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The development of hypothetical learning trajectory (HLT) based on problem-based learning (PBL) in the concept of relations and functions

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Abstract
This study aims to develop a learning process that aligns with the diverse thinking processes of students in Grade VIII at MTsN 9 Padang Pariaman. This is a developmental research study using the Gravemeijer and Cobb model. This development model consists of three stages: 1) Preparing for the experiment, which involves conducting a needs analysis (such as analyzing students and educators, syllabus analysis, literature review, and concept analysis), designing the HLT, and assessing the HLT (self-evaluation and expert review); 2) Conducting the experiment, which includes product trials with students at MTsN 9 Padang Pariaman through one-to-one evaluation and small group evaluation; 3) The retrospective analysis, which involves analyzing data obtained from the previous stages. The results of this analysis are used to plan activities and refine the design of future learning activities. The results of this study indicate that the developed HLT is highly valid, with an average validity score of 89.71%. This research demonstrates that an HLT based on Problem-Based Learning can be validly used in mathematics learning, specifically in the topic of relations and functions.

Keywords: Hypothetical learning trajectory, Problem-based learning, Relations and functions

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Introduction

Mathematics plays a critical role in the context of education. A comprehensive understanding and mastery of mathematics not only provide direct academic benefits but also have far-reaching implications in everyday life. Mathematics learning is a structured process developed by educators to enhance creative thinking and foster students' ability to construct new knowledge (Eliza et al., 2018). However, empirical evidence indicates that numerous challenges persist in mathematics education, such as students' inability to independently and critically solve mathematical problems. Many students also demonstrate a lack of interest in mathematics and actively avoid engaging with the subject. Such issues necessitate scientific methods to address them, specifically systematic approaches designed to resolve encountered problems (Ridha, 2017).

One of the fundamental topics in mathematics education is relations and functions. This topic is notoriously challenging for many students to comprehend (Agustin & Ruli, 2023; Anggreni et al., 2020; Hutagaol et al., 2022; Muhammad et al., 2022; Yanti et al., 2019). Learning difficulties pose a significant barrier that prevents students from effectively following lessons. Many students struggle to differentiate between relations and functions and to apply these concepts in mathematical problem-solving (Rosidah & Hasanah, 2019). According to (Raharjo & Christanti, 2020) students experience difficulties in understanding relations when using arrow diagrams, Cartesian diagrams, and ordered sets. Similarly, in the study of functions, students struggle with identifying appropriate procedural steps, leading to confusion in distinguishing between relations, functions, and non-functions.

One of the factors contributing to the perception that this subject is difficult for students at MTsN 9 Padang Pariaman is the abstract nature of how relations and functions are taught. As a result, students struggle to grasp these concepts effectively. The learning trajectory employed by educators has not yet facilitated students' understanding of relations and functions. Teachers primarily rely on the structured learning sequence provided in textbooks supplied by the school, without independently designing how the material should be delivered. Consequently, mathematics learning becomes less meaningful, previously learned mathematical concepts are easily forgotten, and students' cognitive development remains stagnant. This issue has led to mathematics instruction in schools falling short of expectations, as evidenced by the low academic performance of many students in mathematics (Muthmainnah & Purnamasari, 2019; Siagian, 2016; Woi & Prihatni, 2019).

This situation is inconsistent with the notion that educators should strategically select teaching methods that actively engage students in learning—mentally, physically, and socially—while fostering enthusiasm and enjoyment in mathematics (Khaidir, 2016). Research conducted by (Gouw, 2012) reveals that teachers frequently rely on lecture-based instruction, which results in passive classroom dynamics, as the teacher is perceived as the sole source of knowledge. When instruction strictly follows the textbook's learning sequence, the process becomes teacher-centered. Although students participate in group learning, the interaction during discussions remains suboptimal, meaning that students remain passive despite the group-based approach.

Designing a structured learning trajectory is a potential solution to these challenges. However, planning an ideal learning experience that is easily understood by students is not straightforward. Often, lesson plans (RPP) do not align with actual classroom implementation, as students' responses can be unpredictable. To enhance the quality of mathematics instruction, particularly in relations and functions, educators can develop a learning trajectory, commonly referred to as a Hypothetical Learning Trajectory (HLT), using the Problem-Based Learning (PBL) model.

(Simon, 1995) introduced HLT as a critical component of pedagogical thinking related to mathematical understanding. Specifically, mathematics educators—including curriculum developers, teachers, researchers, and instructional designers—can design and implement mathematical tasks based on constructivist perspectives and learning objectives. Furthermore, (Simon, 2004) explain that HLT is grounded in current knowledge about student engagement in learning. HLT serves as a tool for planning instruction on specific mathematical concepts.

Developmental research on learning trajectories has been extensively explored by previous scholars. For instance, (Aklimawati et al., 2022) examined HLT as a means to streamline the delivery of instructional materials, ensuring that students can comprehend them effectively. By understanding students' learning trajectories, educators can identify key aspects that must be considered to enable students to construct knowledge effectively, leading to meaningful learning experiences. Ultimately, HLT allows educators to enhance the quality of instruction, whether through teaching methods, instructional materials, or pedagogical strategies.

Hypothetical Learning Trajectory (HLT) is a learning trajectory that consists of three key components: learning objectives, a series of tasks, and hypotheses regarding students' thinking and learning processes. The learning objectives refer to the expected conceptual understanding of mathematics. The tasks are designed to assess students' thought processes, while the hypothesis pertains to students' cognitive pathways in understanding mathematical concepts (Fuadiah, 2017).

Several studies have demonstrated that HLT can enhance students' comprehension. Learning trajectories support educators in restructuring their mathematical knowledge for teaching purposes. HLT enables teachers to identify and understand students' thinking patterns, allowing them to anticipate students' actions and responses during mathematical problem-solving and their engagement in subsequent tasks (Wilson et al., 2013). A well-structured learning trajectory provides students with opportunities to discover mathematical models, and with guidance from educators, they can successfully develop these models independently (Lede & Kii, 2018). HLT facilitates students in transitioning from informal to formal mathematical reasoning while fostering interactive engagement between students and educators (Yulia et al., 2020). Therefore, the implementation of an appropriate instructional model tailored to students' needs is essential (Isman et al., 2021)

An instructional model is defined as a systematic procedure for organizing learning experiences to achieve educational objectives (Susanto, 2019). The model used to design HLT in this study is Problem-Based Learning (PBL). The primary goal of PBL is to present complex problems that challenge students beyond their previous experiences. PBL fosters active participation, collaboration, and teamwork among students while developing leadership and analytical skills (Rahimma et al., 2024).

Problem-Based Learning also cultivates students' critical thinking abilities by encouraging them to actively engage in knowledge discovery, conceptual learning, and problem-solving within a group setting. According to (Hosnan, 2014), PBL is an instructional model that employs authentic problem-based learning, enabling students to construct their own knowledge, develop higher-order thinking skills, become more independent, and build self-confidence. The successful implementation of PBL has been shown to enhance students' cognitive learning outcomes (Hasanah et al., 2023; Nafiah & Suyanto, 2014; Utama & Kristin, 2021; Wahyu Ariyani & Prasetyo, 2021).

Several steps are involved in designing HLT based on PBL, including: (1) Needs analysis, which encompasses an examination of students, educators, syllabi, literature, and mathematical concepts; (2) Defining learning objectives to establish clear instructional goals; (3) Determining learning activities that align with students' cognitive development; (4) Formulating hypotheses about students' thinking, including predicting students' responses and anticipating educators' interventions.

HLT consists of three components: learning objectives, a series of tasks, and hypotheses regarding students' thinking and learning processes (Fuadiah, 2017). In designing PBL-based learning activities, educators must adhere to the five essential steps of the PBL model (M & Rusman, 2012) including (1) Orienting students to the problem, (2) Organizing students for learning, (3) Guiding individual and group investigations, (4) Developing and presenting student work, (5) Analyzing and evaluating the learning process.

To design and develop PBL-based HLT, this study reviews previous research on learning trajectory development. Several studies have explored HLT based on Realistic Mathematics Education (RME), including (Aklimawati et al., 2022), who designed RME-based HLT for geometry. The use of contextual problems in Aklimawati's study significantly improved students' understanding of geometry. Similarly, (Sujiani et al., 2022) developed RME-based HLT for probability, demonstrating that the RME approach effectively supports mathematics instruction and enhances students' mathematical abilities (Gravemeijer et al., 2013; Hadi, 2002). However, research on HLT using PBL remains limited. Therefore, the objective of this study is to develop an instructional design in the form of PBL-based HLT for the topic of relations and functions.

In designing the learning trajectory, this study also adopts the framework proposed by (Gravemeijer & Cobb, 2006). Their model consists of three essential stages that must be implemented in the development process. By designing HLT using Problem-Based Learning (PBL), the resulting HLT will be valid, meaning it can be effectively used by students to achieve the expected competencies and can be integrated into instructional activities. Additionally, the developed HLT is designed to be user-friendly, easily comprehensible, and structured in a logical and harmonious sequence. Furthermore, it serves as a valuable tool for educators, enabling them to facilitate learning, guide students, and support them in achieving learning objectives.

Methods

This study employs a developmental research design based on the (Gravemeijer & Cobb, 2006) model, which consists of three stages: (1) Preparing for the experiment – This stage involves conducting a needs analysis, including an examination of students and educators, syllabus analysis, literature review, and concept analysis. Additionally, it includes the design of HLT and its evaluation through self-assessment and expert review. (2) Conducting the experiment – This stage involves product trials with students at MTsN 9 Padang Pariaman, including one-to-one evaluation and small group evaluation. (3) The retrospective analysis – This stage focuses on analyzing data obtained from the previous stages. The results of this analysis are used to plan future learning activities and refine instructional designs for subsequent lessons.

The collected data is analyzed descriptively, and the process is iterative, ensuring that the HLT product applied in student worksheets (LKPD) is both valid and practical. Data collection in this study is based on HLT validation and practicality sheets, observation records, interviews, and documentation. The data consists of qualitative information, including comments, critiques, and suggestions from validators, educators, students, and observers, as well as documentation of the research process. The collected data serves as the basis for revising and improving the designed HLT. The data collection techniques used in this study include documentation, interviews, and field notes. The data analysis method employs a modified Likert scale, adapted to meet the specific needs of the research (Riduwan, 2013).

Results

This study was conducted at MTsN 9 Padang Pariaman, involving three subject matter experts, one media expert, and one language expert. The developed instructional model is a Hypothetical Learning Trajectory (HLT) based on Problem-Based Learning (PBL) for the topic of relations, applied in mathematics student worksheets (LKPD) for Grade VIII SMP/MTs, specifically covering relations and functions. The research findings are based on the type and model used by the researchers, namely design research following the (Gravemeijer & Cobb, 2006) model, which consists of three stages: Preparation for the experiment, Conducting the experiment and analyzing the results, Retrospective analysis of the previous stages.

Preparing for the experiment

Observations conducted at MTsN 9 Padang Pariaman indicate that the developed HLT product is suitable for use in the learning process. The analysis of students and educators involved six students and two mathematics teachers to assess students' learning conditions and educators' responses during instruction. The resulting product is a learning trajectory for relations and functions, designed using the Problem-Based Learning (PBL) model at MTsN 9 Padang Pariaman.

The HLT design includes learning objectives, instructional activities, a series of tasks, and hypotheses about students' thinking, all of which align with competency indicators. The validity of the HLT was then assessed through expert validation, involving subject matter experts, media experts, and language experts (see Table 1).

Table 1. Average Scores and Validity Criteria of HLT

Assessment Aspect	Average Score	Category
Content Validation	85,78%	Highly Valid
Media Validation	91,67%	Highly Valid
Language Validation	91,67%	Highly Valid
Overall Score	**89,71%**	**Highly Valid**

Based on the validation results across the three aspects, the average rating of the HLT components is 89.71%, indicating that the Problem-Based Learning (PBL)-based HLT falls under the "Highly Valid" category. This means that the HLT is ready for implementation, allowing students to achieve the expected competencies and be effectively utilized in instructional activities.

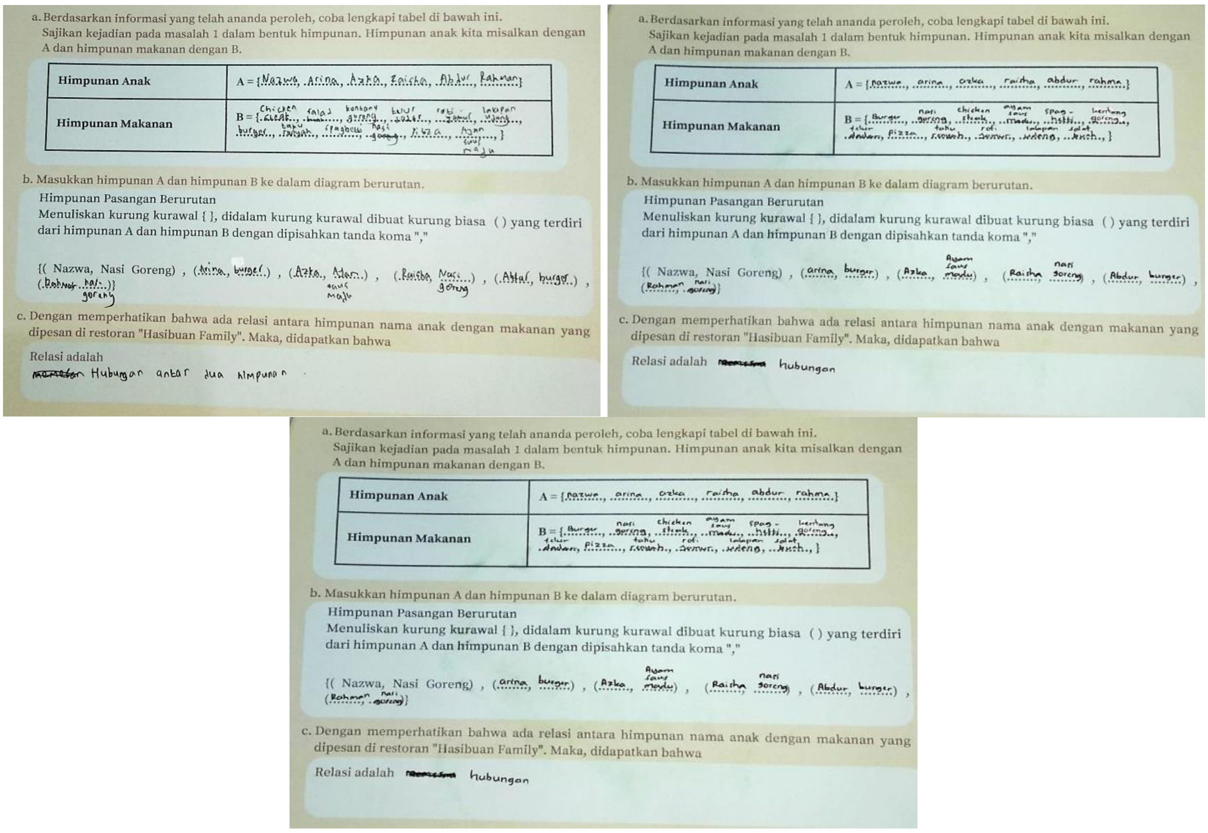
Conducting the experiment

Following the validation of HLT and student worksheets (LKPD) for relations and functions using Problem-Based Learning (PBL) in the preparing for the experiment phase—specifically during the expert review stage—the next step involved product trials through one-to-one evaluation and small group evaluation.

During this phase, three Grade VIII students from MTsN 9 Padang Pariaman, representing high, medium, and low ability levels, participated in the trial implementation of HLT and LKPD. The primary objective was to identify instructional elements that were difficult to understand, record students' feedback and suggestions, and analyze students' thought processes to refine the HLT.

The researcher also conducted informal interviews and direct observations of students' activities. Their comments and responses contributed to improving the LKPD, ensuring that it would be more accessible to other students in the small group evaluation phase. Additionally, the HLT was refined by incorporating a broader range of hypotheses regarding students' cognitive processes.

An example of students' responses in the worksheet can be seen in Figure 1.


a. Berdasarkan informasi yang telah ananda peroleh, coba lengkapi tabel di bawah ini. Sajikan kejadian pada masalah 1 dalam bentuk himpunan. Himpunan anak kita misalkan dengan A dan himpunan makanan dengan B.

Himpunan Anak	A = {Nazwa, Arina, Azka, Raisha, Abdul, Rahman}
Himpunan Makanan	B = { ... }

b. Masukkan himpunan A dan himpunan B ke dalam diagram berurutan.
Himpunan Pasangan Berurutan
Menuliskan kurung kurawal { }, didalam kurung kurawal dibuat kurung biasa ( ) yang terdiri dari himpunan A dan himpunan B dengan dipisahkan tanda koma ","
{(Nazwa, Nasi Goreng) , ...

c. Dengan memperhatikan bahwa ada relasi antara himpunan nama anak dengan makanan yang dipesan di restoran "Hasibuan Family". Maka, didapatkan bahwa
Relasi adalah ~~memesan~~ Hubungan antar dua himpunan

a. Berdasarkan informasi yang telah ananda peroleh, coba lengkapi tabel di bawah ini. Sajikan kejadian pada masalah 1 dalam bentuk himpunan. Himpunan anak kita misalkan dengan A dan himpunan makanan dengan B.

Himpunan Anak	A = {nazwa, arina, azka, raisha, abdur, rahma}
Himpunan Makanan	B = { ... }

b. Masukkan himpunan A dan himpunan B ke dalam diagram berurutan.
Himpunan Pasangan Berurutan
Menuliskan kurung kurawal { }, didalam kurung kurawal dibuat kurung biasa ( ) yang terdiri dari himpunan A dan himpunan B dengan dipisahkan tanda koma ","
{(Nazwa, Nasi Goreng) , (arina, burger) , (Azka, Ayam saus medu) , (Raisha, nasi goreng) , (Abdur, burger) , (Rahman, nasi goreng)}

c. Dengan memperhatikan bahwa ada relasi antara himpunan nama anak dengan makanan yang dipesan di restoran "Hasibuan Family". Maka, didapatkan bahwa
Relasi adalah ~~memesan~~ hubungan

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Menuliskan kurung kurawal { }, didalam kurung kurawal dibuat kurung biasa ( ) yang terdiri dari himpunan A dan himpunan B dengan dipisahkan tanda koma ","
{(Nazwa, Nasi Goreng) , (arina, burger) , (Azka, Ayam saus medu) , (Raisha, nasi goreng) , (Abdur, burger) , (Rahman, nasi goreng)}

c. Dengan memperhatikan bahwa ada relasi antara himpunan nama anak dengan makanan yang dipesan di restoran "Hasibuan Family". Maka, didapatkan bahwa
Relasi adalah ~~memesan~~ hubungan

Figure 1. The possibility of 1,2, and 3 answer from the Worksheet of Activity 1

Observing these three possible responses, it can be concluded that students may have different answers, varying from one another. This indicates that students have unique approaches to solving the given problems, and their responses align with the predicted answers in the developed HLT. As a result, the range of possible answers within the HLT expands. However, after a classroom discussion, all students will ultimately adopt a common strategy in interpreting the concept of relations (see Figure 2).

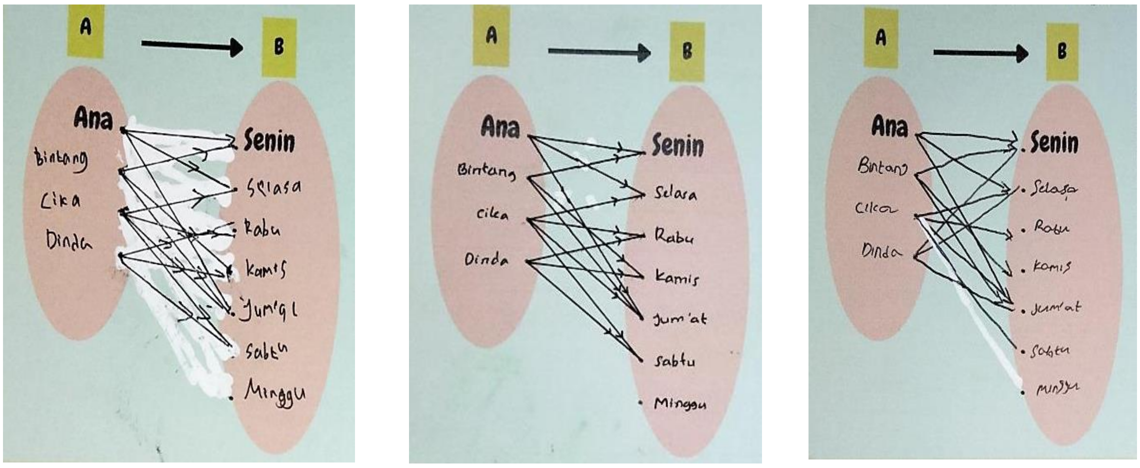


Figure 2. The possibility of 1,2, and 3 from the Worksheet of the Activity 2

Observing these three possible responses, it can be concluded that students do not have differing answers from one another, as their responses align with the predicted answers in the developed HLT. As a result, there is no expansion of possible answers within the HLT. However, after a classroom discussion, all students will ultimately adopt a common strategy in representing relations using arrow diagrams (see Figure 3).

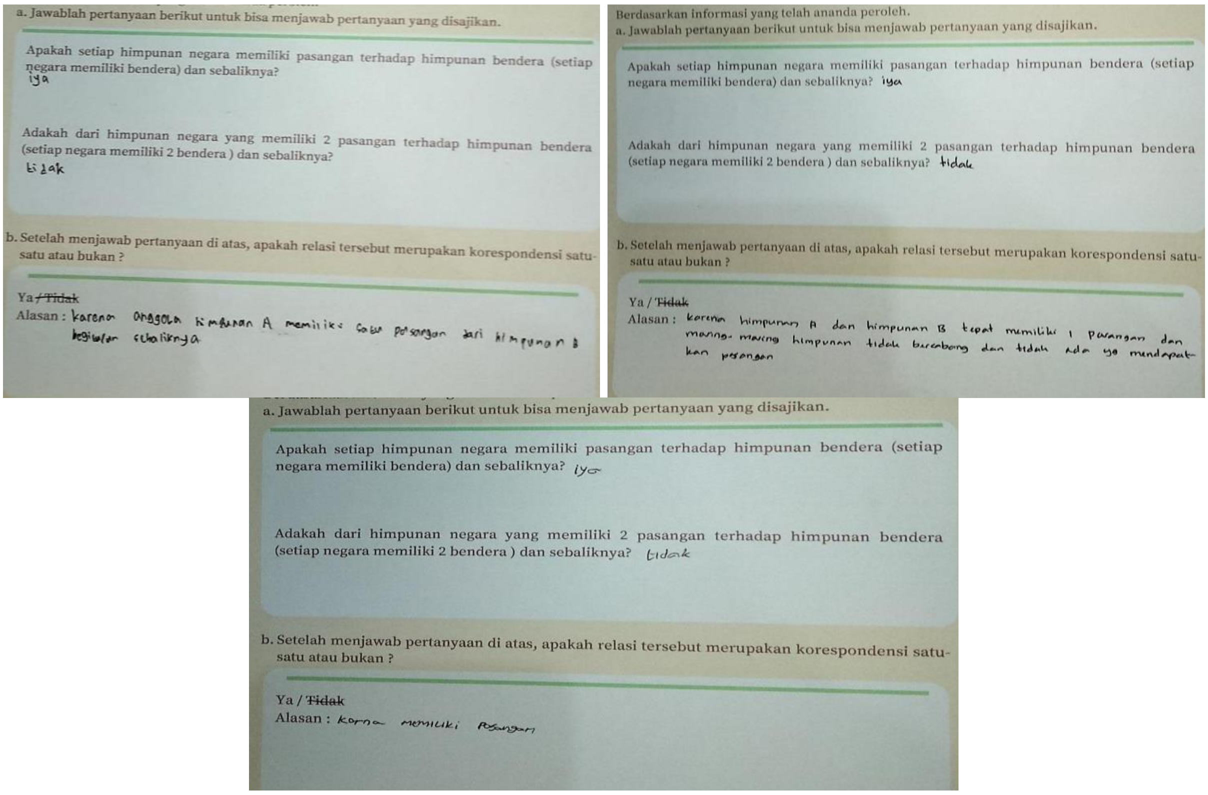


Figure 3. The possibility of 1,2, and 3 answers from the Worksheet of Activity 3

Observing these three possible responses, it can be concluded that students have different answers, varying from one another. This indicates that students have unique approaches to solving the given problems, which expands the range of possible answers within the developed

HLT. However, after a classroom discussion, all students will ultimately adopt a common strategy in interpreting the concept of one-to-one correspondence.

The small group evaluation phase involved six Grade VIII students from MTsN 9 Padang Pariaman, consisting of two high-achieving students, two average-achieving students, and two low-achieving students. The students were divided into two heterogeneous groups. Additionally, three mathematics educators were provided with HLT and LKPD to assess their practicality.

Retrospective analysis

The results of the retrospective analysis in the small group evaluation phase showed better outcomes compared to the retrospective analysis in the one-to-one evaluation phase. The findings from this stage indicate that the HLT product (see Figure 4, Figure 5, and Figure 6) has been successfully tested, as evidenced by the effective implementation of the designed HLT, which was developed based on the Problem-Based Learning model throughout the learning process.

No.	Tujuan Pembelajaran	Aktivitas Pembelajaran	Hipotesis	
			Prediksi	Antisipasi
1.	Menemukan Konsep Relasi	**Aktivitas 1** **Masalah 1** *Coba ananda perhatikan cerita berikut.* Nazwa sedang berulang tahun ke-17. Ia mengajak teman-temannya, yaitu Arina, Azka, Raisha, Abdur, dan Rahman pergi ke restoran "Hasibuan Family". Menu yang disediakan oleh restoran "Hasibuan Family" adalah Chicken Steak, Salad Buah, Kentang Goreng, Telur Dadar, Roti Jamur, Lalapan Udang, Burger, Tahu Renyah, Spaghetti, Nasi Goreng, Pizza, dan Ayam Saus Madu.  	Kemungkinan jawaban peserta didik. a) Relasi adalah hubungan. b) Relasi adalah suatu aturan yang menghubungkan anggota himpunan satu terhadap anggota himpunan lainnya.	Jawaban **a)** dan **b)** merupakan jawaban yang diharapkan oleh pendidik. Perlu diperhatikan, untuk jawaban **a)** Pendidik tidak menyalahkan jawaban dari peserta didik. Hanya saja untuk lebih memahami konsep relasi, pendidik membimbing dengan memberikan pertanyaan, seperti : "*Jika relasi adalah hubungan, lalu hubungan seperti apa yang terjadi antara himpunan anak dengan himpunan makanan?*" Dan jika tidak ada jawaban yang diharapkan pendidik, maka pendidik memberikan arahan dan berbagai pertanyaan yang bisa membantu peserta didik dalam menemukan konsep dari relasi dengan bahasa yang lebih mudah dipahami/dikaitkan dengan kehidupan

Figure 4. HLT of Activity 1

Hypotetical Learning Trajectory **(HLT) Materi Relasi dan Fungsi Kelas VIII**

Pertemuan 2 (3 x 40' menit)

Rencana Aktivitas : Menyajikan Relasi Dalam Bentuk Diagram Panah, Diagram Kartesius, dan Pasangan Berurutan, menemukan konsep Domain, Kodomain, Range, Fungsi, membedakan fungsi dan bukan fungsi.

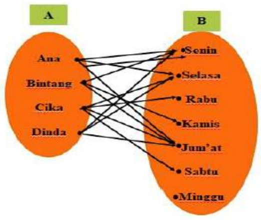
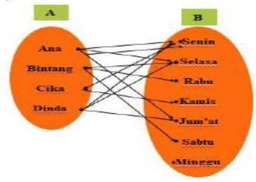
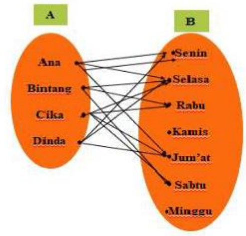
No.	Tujuan Pembelajaran	Aktivitas Pembelajaran	Hipotesis	
			Prediksi	Antisipasi
1.	Menemukan Konsep Fungsi, membedakan fungsi dan buka fungsi, serta memahami konsep korespondensi satu-satu.	**Aktivitas 2** **Masalah 3** *Coba ananda perhatikan masalah berikut.*  Sepulang sekolah Ana, Bintang, Cika dan Dinda berbincang-bincang kapan waktu luang mereka untuk berlatih renang bersama-sama. Ana dapat bermain pada hari senin, selasa, dan jumat. Bintang tidak dapat bermain hari selasa, rabu, dan sabtu. Cika harus tinggal di rumah pada hari senin dan kamis. Dinda dapat bermain pada hari senin, selasa dan jumat. Dan tidak seorang pun dapat bermain hari minggu. 1. Sajikan dalam diagram panah !	Kemungkinan jawaban peserta didik. a)  b)  c)  	Jawaban **b)** merupakan jawaban yang diharapkan oleh pendidik. Jika tidak terdapat jawaban yang diharapkan pendidik, maka pendidik membimbing dengan memberikan kepada peserta didik, seperti : *"Perhatikan kembali soal yang diberikan,! Apakah ananda tahu bagaimana cara membuat diagram panah jika ada kata* ***tidak*** *dapat bermain pada soal?, pernahkah ananda mendengar kata selain atau kecuali pada soal ?"* Kemudian, pendidik meminta peserta didik untuk memeriksa kembali jawabannya.
		2. Tentukan domain, kodomain, dan range dari diagram panah tersebut !	Kemungkinan jawaban peserta didik. a) Domain = {Ana, Bintang, Cika, Dinda} Kodomain = {Senin, Selasa, Rabu, Kamis, Jum'at, Sabtu, Minggu} Range = {Senin, Selasa, Rabu, Kamis, Jum'at, Sabtu} b) Domain = {Ana, Bintang, Cika, Dinda} Kodomain = {Senin, Selasa, Rabu, Kamis, Jum'at, Sabtu, Minggu} Range = {Senin, Selasa, Rabu, Kamis, Jum'at, Sabtu, Minggu} c) Domain = {Ana, Bintang, Cika, Dinda} Kodomain = {Senin, Selasa, Rabu, Kamis, Jum'at, Sabtu, Minggu} Range = {Senin, Selasa, Rabu, Jum'at, Sabtu}	Jawaban **a)** merupakan jawaban yang diharapkan oleh pendidik. Pendidik memberikan apresiasi kepada peserta didik. Diantara sebagian jawaban dari peserta didik telah benar. hanya saja terdapat kesalahan pada saat menentukan daerah hasil (range) dari himpunan tersebut. Peserta didik cenderung membuat jawaban dengan fokus pada himpunan A dan B saja. Artinya peserta terfokus bahwa himpunan A adalah daerah asal dan himpunan B adalah daerah hasil tanpa mengetahui bahwa konsep dari daerah hasil (range). Sehingga, jika tidak terdapat jawaban yang diharapkan pendidik, maka pendidik mengarahkan dengan memberikan pertanyaan, seperti :*"Perhatikan diagram yang telah ananda buat! Apakah ada dari himpunan A yang menunjuk terhadap himpunan B? Perhatikan tanda panah, lihatlah kemana arah tanda panah menunjuk!* Kemudian, pendidik meminta peserta didik untuk memeriksa kembali jawabannya.

Figure 5. HLT of Activity 2

No.	Tujuan Pembelajaran	Aktivitas Pembelajaran	Hipotesis	
			Prediksi	**Antisipasi**
1.	Memahami konsep korespondensi satu-satu.	**Kegiatan 3** **Masalah 6** *Coba ananda perhatikan cerita berikut.* Hampir semua negara di dunia termasuk dalam Perserikatan Bangsa-Bangsa (PBB). PBB didirkan untuk memfasilitasi dalam hukum internasional, hak asasi, dan mempertahankan perdamaian, serta  keamanan dunia. Di depan kantor pusat PBB di New York, semua bendera negara anggota dikibarkan.	Kemungkinan jawaban peserta didik. **Ya, korespondensi satu-satu** Alasannya : Berdasarkan syarat dikatakan korespondensi satu-satu tercapai. Seperti setiap Negara memiliki pasangan terhadap himpunan bendera negara (setiap Negara memiliki bendera). Dan tidak ada dari himpunan negara yang memiliki 2 pasangan terhadap himpunan bendera Negara (tidak ada Negara yang memiliki 2 bendera negara).	Jika jawaban peserta didik seperti demikian, maka pendidik membenarkannya. Pendidik mengarahkan dan memberi penguatan kepada peserta didik. Jika tidak ada jawaban yang diharapkan, maka pendidik membimbing dan memberikan pertanyaan, seperti : *"Apakah setiap himpunan Negara memiliki pasangan terhadap himpunan bendera negara (setiap Negara memiliki bendera Negara) dan sebaliknya? Adakah dari himpunan Negara yang memiliki 2 pasangan terhadap himpunan bendera (setiap Negara memiliki 2 bendera Negara) dan sebaliknya ?"* Kemudian, pendidik meminta peserta

Figure 6. HLT of Activity 3

Discussion

Educator-centered learning has begun to haunt the world of education and reduce the optimal educational outcomes for students, leading to passive learning, lack of student participation in class, absence of meaningful learning, and a learning process that merely involves knowledge transfer (Rozali et al., 2022). This situation certainly impacts students and makes the learning process less meaningful. One effort to improve the quality of mathematics learning, particularly in the topic of relations and functions, is to design a learning trajectory, also known as a hypothetical learning trajectory (HLT), using the Problem-Based Learning (PBL) model.

This study employs a research design based on the development model of (Gravemeijer & Cobb, 2006) which consists of three stages: preparing for the experiment, conducting the experiment, and retrospective analysis. HLT serves as a learning guideline that helps teachers implement appropriate models, strategies, teaching materials, and assessments aligned with students' thinking stages (Lantakay et al., 2023). The PBL model is a learning approach in which students work on authentic problems to construct their own knowledge, develop inquiry skills and higher-order thinking, foster independence, and build self-confidence. This model aligns with the characteristics of existing problems (Arends, 1997)

From the validity test analysis, the designed HLT includes learning objectives, learning activities, and hypotheses about students' thinking processes, all of which refer to competency indicators. The HLT will then be assessed for feasibility through validation by subject matter experts, media experts, and language experts. In terms of content/material, the feasibility of HLT is evaluated based on its alignment with Core Competencies (KI) and Basic Competencies (KD), its relevance to students' needs, and the accuracy of the learning substance

The average rating of HLT from the media aspect shows a result of 85.78%, indicating that PBL-based HLT in terms of content falls into the "Highly Valid" category. Thus, it can be concluded that the developed product aligns with KI and KD, meets students' needs, and presents learning materials that are consistent with the Problem-Based Learning (PBL) model

(Depdiknas, 2008). Regarding the presentation/media aspect, the feasibility of HLT in the topic of relations and functions using the PBL model can be assessed based on readability—the font used in printed teaching materials should not be too small and must be easy to read. Additionally, the background color should contrast with the text to enhance readability

The average rating of HLT from the media aspect is 91.67%, indicating that PBL-based HLT in terms of media falls into the "Highly Valid" category. Thus, it can be concluded that the developed product features clear and attractive font types and sizes, clear usage instructions, balanced layout, relevant illustrations and images, and an appealing design (Depdiknas, 2008)

From the language validation aspect, teaching materials must contain clear and concise sentences (Depdiknas, 2008). The average rating of HLT from the language aspect is 91.67%, indicating that PBL-based HLT in terms of language falls into the "Highly Valid" category. Thus, it can be concluded that the developed product uses proper, clear, and concise Indonesian language, avoiding ambiguity (double meanings) for students, ensuring that the information conveyed in HLT and Student Worksheets (LKPD) is clearly understood by readers.

Based on the validation results from the three aspects above, the average rating of HLT across all aspects is 89.71%, indicating that PBL-based HLT falls into the "Highly Valid" category. This means that HLT and LKPD can be used by students to achieve the expected competencies and can be implemented in learning activities. Based on the validity test analysis of the developed HLT, it can be concluded that PBL-based HLT for the topic of relations and functions is categorized as highly valid for use in mathematics learning activities.

Conclusion

From the explanation above, it can be concluded that before developing HLT, several stages were carried out, namely preparing for the experiment, conducting the experiment, and retrospective analysis, all of which were based on the curriculum implemented at MTsN 9 Padang Pariaman. The findings indicate that the HLT developed for the topic of relations and functions can serve as a solution to existing learning challenges. This aligns with the research results, which show that the developed HLT meets the "Highly Valid" criteria with an average validity score of 89.71%. This study proves that Problem-Based Learning (PBL)-based HLT can be effectively used in mathematics learning, particularly in teaching relations and functions to eighth-grade students.

Declarations

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