check answers - R2 does not find other alternative solutions

The results of the study show that students with low problem-solving skills can only identify important information in the early stages, which include the reading and thinking stages. They have difficulty organizing important information and developing effective

strategies during the investigation and planning stages. These findings are in line with the research by Suryaningsih (2019) and Laila Sulistiowati (2022), which highlight students' weaknesses in identifying important information. Therefore, this study confirms that the limitations of students in the low category consistently appear in the planning and strategy selection stages. In order to design strategies, students need a basic understanding and skills in the problem-solving process (Kaitera & Harmoinen, 2022).

Students with low problem-solving skills also show limitations in finding and concluding answers. Several factors that influence this include errors in calculation, inability to write conclusions, not double-checking answers, and lack of habit in checking or proving the correctness of answers (Sofia et al., 2021). Students are often only able to carry out the initial stages of problem solving and experience difficulties in carrying out or continuing to the next stage. This can hinder students' ability to solve problems effectively.

This analysis can be better understood through the problem-solving theory framework. According to Polya (1945), the main weaknesses of low-category students are seen in the stages of understanding the problem and making a plan. Based on Schoenfeld's (1985) model, student problems include resource limitations (limited knowledge), heuristics (lack of alternative strategies), and control (little practice in monitoring and reviewing their processes). Therefore, student weaknesses lie not only in technical skills but also in the areas of metacognition and self-regulation.

The uniqueness of this study, compared to Sesa et al. (2022) and similar studies, lies in the detailed mapping of low problem-solving ability students' responses at each stage of problem solving. An important finding is the variation within the low category group; for example, only subject R1 was able to review their answers and consider alternative solutions. This shows that, despite being categorized as having low problem-solving abilities, there are differences in the quality of problem solving that have not been highlighted in previous studies. These findings have important implications for learning practices. Teachers can apply PISA-based mathematical problem-solving questions in their teaching because studies show that higher-order thinking skills have been proven to improve students' problem-solving abilities (Mashuri & Jahring, 2024). In addition, teachers can also provide formative feedback not only on the final answer, but also on the students' thinking process. With this strategy, it is hoped that the problem-solving abilities of low-category students can be trained more systematically and continuously.

Conclusion

Based on the results of data analysis after using HOTS questions to analyze problem-solving abilities, several important insights were obtained. First, students with lower problem-solving abilities were able to identify important information and represent problems verbally and symbolically, but had difficulty connecting important information with relevant concepts. Second, during the investigation and planning stages, patterns of failure varied depending on the level of difficulty of the questions: students were generally able to organize information in questions C4 and C6, but experienced difficulties in question C5 due to their inability to sort relevant information. This shows that the challenges faced by students are not only related to the complexity of the question, but also how they interpret a problem. Third, in the strategy

selection stage, students often show uncertainty and tend to rely on peers for help, highlighting limitations in self-regulation and self-confidence. Fourth, at the stage of finding an answer, basic computational skills can still be applied, but the results are often incomplete or inaccurate due to a lack of reflection on the process. Fifth, at the stage of describing and communicating, students are able to draw conclusions even if they are not entirely correct, and often skip the steps of double-checking and searching for alternative solutions.

The contribution of this study lies in mapping the thinking patterns of low-performing students at each stage of problem solving, while highlighting the diversity within the low-performing student group; for example, only a small proportion of students were able to review their answers, something that has not been widely discussed in previous studies. The results show that the challenges faced by students stem not only from conceptual difficulties, but also involve cognitive, affective, and learning strategy factors, thereby clarifying the role of these three factors in influencing their problem-solving abilities. Further research is recommended involving more subjects to improve generalization and compare the problem-solving patterns of low- and high-performing students.

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Declarations

Author Contribution : ARAG: Conceptualization, Writing - Original Draft, Editing and Visualization.
NDSL: Writing - Review & Editing, Formal analysis and Methodology.
EO: Writing - Review & Editing, Formal analysis and Methodology.
DT: Supervision, Review & Editing.
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